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ROBUST PERFORMANCE OF DECENTRALIZED CONTROL SYSTEMS BY INDEPENDENT DESIGNS

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Abstract

Decentralized control systems have fewer tuning parameters, are easier to understand and tune, and are more easily made failure tolerant than general multivariable control systems. In this paper the decentralized control problem is formulated as a series of independent designs. Simple bounds on these individual designs are derived, which when satisfied, guarantee robust performance of the overall system. The results provide a generalization of the μ-Interaction Measure introduced by Grosdidier and Morari (1986).

1. INTRODUCTION

Robust Performance

The goal of any controller design is that the overall system is stable and satisfies some minimum performance requirements. These requirements should be satisfied at least when the controller is applied to the nominal plant (G), that is, we require nominal stability (NS) and nominal performance (NP). In addition, when a decentralized controller is used, it is desirable that the system be failure tolerant. This means that the system should remain stable as individual loops are opened or closed.

In practice the real (or "perturbed") plant G_p is not equal to the model G. The term "robust" is used to indicate that some property holds for a set Π of possible plants G_p as defined by the uncertainty description. In particular, by robust performance (RP) we mean that the performance requirements are satisfied for all $G_p \in \Pi$. Mainly for mathematical convenience, we choose to define performance using the H_∞- norm. Define

$$NP \quad \Leftrightarrow \quad \bar{\sigma}(\Sigma) \leq 1, \quad \forall\omega \tag{1a}$$

$$RP \quad \Leftrightarrow \quad \bar{\sigma}(\Sigma_p) \leq 1, \quad \forall\omega, \quad \forall G_p \in \Pi \tag{1b}$$

In most cases Σ is the weighted sensitivity operator

$$\Sigma = W_1 S W_2, \quad S = (I + GC)^{-1} \tag{2a}$$

$$\Sigma_p = W_1 S_p W_2, \quad S_p = (I + G_p C)^{-1} \tag{2b}$$

The input weight W_2 is often equal to the disturbance model. The output weight W_1 is used to specify the frequency range over which the sensitivity function should be small and to weight each output according to its importance.

The definition of Robust Performance is of no value without simple methods to test if conditions like (1b) are satisfied for all G_p in the set Π of possible plants. Doyle et al. (1982) have derived a computationally useful condition for (1b) involving the Structured Singular Value μ. To use μ we must model the uncertainty (the set Π of possible plants G_p) as normbounded perturbations (Δ_i) on the nominal system. Through weights each perturbation is normalized to be of size one:

$$\bar{\sigma}(\Delta_i) \leq 1 \quad ,\forall\omega \tag{3}$$

The perturbations, which may occur at different locations in the system, are collected in the diagonal matrix Δ_U (the subscript U denotes uncertainty)

$$\Delta_U = diag\{\Delta_1, \ldots, \Delta_n\} \tag{4}$$

and the system is rearranged to match the structure in Fig.

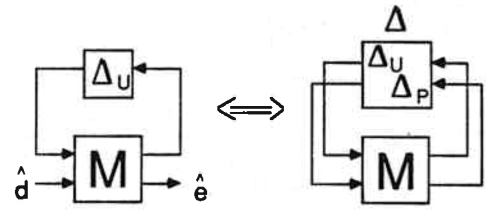

Fig. 1 General structure for studying effect of uncertainty (Δ_U) on performance. M is a function of the plant model (G) and the controller. $\hat{d}$: external inputs (disturbances, reference signals), $\hat{e}$: external outputs (weighted errors $y - r$), $\hat{e} = \Sigma_p \hat{d}$ (Eq. (1)).

1. The interconnection matrix M in Fig.1 is determined by the nominal model (G), the size and nature of the uncertainty, the performance specifications and the controller. For Fig.1 the robust performance condition (1b) becomes (Doyle et al., 1982)

$$RP \quad \Leftrightarrow \quad \mu(M) < 1, \quad \forall\omega \tag{5}$$

$\mu(M)$ depends on both the elements in the matrix

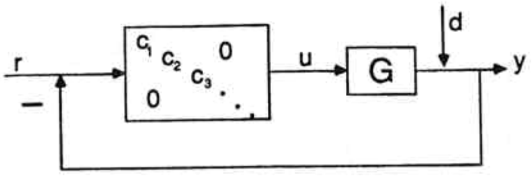

Fig. 2 Decentralized control structure.

M and the structure of the pertubation matrix $\Delta = diag\{\Delta_U, \Delta_P\}$. Sometimes this is shown explicity by using the notation $\mu(M) = \mu_\Delta(M)$. Δ_P is a full square matrix with dimension equal to the number of outputs (the subscript P denotes performance). In addition to satisfying (5), the system must be nominally stable (i.e., M is stable). Also note that within this framework, the issue of robust stability (RS) is simply a special case of robust performance.

Decentralized Control

Decentralized control involves using a diagonal or block-diagonal controller (Fig. 2)

$$C = diag\{c_i\}$$

Some reasons for using a decentralized controller are

- tuning and retuning is simple
- they are easy to understand
- they are easy to make failure tolerant

The design of a decentralized control system involves two steps

A) Choice of pairings (control structure)

B) Design of each SISO-controller c_i (or block).

The best way to proceed for each of these steps is still an active area of research. The RGA has proven to be an efficient tool for eliminating undesirable pairings in Step A. This paper deals with the Step B. Two design methods which may be applied for this step are 1) Sequential loop-closing and 2) Independent design of each loop.

1) Sequential loop-closing. This design approach involves designing each element (or block) in C sequentially. Usually the controller corresponding to a fast loop is designed first. This loop is then closed before the design proceeds with the next controller. This means that the information about the "lower-level" controllers is directly used as more loops are closed. The final step in the design procedure is to test if the overall system satisfies the RP-condition (5). The main disadvantages of this design method are

- Failure tolerance is not guaranteed when "lower-level" loops fail.
- The method depends strongly on which loop is designed first and how this controller is designed.
- There are no guidelines on how (and in which order) to design the controllers for each loop in order to guarantee robust performance of the overall system. Therefore the design proceeds by "trial-and-error".

2) Independent design of each loop. This is the design approach used in this paper. In this case each controller element (or block) is designed independently of the others. We present a procedure for these designs which guarantees robust performance of the overall system. The proposed method has the following advantages

- Failure tolerance: Nominal stability (of the remaining system) is guaranteed if any loop fails.
- Each controller is designed directly with no need for trial-and- error.

The main limitation of the approach is the assumption of independent designs, which means that we do not exploit information about the controllers used in the other loops. Therefore the derived bounds are only sufficient for robust performance.

Problem definition.

This paper addresses the following problem: Let $\tilde{G}$ denote the diagonal (or block-diagonal) version of the plant corresponding to the chosen structure of C (i.e., $\tilde{G}$ is found from G by deleting the off-diagonal elements). Assume that uncertainty and "interactions" are neglected when designing the controller C, that is, design each element (or block) of C independently based on the information contained in $\tilde{G}$ only. What constraints have to be placed on the individual designs in order to guarantee robust performance of the overall system (which can be any plant G_p from the set Π)?

The constraints on the individual designs are chosen to be in terms of bounds on $|\tilde{h}_i|$ and $|\tilde{s}_i|$ where $\tilde{h}_i$ and $\tilde{s}_i$ are the closed-loop transfer functions for loop i:

$$\tilde{h}_i = g_{ii}c_i(1+g_{ii}c_i)^{-1} \qquad \tilde{H} = diag\{\tilde{h}_i\} \qquad (6a)$$

$$\tilde{s}_i = (1+g_{ii}c_i)^{-1} \qquad \tilde{S} = diag\{\tilde{s}_i\} \qquad (6b)$$

(In general, if C is block-diagonal, $\tilde{h}_i$, $\tilde{s}_i$ and g_{ii} are matrices corresponding to the block-structure of C, and $|\tilde{s}_i|$ and $|\tilde{h}_i|$ are replaced by $\bar{\sigma}(\tilde{H}_i)$ and $\bar{\sigma}(\tilde{S}_i)$).

We solve the decentralized problem as defined above, by deriving the tightest possible bounds on

$$\bar{\sigma}(\tilde{H}) = \max_i |\tilde{h}_i| \quad \text{and} \quad \bar{\sigma}(\tilde{S}) = \max_i |\tilde{s}_i|$$

which guarantee robust performance:

$$RP \quad \Leftarrow \quad \bar{\sigma}(\tilde{H}) < \tilde{c}_H \quad \text{or} \quad \bar{\sigma}(\tilde{S}) < \tilde{c}_S, \quad \forall\omega \qquad (7)$$

In addition to satysfying (7) the system has to be nominally stable. The μ-interaction measure, introduced by Grosdidier and Morari (1986), gives a sufficient condition for nominal stability:

$$NS \quad \Leftarrow \quad \bar{\sigma}(\tilde{H}) \le \mu_C(E_H), \quad \forall\omega, \quad E_H = (G-\tilde{G})\tilde{G}^{-1} \qquad (8)$$

(μ is computed with respect to the structure of C which is equal to the structure of $\tilde{G}, \tilde{H}$ and $\tilde{S}$). This paper provides a generalization of the μ-interaction measure from the case of nominal stability (NS) to the case of robust performance (RP). The results derived here also apply to robust stability (RS) or nominal performance (NP) if the μ-condition (5) is a RS- or NP-condition rather than a RP-condition.

Notation

The most important notation is summarized below.

G - model of the plant
$\tilde{G} = diag\{g_{ii}\}$ (corresponding to structure of C)
$G_p = f(G, \Delta_U)$, $G_p = G$ when $\Delta_U = 0$

$$\begin{aligned} \tilde{S} &= (I + \tilde{G}C)^{-1}, & \tilde{H} &= I - \tilde{S} \\ S &= (I + GC)^{-1}, & H &= I - S \\ S_p &= (I + G_pC)^{-1}, & H_p &= I - S_p \end{aligned} \tag{9}$$

Stability of individual loops $\Leftrightarrow$ $\tilde{H}$ (and $\tilde{S}$) is stable
NS $\Leftrightarrow$ H (and S) is stable
RS $\Leftrightarrow$ H_p (and S_p) is stable (for all $G_p \in \Pi$).
NP $\Leftrightarrow$ S satisfies the performance specification
RP $\Leftrightarrow$ S_p satisfies the performance specification (for all $G_p \in \Pi$).

2. NOMINAL STABILITY (OF H AND S)

To apply the general robust performance condition $\mu(M) < 1$ (5) we must require that the system is nominally stable, that is, that the interconnection matrix M is stable. Nominal stability is satisfied if H (and S) is stable. However, note that nominal stability (i.e, stability of H and S) is not necessarily implied by the stability of the individual loops (i.e., stability of $\tilde{H}$ and $\tilde{S}$). The "interactions" (difference between G and $\tilde{G}$) may cause stability problems as discussed by Grosdidier and Morari (1986). If either one of the following conditions on $\bar{\sigma}(\tilde{H})$ and $\bar{\sigma}(\tilde{S})$ is satisfied, then the stability of $\tilde{H}$ (or $\tilde{S}$) implies nominal stability.

Condition 1 for NS (Grosdidier and Morari, 1986). *Assume $\tilde{H}$ is stable (each loop is stable by itself), and that G and $\tilde{G}$ have the same number of RHP (unstable) poles. Then H is stable (the system is stable when all loops are closed) if*

$$\bar{\sigma}(\tilde{H}) \leq \mu_C^{-1}(E_H) \quad \forall\omega \tag{10}$$

$$\textit{where} \quad E_H = (G - \tilde{G})\tilde{G}^{-1} \tag{11}$$

$\mu_C(E_H)$ is the μ-interaction measure and μ is computed with respext to the structure of the decentralized controller C. Note that the condition that G and $\tilde{G}$ have the same number of RHP poles, is generally satisfied only when G and $\tilde{G}$ are stable. In order to allow integral action ($\tilde{H}(0) = I$), we have to require that $\mu(E_H) < 1$ at $\omega = 0$, that is, we need diagonal dominance at low frequencies. If this is not the case the following alternative condition may be used:

Condition 2 for NS (Postlethwaite and Foo, 1985, Grosdidier, 1985). *Assume $\tilde{S}$ is stable, and that G and $\tilde{G}$ have the same number of RHP zeros. Then S (and H) is stable if*

$$\bar{\sigma}(\tilde{S}) \leq \mu_C^{-1}(E_S) \quad \forall\omega \tag{12}$$

$$\textit{where} \quad E_S = (G - \tilde{G})G^{-1} \tag{13}$$

Since we have to require $\tilde{S} = I$ as $\omega \to \infty$ for any real system, we have to require $\mu(E_S) < 1$ as $\omega \to \infty$, in order to be able to satisfy (12), that is, we must have diagonal dominance at high frequencies. Conditions 1 and 2 cannot be combined over different frequency ranges. The reason is that the "uncertainty" $G - \tilde{G}$ is not a norm-bounded set and therefore is not "connected in the graph topology" (Postlethwaite, et al., 1985).

3. ROBUST PERFORMANCE

Having derived conditions for nominal stability, we can now proceed to the case of robust performance. The objective of this section is to derive bounds on the individual designs ($\tilde{H}$ and $\tilde{S}$), which when satisfied guarantee robust performance of the overall system (that is, $\mu(M) < 1$). This is accomplished in two steps:

1. Sufficient conditions for RP in terms of bounds on $\bar{\sigma}(H)$ and $\bar{\sigma}(S)$ are derived by writing M as a linear fractional transformation (LFT) of H and S.

2. These bounds are used to derive sufficient conditions for RP in terms of bounds on $\bar{\sigma}(\tilde{H})$ and $\bar{\sigma}(\tilde{S})$.

3.1 Robust Performance Condition in Terms of H and S

The robust performance condition

$$RP \quad \Leftrightarrow \quad \mu_\Delta(M) \leq 1, \quad \forall\omega \tag{5}$$

may be used to derive sufficient conditions for RP in terms of bounds on $\bar{\sigma}(H)$ and $\bar{\sigma}(S)$ (Skogestad and Morari, 1987a). To this end write M as a LFT of H (Fig. 3)

$$M = N_{11}^H + N_{12}^H H(I - N_{22}^H H)^{-1} N_{21}^H \tag{16}$$

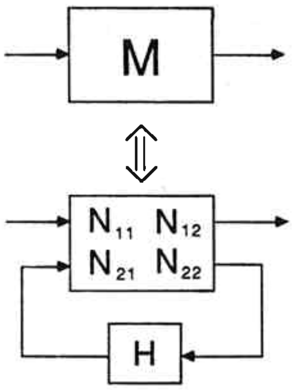

Fig. 3 M written as a LFT of H.

The matrix N^H, which is independent of C, can be obtained from M by inspection in many cases. Otherwise, the procedure given by Skogestad and Morari (1987a) can be used. They also point out that in general M is affine in H, that is, $N_{22}^H = 0$. Applying Theorem 1 of Skogestad and Morari (1987a) (the theorem is reproduced in the Appendix) the following sufficient condition for (5) is derived:

RP-condition in terms of H. *Assume M is given as a LFT of H (Eq. 16). Then at any given freqency*

$$\mu_\Delta(M) \leq 1 \quad \textit{if} \quad \bar{\sigma}(H) \leq c_H \tag{17a}$$

where at this frequency c_H solves

$$\mu_{\hat{\Delta}}\begin{pmatrix} N_{11}^H & N_{12}^H \\ c_H N_{21}^H & c_H N_{22}^H \end{pmatrix} = 1 \qquad (17b)$$

and μ is computed with respect to the structure $\hat{\Delta} = diag\{\Delta, H\}$.

Note that H is generally a "full" matrix if the controller is diagonal. A similar bound in terms of S is derived by replacing H by S in Eq. (16) and (17). (17) applies on a freqency-by-frequency basis. This implies that $\mu(M) \leq 1$ at a given frequency is guaranteed if either $\bar{\sigma}(H) < c_H$ or $\bar{\sigma}(S) < c_S$ at this frequency. Consequently, the bounds on $\bar{\sigma}(H)$ and $\bar{\sigma}(S)$ can be combined over different frequency ranges. In particular, the following holds

$$RP \quad \Leftarrow \quad \bar{\sigma}(H) \leq c_H \quad \text{or} \quad \bar{\sigma}(S) \leq c_S, \quad \forall\omega \qquad (18)$$

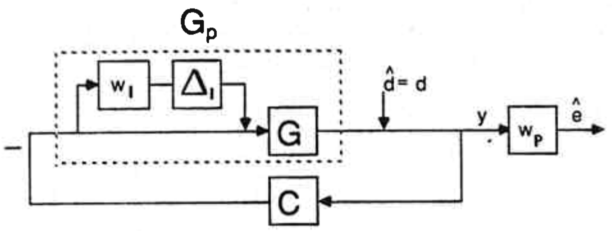

Fig. 4 Plant with input uncertainty (Δ_I)

Example. RP with input uncertainty (Fig. 4).

Let the set Π of possible plants be given by

$$G_p = G(I + w_I\Delta_I), \quad \bar{\sigma}(\Delta_I) \leq 1, \quad \forall\omega \qquad (19)$$

Here w_I is the magnitude of the relative (multiplicative) uncertainty at the plant inputs. For robust performance we require that the magnitude of the sensitivity operator is bounded by $|w_p|^{-1}$:

$$RP \quad \Leftrightarrow \quad \bar{\sigma}(w_P(I + G_pC)^{-1}) \leq 1 \; \forall\omega, \; \forall G_p \in \Pi \qquad (20)$$

This condition is most easily checked using μ (Eq. (5)):

$$RP \quad \Leftrightarrow \quad \mu_\Delta(M) \leq 1 \quad \forall\omega \qquad (21a)$$

where the interconnection matrix M is (Skogestad and Morari, 1986):

$$M = \begin{pmatrix} -w_I CSG & -w_I CS \\ w_p SG & w_p S \end{pmatrix} \qquad (21b)$$

and $\mu(M)$ is computed with respect to the structure $\Delta = diag\{\Delta_I, \Delta_P\}$. Δ_P is always a "full" matrix of the same dimension as S. Δ_I is often a diagonal matrix (if the inputs do not affect each other). Rewrite M in terms of S and H such that C does not appear

$$M = \begin{pmatrix} -w_I G^{-1} HG & -w_I G^{-1} H \\ w_P SG & w_P S \end{pmatrix} \qquad (22)$$

By inspection M may be written as a LFT (16) of H (recall S = I - H)

$$M = N_{11}^H + N_{12}^H H N_{21}^H$$

$$= \begin{pmatrix} 0 & 0 \\ w_P G & w_P I \end{pmatrix} + \begin{pmatrix} -w_I G^{-1} \\ -w_P I \end{pmatrix} H (G \quad I) \qquad (23)$$

We derive from (23) and (17)

$$RP \quad \text{if} \quad \bar{\sigma}(H) \leq c_H \quad \forall\omega \qquad (24a)$$

where at each frequency c_H solves

$$\mu\begin{pmatrix} 0 & 0 & -w_I G^{-1} \\ w_P G & w_P I & -w_P I \\ c_H G & c_H I & 0 \end{pmatrix} = 1 \qquad (24b)$$

where μ is computed with respect to the structure $diag\{\Delta_I, \Delta_P, H\}$. A similar condition on $\bar{\sigma}(S)$ is derived by writing M as a LFT of S.

3.2 Robust Performance Condition in Terms of $\tilde{H}$ and $\tilde{S}$

Sufficient conditions for RP in terms of $\bar{\sigma}(\tilde{H})$ and $\bar{\sigma}(\tilde{S})$ may now be derived using the identities (Grosdidier, 1985)

$$H = G\tilde{G}^{-1}\tilde{H}(I + E_H\tilde{H})^{-1} \qquad (25)$$

$$S = \tilde{S}(I - E_S\tilde{S})^{-1}\tilde{G}G^{-1} \qquad (26)$$

Note that (25) and (26) both are LFT's of H (and S) in terms of $\tilde{H}$ (and $\tilde{S}$). In Section 3.1 we pointed out that in general M can be written as a LFT of H with $N_{22}^H = 0$:

$$M = N_{11}^H + N_{12}^H H N_{21}^H \qquad (27)$$

Substituting (25) into (27) yields

$$M = N_{11}^H + N_{12}^H G\tilde{G}^{-1}\tilde{H}(I + E_H\tilde{H})^{-1}N_{21}^H \qquad (28)$$

which is a LFT of M in terms of $\tilde{H}$. Using Theorem 1 (Appendix 1) and (28) we derive:

RP-condition in terms of $\tilde{H}$. *Let $M = N_{11}^H + N_{12}^H H N_{21}^H$. Then at any frequency*

$$\mu_\Delta(M) \leq 1 \quad \text{if} \quad \bar{\sigma}(\tilde{H}) \leq \tilde{c}_H \qquad (29a)$$

where at this frequency $\tilde{c}_H$ solves

$$\mu_{\hat{\Delta}}\begin{pmatrix} N_{11}^H & N_{12}^H G\tilde{G}^{-1} \\ \tilde{c}_H N_{21}^H & -\tilde{c}_H E_H \end{pmatrix} = 1 \qquad (29b)$$

and μ is computed with respect to the structure $\hat{\Delta} = diag\{\Delta, C\}$.

Note that the structure of C is block-diagonal and equal to that of $\tilde{H}$. An entirely equivalent condition may be derived in terms of $\bar{\sigma}(\tilde{S})$:

RP-condition in terms of $\tilde{S}$. *Let $M = N_{11}^S + N_{12}^S S N_{21}^S$. Then at any frequency*

$$\mu_\Delta(M) \leq 1 \quad \text{if} \quad \bar{\sigma}(\tilde{S}) \leq \tilde{c}_S \qquad (30a)$$

where $\tilde{c}_S$ solves

$$\mu_{\hat{\Delta}}\begin{pmatrix} N_{11}^S & N_{12}^S \\ \tilde{c}_S\tilde{G}G^{-1}N_{21}^S & \tilde{c}_S E_S \end{pmatrix} = 1 \qquad (30b)$$

and μ is computed with respect to the structure $\hat{\Delta} = diag\{\Delta, C\}$.

Again, the bounds (29) and (30) may be combined over different frequency ranges:

$$RP \quad \text{if} \quad \bar{\sigma}(\tilde{H}) \leq \tilde{c}_H \quad \text{or} \quad \bar{\sigma}(\tilde{S}) < \tilde{c}_S \quad \forall \omega \qquad (31)$$

Example. RP with Input Uncertainty (Continued)

Consider the same example as above (Fig. 4). However, in this case we will derive bounds in terms of $\bar{\sigma}(\tilde{H})$ and $\bar{\sigma}(\tilde{S})$. A RP-condition in terms of $\bar{\sigma}(\tilde{H}) = |\tilde{h}_i|$ is derived by combining (29) and (23):

$$RP \quad \text{if} \quad \bar{\sigma}(\tilde{H}) \leq \tilde{c}_H \quad \forall \omega \qquad (32a)$$

where at each frequency $\tilde{c}_H$ solves

$$\mu_{\hat{\Delta}} \begin{pmatrix} 0 & 0 & -w_I\tilde{G}^{-1} \\ w_P G & w_P I & -w_P G\tilde{G}^{-1} \\ \tilde{c}_H G & \tilde{c}_H I & -\tilde{c}_H E_H \end{pmatrix} = 1 \qquad (32b)$$

Similarly, the RP-condition in terms of $\bar{\sigma}(\tilde{S}) = |\tilde{s}_i|$ is

$$RP \quad \text{if} \quad \bar{\sigma}(\tilde{S}) \leq \tilde{c}_S \quad \forall \omega \qquad (33a)$$

where at each frequency $\tilde{c}_S$ solves

$$\mu_{\hat{\Delta}} \begin{pmatrix} -w_I I & -w_I G^{-1} & w_I G^{-1} \\ 0 & 0 & w_P I \\ \tilde{c}_S \tilde{G} & \tilde{c}_S G\tilde{G}^{-1} & \tilde{c}_S E_S \end{pmatrix} = 1 \qquad (33b)$$

In both (32b) and (33b) μ is computed with respect to the structure $\hat{\Delta} = diag\{\Delta_I, \Delta_P, C\}$. Conditions (32) and (33) can be combined as shown in (31).

4. DESIGN PROCEDURE

The following design procedure for decentralized control systems based on the "independent designs"-assumption is proposed: Find a decentralized controller which yields individual loops ($\tilde{H}$ and $\tilde{S}$) which are stable and in addition satisfy

1) Nominal Stability: Satisfy $\bar{\sigma}(\tilde{H}) \leq \mu^{-1}(E_H)$ (10) at all frequencies or satisfy $\bar{\sigma}(\tilde{S}) \leq \mu^{-1}(E_S)$ (12) at all frequencies. It is not allowed to combine (10) and (12).
2) Robust performance: At each frequency satisfy either $\bar{\sigma}(\tilde{H}) \leq \tilde{c}_H$ (29) or $\bar{\sigma}(\tilde{S}) \leq \tilde{c}_S$ (30). Combining (29) and (30) over different frequency ranges is allowed.

Consequently, two separate conditions must be satisfied by the individual designs: One for nominal stability and one for robust performance.

5. NUMERICAL EXAMPLE

In this section we continue the previous example of RP with diagonal input uncertainty (Fig.4). Consider the plant (time is in minutes)

$$G = \frac{1}{1+75s} \begin{bmatrix} -0.878 & 0.014 \\ -1.082 & -0.014 \end{bmatrix} \qquad (34)$$

Physically, this may correspond to a high-purity distillation column using distillate (D) and boilup (V) as manipulated inputs to control composition (Skogestad and Morari, 1986). We want to design a decentralized controller for this plant such that robust performance is guaranteed when there is uncertainty on the manipulated inputs. The performance and uncertainty weights are

$$w_P(s) = 0.25\frac{7s+1}{7s} \qquad (35a)$$

$$w_I(s) = 0.1\frac{5s+1}{0.25s+1} \qquad (35b)$$

A particular sensitivity function which matches the performance condition (29) exactly at low frquency and easily at high frequency is $S = \frac{28}{28s+1}I$. This corresdonds to a first order response with time constant 28 min. The uncertainty weight (35b) corresponds to 10% uncertainty on each input at low frequency. $||w_I(j\omega)||$ reaches a value at one at about $\omega = 2\ min^{-1}$. This allows for a uncertain time delay of up to 0.5 min.

We find $\mu(E_H(0)) = 1.11$ and the NS-condition (10) is impossible to satisfy. However, the NS-condition (12) on $\bar{\sigma}(\tilde{S})$ is easily satisfied since G and $\tilde{G}$ have the same number of RHP- zeros (none), and $\mu(E_S) = 0.743$ at all frequencies. The only restriction this imposes on $\tilde{S}$ is that the maximum peaks of $|\tilde{s}_1|$ and $|\tilde{s}_2|$ must be less than $1/0.743 = 1.35$. This is easily satisfied since both $\tilde{g}_{11} = \frac{-0.878}{1+75s}$ and $\tilde{g}_{22} = \frac{-0.014}{1+75s}$ are minimum phase.

Robust Performance (RP)

Bound on $\bar{\sigma}(\tilde{H})$). The bound $\tilde{c}_H$ on $\bar{\sigma}(\tilde{H})$ is given by Eq. (32) and is shown graphically in Fig. 5. (μ of the matrix in (34b) is computed with respect to the structure $\hat{\Delta} = diag\{\Delta_I, \Delta_P, C\}$, where Δ_I is a diagonal 2×2 matrix, Δ_P is a full 2×2 matrix and C is a diagonal 2×2 matrix). It is clearly not possible to satisfy the bound $\bar{\sigma}(\tilde{H}) < \tilde{c}_H$ at low frequencies.

Bound on $\bar{\sigma}(\tilde{S})$. The bound $\tilde{c}_S$ on $\bar{\sigma}(\tilde{S})$ is given by Eq. (33) and is shown graphically in Fig. 6 (μ is computed with respect to the same structure as above). It is not possible to satisfy this bound at high frequencies.

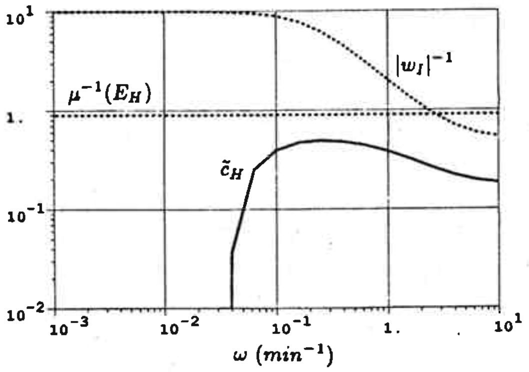

Fig. 5 Bounds $\mu^{-1}(E_H)$ and $\tilde{c}_H$ on $\bar{\sigma}(\tilde{H})$.

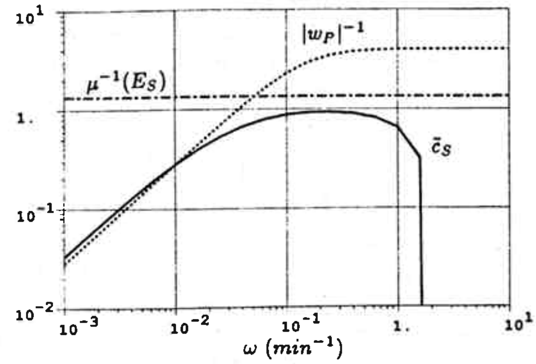

Fig. 6 Bounds $\mu^{-1}(E_S)$ and $\tilde{c}_S$ on $\bar{\sigma}(\tilde{S})$.

Combining bounds on $\bar{\sigma}(\tilde{H})$ and $\bar{\sigma}(\tilde{S})$. The bound on $\bar{\sigma}(\tilde{S})$ is easily satisfied at low frequencies, and the bound on $\bar{\sigma}(\tilde{H})$ is easily satisfied at high frequencies. The difficulty is to find a $\tilde{S} = I - \tilde{H}$ which satisfy either one of the conditions in the frequency range from 0.1 to 1 min^{-1}. The following

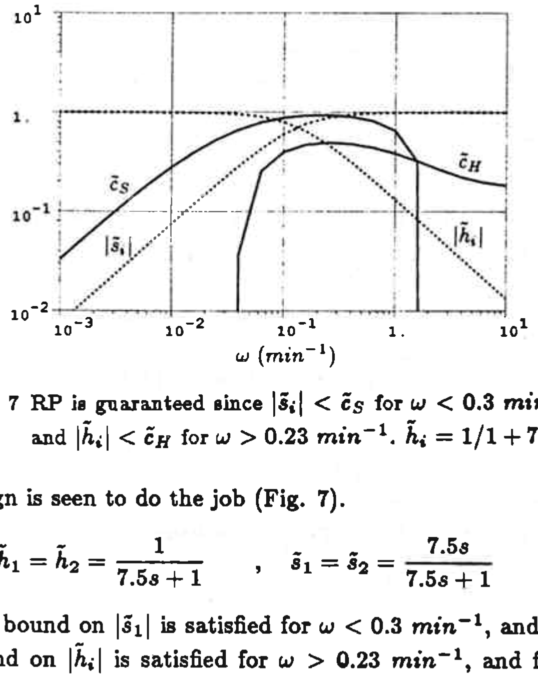

Fig. 7 RP is guaranteed since $|\tilde{s}_i| < \tilde{c}_S$ for $\omega < 0.3\ min^{-1}$ and $|\tilde{h}_i| < \tilde{c}_H$ for $\omega > 0.23\ min^{-1}$. $\tilde{h}_i = 1/1 + 7.5s$.

design is seen to do the job (Fig. 7).

$$\tilde{h}_1 = \tilde{h}_2 = \frac{1}{7.5s+1} \quad , \quad \tilde{s}_1 = \tilde{s}_2 = \frac{7.5s}{7.5s+1} \qquad (36)$$

The bound on $|\tilde{s}_1|$ is satisfied for $\omega < 0.3\ min^{-1}$, and the bound on $|\tilde{h}_i|$ is satisfied for $\omega > 0.23\ min^{-1}$, and from (31) we get that RP of the overall system is guaranteed.

6. CONCLUSION

This paper solves the problem of robust performance using independent designs as introduced in the Introduction. The example illustrates that this design approach may be useful for designing decentralized controllers.

The main limitation of the approach stems from the initial assumption regarding independent designs: Since each loop is designed separately, we cannot make use of information about the controllers used in the other loops. The consequence is that the bounds on $\bar{\sigma}(\tilde{S})$ and $\bar{\sigma}(\tilde{H})$ are only sufficient for robust performance; there will exist decentralized controllers which violate the bounds on $\bar{\sigma}(\tilde{S})$ and $\bar{\sigma}(\tilde{H})$, but which satisfy the robust performance condition. However, the derived bounds on $\bar{\sigma}(\tilde{S})$ and $\bar{\sigma}(\tilde{H})$ are the tightest norm bounds possible, in the sense that in such cases there will exist another controller with the same values of $\bar{\sigma}(\tilde{H})$ and $\bar{\sigma}(\tilde{S})$ which does not yield robust performance.

The bounds on $\bar{\sigma}(\tilde{H})$ and $\bar{\sigma}(\tilde{S})$ tend to be most conservative in the frequency range around crossover where $\bar{\sigma}(\tilde{H})$ and $\bar{\sigma}(\tilde{S})$ are both close to one. If, for a particular case, it is not possible to satisfy either $\bar{\sigma}(\tilde{H}) < \tilde{c}_H$ or $\bar{\sigma}(\tilde{S}) < \tilde{c}_S$ in this frequency range, then try the following: Design a controller for which the frequency range where both bounds are violated is as small as possible. Since the bounds are only sufficient for RP, this may still yield an acceptable design with robust performance. This may be checked using the tight RP-condition $\mu(M) < 1$ (5).

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References

Doyle, J. C., Analysis of Feedback Systems with Structured Uncertainties, Inst. Electrical Engrs. Proc., **129**, Pt. D, 6, 242-250 (1982).

Doyle, J. C., J. E. Wall and G. Stein, Performance and Robustness Analysis for Structured Uncertainty, Proc. IEEE Conf. on Decision and Control, Orlando, FL (1982).

Grosdidier, P., Personal communication (1985).

Grosdidier, P. and M. Morari, Interaction Measures for Systems Under Decentralized Control, Automatica, **22**, 3, 309-319 (1986).

Postlethwaite, I. and V. K. Foo, Robustness with Simultaneous Pole and Zero Movement Across the $j\omega$-Axis, Automatica, **21**, 433-443 (1985).

Skogestad, S. and M. Morari, Control of Ill-Conditioned Plants: High Purity Distillation, AIChE Annual Meeting, Miami Beach, FL (1986).

Skogestad, S. and M. Morari, Some New Properties of the Structured Singular Value, submitted to IEEE Trans. on Automatic Control (1987a).

APPENDIX.

Theorem 1. *Let M be written as a LFT of T:*

$$M = N_{11} + N_{12}T(I - N_{22}T)^{-1}N_{21}$$

and let k be a given constant. Assume $\mu_\Delta(N_{11}) < k$ and $det(I - N_{22}T) \neq 0$. Then

$$\mu_\Delta(M) \leq k$$

if

$$\bar{\sigma}(T) \leq c_T$$

where c_T solves

$$\mu_{\hat{\Delta}} \begin{bmatrix} N_{11} & N_{12} \\ kc_T N_{21} & kc_T N_{22} \end{bmatrix} = k$$

and $\hat{\Delta} = diag\{\Delta, T\}$.