Control Configuration Selection for Distillation Columns

Most two-product distillation columns can be described as 5 × 5 plants, but the control system design is usually simplified by means of the following procedure:

1. Choose two manipulated inputs for composition control (corresponding to a specific control configuration).
2. Design the level and pressure control system (usually three SISO controllers).
3. Design a 2 × 2 controller for composition control.

This paper provides guidelines for step 1, which is considered the most important. Ratios (e.g., L/D or V/B) are frequently chosen as manipulated inputs in step 1. It is shown that the ratio configurations are effectively complex multivariable controllers that provide, among other features, improved flow disturbance rejection.

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Introduction

Distillation columns constitute a major part of most chemical processing plants. The purpose of a distillation column is to split the feed into two or more products with compositions different from that of the feed. The desired composition of the products may be fixed by product requirements or may result from some plantwide optimization. An important objective of the control system should be to keep these product compositions at their desired levels. In practice, very few industrial columns maintain dual composition control, and it is still common to find that both compositions are controlled manually. Reports from industry indicate energy savings of 10–30% (Ryskamp, 1980; Stanley and McAvoy, 1985) if dual composition control is used instead of manual control, which usually results in overpurification or loss of valuable product. Also, a recent survey among plant managers (Dartt, 1985) cites distillation as the unit operation that could benefit most significantly from improved control.

A main reason dual composition control is not widely applied in industry is the stability problem often encountered when such a system is tuned to get a reasonably fast response. Some reasons usually cited in the literature for the problems with dual compositions control are

- Strongly nonlinear behavior
- Very sluggish response
- Measurement problems, dead times for composition measurements
- Difficulty in choosing appropriate manipulated variables for composition control
- Strongly interactive system

These problems do not apply to all columns. Columns with low-purity products tend to be simpler to control. Ironically, simple columns are the ones usually studied experimentally in university laboratories. Another reason for the infrequent use of dual composition control is the lack of systematic guidelines in the literature on how to design such control systems.

This paper is concerned with the issue of control configuration (or structure) selection. Although it is probably the most important step in the design of a distillation control system, this issue has not been treated systematically in the literature. Consider the schematic picture of a distillation column in Figure 1. The column has five inputs (valves) that can be manipulated and five controlled variables. Three of these controlled variables (condenser and reboiler holdups, and pressure) have to be controlled carefully to maintain stable operation; this leaves two degrees of freedom for control of the top and bottom compositions y_D and x_B. The issue of control configuration selection is to decide which two independent inputs (or combinations of inputs) to use for composition control. One common choice is to use reflux, L, and boilup, V, (LV configuration). In a manually controlled column these are the two flows that the operator will set in order to maintain the desired product separation.

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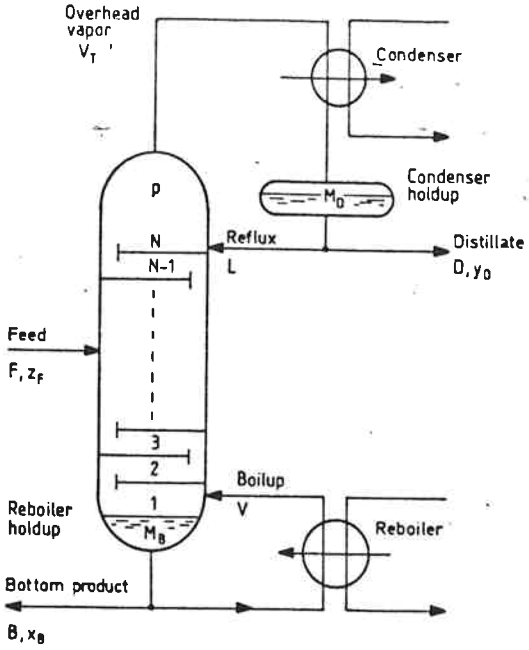

Figure 1. Five-input, five-output distillation column.
Manipulated inputs: L, V, D, B, V_T
Controlled outputs: y_D, x_B, M_D, M_B, p

Most industrial columns have automatic control of one product, usually the top product, y_D. This is often denoted as single or one-point composition control. In this case one of the flows is set manually by the operator. An early discussion on the control configuration selection for one-point composition control is presented by Hills (1948). However, to maintain tight product specifications, it is obviously desirable to have automatic control of both products. This is denoted as dual or two-point composition control. This paper is mainly concerned with dual composition control, but manual and one-point control are also addressed.

A number of important issues not directly related to the control configuration problem are not discussed in this paper. We assume that we can manipulate boilup, V, and overhead vapor, V_T, but we do not discuss how this should be done physically. For example, V_T may be manipulated by changing the flow of cooling water or by adjusting the area available for heat transfer (flooding the condenser). Furthermore, we assume that measurements of the product compositions are available, and do not discuss at all the important problem of how to estimate compositions from measurements of the temperature profile. Also, we do not discuss how to pair the two manipulated variables with the compositions, an issue that arises when we choose single-loop controllers.

Most readers are probably familiar with the LV and DV configurations that are most often analyzed in the literature. These configurations are sometimes designated "indirect material balance," and "direct material balance" (McCune and Gallier, 1973), but since there does not seem to be any consensus in the literature on these names (Luyben, 1979), we will not use them in the following. Rosenbrock (1962) discusses both the LV and the DV configurations (the latter combined with a steady-state decoupler), but indicates no preference. Nisenfeld (1969) recommends the DV configuration, and McCune and Gallier (1973) in a simulation study come out strongly in favor of the DV configuration. Lubyen (1979), however, disputes these findings, and the LV configuration seems to be preferred in industrial practice (Buckley et al., 1985, p. 479). Ryskamp (1980) recommends the LV configuration for columns with low reflux ratios ($L/D < 1$), and the DV configuration for high reflux ratios, ($L/D > 5$). In his earlier work Shinskey (1967, p. 306) came out strongly in favor of the DV configuration, but he seems to have reconsidered his position (Shinskey, 1984) and now recommends using the relative gain array (RGA) to choose the best structure.

More recently, ratio control schemes have become increasingly popular (Ryskamp, 1980; Shinskey, 1984). Rademaker et al. (1975) present an extensive overview of proposed ratio control schemes. (Note that their ratios are inverted compared to the notation in this paper. For example, what they call D/R (they use R for L) we denote by L/D.) They also include schemes using a ratio with the feed rate, F. Since F is generally not a manipulated flow, we classify these as feedforward schemes, which is a separate issue from that of choosing a control configuration for *feedback* control. The earliest reference on the use of ratio control seems to be by Hills (1948), who discusses the $(D/L)V$ configuration. An early reference on the use of ratio schemes for two-point control is by Rijnsdorp (1965), who suggests using the $(L/V_T)V$ configuration instead of the LV configuration in order to reduce the interaction between the control loops. Shinskey (1984) has proposed the $(L/D)(V/B)$ configuration to be the configuration that suits most columns.

This paper is aimed at assisting the engineer in choosing the best control configuration. There is much written in the literature on the pros and cons of various configurations, but case studies predominate over analysis, and theoretically founded guidelines are lacking. Take for example the loop pairing recommendations based on the RGA given by Shinskey (1984). From his book the reader is led to believe that the RGA is just an interaction measure indicating the difficulties one can expect when tuning single loops for a multivariable system. But if this were true the RGA recommendations regarding configuration selection would be of no use if a multivariable controller were chosen. Practical evidence suggests however, that an RGA evaluation is very useful even for the design of multivariable systems. Thus, while the RGA has proven to be a very useful tool for *categorizing* experience, it has not helped to *explain* the observed phenomena. What is clear from our work is that a number of generally conflicting considerations have to be taken into account when choosing the control configuration. Looking at only one of these may give misleading results.

Much of the material in this paper has been presented elsewhere. One of our goals is to present it here in a systematic manner. However, we would like to focus attention on some issues that we believe are novel contributions.

1. A new viewpoint on ratio control systems is presented in which the systems are interpreted in terms of linear combinations of the flows L, V, D, and B. For example, we show that if a multivariable controller were used, then there would be not much difference in the expected performance of the $(L/V)V$ and LV configurations. On the other hand, the $(L/D)V$ or $(L/V_T)V$ configurations may behave entirely differently from the LV configuration.

2. The effect of uncertain manipulated inputs (e.g., valve position errors) is demonstrated and simple physical interpreta-

tions are given. For example, the sensitivity to input uncertainty is reduced by a factor of about $1 + (L/D) + (V/B)$ for the $(L/D)(V/B)$ configuration compared to the LV configuration.

3. The difference between the configurations with regard to open-loop rejection of disturbances is demonstrated. Table 3 gives in a compact form the open-loop (the two manipulated inputs for composition control are constant) effect of various flow disturbances on D/B. Variations in D/B have a large effect on product compositions and should be avoided. Configurations with small entries in Table 3 are therefore preferable. In particular, this is the case if manual control is used, but it also applies to two-point control since then it is advantageous to keep the effect of the disturbances on the product compositions as small as possible. For example, Table 3 shows that the effect of disturbances in L, V, and feed enthalpy on D/B is reduced by a factor of about $1 + (L/D) + (V/B)$ for the $(L/D)(V/B)$ configuration compared to the LV configuration, and in addition the ratio configuration is insensitive to disturbances in the feed rate.

All these issues are discussed in detail below. However, first we will present the distillation control problem from a slightly more general point of view.

Distillation Column from a System Point of View

A schematic picture of a two-product distillation column is shown in Figure 1. Conventional notation is used. A total condenser has been assumed, but this has little significance on the results that follow.

Input and output signals

Viewed from a systems point of view, the distillation column is a box that takes some input functions and maps them into a set of output functions. The inputs are divided into those that can be adjusted (manipulated variables u, usually corresponding to valves) and those that cannot be affected within the system (disturbances d, and set points y_s). Similarly, the outputs are divided into those of interest (controlled variables y) and the known or measured signals (y_m). Obviously, in many cases an output will be both a controlled variable and a measurement, but this is not necessarily the case. The distillation column in Figure 1 has five manipulated inputs u and five controlled outputs y.

Controlled Outputs (y). The five controlled outputs in Figure 1 are:

- Vapor holdup M_V (expressed by the pressure p)
- Liquid holdup in accumulator (condenser) M_D
- Liquid holdup in column base (reboiler) M_B
- Distillate product composition y_D
- Bottom product composition x_B

The reason for choosing these five variables as controlled outputs is briefly discussed: Since vapor and liquid holdups must always be controlled to ensure stable operation, the pressure and the condenser and reboiler holdups (M_V, M_B, M_D) clearly have to be controlled. The liquid holdup inside the column is self-regulating and does not have to be controlled unless the column is overloaded.

We have chosen to use the mole fractions, y_D and x_B, of the light component in the top and bottom product as our product specification. In general, other choices are possible—for example, ratios between compositions, densities, boiling points, and others—but y_D and x_B are most common. Also note that for a multicomponent system, only one composition variable may be controlled independently for each product. In addition to the five controlled outputs mentioned above, there will also be other signals about which we may be concerned. In particular we want to avoid excessive movements of the manipulated variables, mainly because of constraints. Therefore these signals should also be included as controlled outputs in general.

Manipulated Inputs (u). The five manipulated variables in Figure 1 are:

- Distillate flow D
- Bottom flow B
- Reflux L
- Boilup V (manipulated indirectly through the reboiler duty)
- Overhead vapor flow V_T (manipulated indirectly through the condenser duty)

Essentially, these correspond to the available valves. The flow rates V and V_T are controlled indirectly, usually with the flow rates of the heating and cooling medium. In some cases additional manipulated variables are available, for example, the feed rate F or the feed enthalpy q_F, but this will not be considered here.

Disturbances (d) and Set Points (y_s). The disturbances to the column are often related to the feed: the feed flow rate F, the feed enthalpy expressed in terms of its fraction of liquid q_F, and the feed composition z_F. In addition, there are disturbances on the five manipulated inputs. Of these, the disturbances on V and V_T are most important. Typical sources of the disturbances in V and V_T are temperature or pressure changes of the heating or cooling medium.

Other variations that in effect are disturbances are set point changes for y_D and x_B. Set point changes are not common, but will be encountered if there is a higher level optimization scheme that changes set points based on some overall economic objective. This kind of optimization, it is believed, will be increasingly common in the future and will probably constitute a major driving force toward implementing dual composition control schemes.

Measurements (y_m). The measurements typically include the pressure p (usually at several locations), the liquid holdup (level) in the reboiler and condenser, the top and bottom compositions (often delayed and/or sampled), and temperatures at several locations. Often some of the disturbances are measured; typically these include the flow rate and temperature of the feed and the flow rate, pressure, and temperature of the heating and cooling medium.

Performance specifications

An important factor to consider when designing a control system is the performance specifications. More precisely, these are specifications on how the controlled outputs are to behave in response to certain inputs.

Consider first liquid and vapor holdups (M_D, M_B, M_V) that must be controlled to ensure stability. From a steady state point of view only M_V (i.e., the pressure p) has any bearing on the performance of the column. The set point for the pressure may be based on an optimization of the column performance. Since separation is usually favored by low pressure, the optimal pressure is often the minimum attainable, that is, the pressure determined by the constraint of maximum cooling in the condenser [floating-pressure control (Shinskey, 1984)]. However, the pressure should always be kept slightly above the minimum attainable in order to maintain short-term pressure control. Short-term pressure control is needed to avoid fluctuations in the pressure, for example due to changes in the cooling medium (Shinskey, 1984).

The control of the condenser and reboiler holdups is important not because the holdups themselves have any significance, but because changes in the holdups affect the flows controlled by them. Perfect level control is not desirable since this removes the "smoothing" effect of the holdups. This is the main reason the holdups are there in the first place. We will not go into any detail about the performance specifications here, but only state that if reflux L or boilup V are used for level control, then these level loops should be considerably faster than the composition response.

The most important controlled outputs are the top and bottom compositions, y_D and x_B. Their set points may be given by strict product specifications or as a result of a column optimization. The optimization may involve, for example, a trade-off between the cost of heating medium and the money earned by recovering more of the valuable product. Obviously the error $e = y - y_r$, which expresses the deviation between actual and desired product purity, should be small. We have to define more precisely what we mean by small, that is, what kind of norm should be used for e. The choice of norm depends on the reasons for keeping e small. Assume that there is a fixed product specification, (e.g., $x_B \leq 0.01$) that should never be violated. (The bottom stream may be a feed stream to another unit where $x_B > 0.01$ is not allowed). In this case we might choose the set point to be $x_{B_s} = 0.008$ and use the performance specification:

$$\max_t |e(t)| \triangleq \|e(t)\|_\infty \leq 0.002$$

In other cases the bottom stream might go to a large storage tank, which will average out the composition such that only the average composition matters. In this case it would be desirable to have $\int_0^\infty e(t)\,dt$ as small as possible. This may be achieved even if $e(t)$ is fluctuating wildly. This is not desirable, however, because the cost of separation increases (Shinskey, 1984); the energy saved when $x_B > 0.01$ is less than the extra energy needed when $x_B < 0.01$. Consequently, in this case a more appropriate performance specification may be to keep the Integral Square Error (ISE) or the Integral Absolute Error (IAE) as small as possible (but there may not be a specified upper bound on these norms).

$$\text{ISE} = \left[\int_0^\infty |e(t)|^2\,dt\right]^{1/2} \triangleq \|e(t)\|_2$$

$$\text{or} \quad \text{IAE} = \int_0^\infty |e(t)|\,dt \triangleq \|e(t)\|_1$$

The ISE (2-norm) or IAE (1-norm) may be even more appropriate if the set point is determined by some optimization rather than by a product specification.

Linear model

A distillation column is strongly nonlinear, but for control design we will describe it by a linear model. We will only outline the structure of this model.

One complication in obtaining a linear model is that without the pressure and level loops closed, the distillation column is

Table 1. Approximate Open-loop Transfer Matrix for Distillation Column

Controlled Output	Manipulated Input				
	L	V	D	B	V_T
y_D	$g_{11}(s)$	$g_{12}(s)$	0	0	0
x_B	$g_{21}(s)$	$g_{22}(s)$	0	0	0
M_D	$-\frac{1}{s}$	0	$-\frac{1}{s}$	0	$\frac{1}{s}$
M_B	$\frac{1}{s}e^{-\theta s}$	$-\frac{1-\lambda(1-e^{-\theta s})}{s}$	0	$-\frac{1}{s}$	0
$M_V(p)$	0	$\frac{1}{s+k_p}$	0	0	$\frac{1}{s+k_p}$

Assumptions and conditions:
1. Constant molar flows.
2. The transfer function for M_V is not a pure integrator because of condensation effects included in k_p.
3. $e^{-\theta s}$ with $\theta = \tau_L N$ is an approximation for $1/(1+\tau_L s)^N$. $\tau_L = (\partial M_i/\partial L_i)_{V_i}$ is the hydraulic time constant. N is the total number of trays. (see Rademaker et al., 1975.)
4. $\lambda = (\partial L_i/\partial V_i)_{M_i}$ is the initial change in liquid flow due to a change in vapor flow (V may push liquid off the tray and give $\lambda > 0$). An inverse response occurs if $\lambda \geq 0.5$ (Rijnsdorp, 1965).
5. In addition there will be dynamics involved in order to change L, V, D, B, and V_T (valve dynamics, etc.).
6. Consistent units have been assumed for holdups and flows (e.g., M_D in kmol and D in kmol/min).
7. For derivation of V's effect on M_B see Rademaker et al., (1975).

unstable. It is then difficult to obtain open-loop transfer functions for the composition responses using simulation because the reboiler and condenser overflow or run dry long before the composition response has settled. However, since the composition response is only very weakly dependent on the actual level in the condenser and reboiler (M_D and M_B), and since these levels are usually tightly controlled, a good approximation of the open-loop composition response is found by assuming these levels to be constant. The pressure (i.e., vapor holdup M_V) does have a significant effect on the composition, but since pressure is usually tightly controlled, this effect may be neglected as a first approximation. Approximate open-loop responses are therefore obtained by varying L and V, and assuming V_T, D, and B to be fixed by the requirement of perfect control of M_V, M_D, and M_B. For consistency, we then also must neglect the effect changes in V_T, D, and B have indirectly on the compositions because of their effect on M_V, M_D, and M_B. With these assumptions the structure of the open-loop transfer matrix is as shown in Table 1. Similar transfer matrices may be derived for the disturbances.

General interconnection structure

Schematically, the distillation column may be represented as a box, as shown in Figure 2. In general, P is a nonlinear operator giving the nominal relationship (model) between inputs and outputs. We will be using linear models, in which case P is a transfer matrix. For linear systems, P is conveniently divided into four subsystems. P_{11} and P_{21} represent the disturbance model

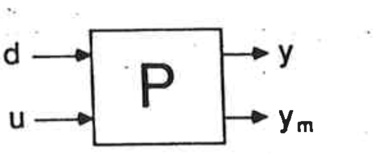

Figure 2. Schematic representation of distillation column.

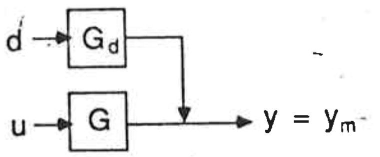

Figure 3. Equivalent representation of Figure 2 for a linear plant with $y = y_m$.

between the disturbances d and the controlled variables y and measured outputs y_m. P_{11} will be denoted by G_d later. P_{12} and P_{22} represent the model between the manipulated inputs u and the controlled outputs y and the measured outputs y_m. P_{12} is what usually is called the process, and will be denoted G. Note that with the usual assumption $y_m = y$, Figure 2 may be represented as in Figure 3.

In Figure 4 we have added two additional blocks to Figure 2. One is the controller C, which computes the appropriate inputs u based on the information about the process y_m. The other block, Δ, represents the model uncertainty (Doyle et al., 1982). Here we will not dwell on this particular way of representing uncertainty, but simply note that it clearly shows that $\hat{P}$ and P are models only, and that the actual plant is different depending on Δ. Based on the measurements y_m, the objective of the controller C is to generate inputs u that keep the outputs y as close as possible to their set points y_s, in spite of disturbances d and model uncertainty Δ. The controller C is often nonsquare, as there are usually more measurements than manipulated variables. For the design of the controller C, information about the expected model uncertainty should be taken into account. The case when $y_m \neq y$ is often called inferential control. It is seen to be handled automatically in this framework.

Figure 4 was introduced by Doyle et al. (1982) and represents a unifying framework for studying linear control problems. The interconnection matrix $\hat{P}$ includes all information needed in order to design the optimal C. In particular $\hat{P}$ includes the matrix P in Figure 2, that is, the process G and the disturbance model G_d. Furthermore, performance weights are included in $\hat{P}$ in order to be able to compare mathematically the controlled variables, which have different physical significance, and in order to decide on the type of desired response. Finally, $\hat{P}$ contains information on how the uncertainty affects the overall system.

Above we have outlined a unifying framework for control problems, Figure 4, and we have tried to give some indication on how distillation fits into it. Clearly, our treatment has been very brief, and for more details the reader should consult Doyle et al. (1982). The main objective is to show that the distillation control problem may be put into a systematic framework and to

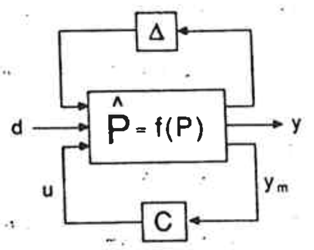

Figure 4. General structure for studying any linear control problem.

point out what information is needed about the process. The problem with this approach is that it results in a controller that is complicated and difficult to design and understand. Engineering judgment cannot easily be brought into this design process. Below, we will outline a stepwise procedure that leads to a much simpler design. The basic idea is to use only two independent manipulated variables for composition control. This is the approach used in practice (Shinskey, 1984). The first and most important step of this design approach is to choose the best control configuration, that is, to decide which two manipulated inputs to use for composition control.

A Simplified Approach

Assume as a simplification that all five controlled outputs (including y_D and x_B) are measured. Given the open-loop model for the distillation column, and information about disturbances, performance, uncertainty, and the like, we can then imagine designing the optimal 5 × 5 controller for the column. While it is certainly of theoretical interest to find this optimal controller, it is very unlikely that such a controller would ever be implemented in practice. In order to make the control system failure-tolerant and easier to understand and tune, simpler control structures are used. This will be the topic of the remainder of this paper.

More specifically, we will not use all five flows L, V, D, B, and V_T for composition control, but only two independent combinations. The overall control system will then consist of a 2 × 2 controller (denoted by K) for composition control plus a control system for level and pressure control.

Inventory control leaves only two degrees of freedom for composition control

The task of subdividing the problem is simplified by the observation that the pressure and level controls are almost always much faster than the composition control because the flow dynamics are usually significantly faster than the composition dynamics. As a first approximation, assume that the pressure and level loops are so fast that they effectively give three static relationships between the five manipulated inputs (L, V, V_T, D, and B) that have to be satisfied at any given time. This implies that there are only two degrees of freedom left for composition control.

In practice, the pressure and level loops are not immediate and, at least on a short time scale, more than two independent inputs could be used for composition control. However, as a first step it is reasonable to design a composition control system using only two manipulated inputs. This system may subsequently be modified to reduce the effect of the lags introduced by the level loops.

Design of a simplified control system

Step 1. Choose two manipulated inputs for composition control. Each choice of manipulated inputs corresponds to a specific control configuration. For example, the choice of L and V for composition control is referred to as the LV configuration.

Step 2. Design the level and pressure control system. Shinskey (1984) calls this closing the material and energy balance. In most cases a simple control system using SISO controllers is chosen, and the choice of pairings is usually obvious once the choice in step 1 is made. Note, however, that the level control system can affect the composition control significantly. The importance of this step in our context is to derive new open-loop composition responses (assuming the pressure and level loops are closed), which may be used to design the controller in step 3. In many cases step 2 is simplified by assuming that the level and pressure controls are "perfect."

Step 3. Design the 2 × 2 controller K for composition control. This is not a trivial step, but it is certainly much simpler than designing a 5 × 5 controller including all inputs and outputs. In many cases K is restricted to be diagonal (decentralized control).

The most important step in the above procedure for designing a simplified dual composition control system is to decide on the control configuration, step 1. Which variables should be "manipulated" in order to maintain composition control? We have put "manipulate" in quotation marks because we are going to define new manipulated variables different from the real ones (which are the valve positions). In fact, we have already implicitly redefined the manipulated variables by assuming that we can actually manipulate the flows L, V, V_T, D, and B directly instead of their valve positions. In practice, for L, D, and B this may be implemented by measuring the actual flow rate and using a very fast inner loop to adjust this measured rate to match the desired flow. By this we also remove the nonlinear relationship between the valve position and the flow rate. It is usually not possible to measure V and V_T, and these flows must be estimated in some other way, for example by enthalpy balance calculations.

There is clearly an infinite number of relationships between L, V, V_T, D, and B that can be defined as new "manipulated" variables. Of these, we will only consider the flows themselves and ratios between the flows. A further simplification results because the condenser duty (i.e., V_T) is almost never used for controlling composition (Shinskey, 1984). The reason for this is probably that V and V_T have almost the same effect on composition, and cannot be used independently for composition control. Furthermore:

- V has a more direct effect on bottom composition x_B, and is therefore preferable over V_T from a dynamic point of view.
- V_T is generally better for pressure control since the primary and secondary effects on pressure are always in the same direction. On the other hand, V may yield an inverse response. Initially, pressure increases in response to an increase in V. However, if composition is uncontrolled the temperature in the column will start rising, thereby decreasing ΔT in the reboiler and increasing ΔT in the condenser, resulting in reduced pressure (Rademaker et al., 1975).

The problem of dual composition control is then reduced to controlling the compositions y_D and x_B using two independent combinations of the inputs L, V, D, and B (Shinskey, 1984).

L, V, D, and B as manipulated inputs

Let us first consider the case when the flows L, V, D, and B themselves are used as manipulated variables for composition control. There are $\binom{4}{2} = 6$ independent pair combinations. However, only five of these are possible since D and B cannot be used together for composition control, because of the steady state material balance constraint $D + B = F$. Having chosen one of the remaining five pairs (LV, LD, LB, DV, or VB) for composition control, the control structure for the level loops follows easily.

Example: LV Configuration. Assume that L and V have been chosen for composition control. [This is the configuration most commonly used (Rademaker et al.., 1975).] The condenser level may be controlled by D, pressure by V_T, and the reboiler level by B, resulting in the following control structure:

$$\begin{bmatrix} dL \\ dV \\ dD \\ dB \\ dV_T \end{bmatrix} = \begin{bmatrix} \multicolumn{2}{c}{\multirow{2}{*}{K}} & 0 & 0 & 0 \\ & & 0 & 0 & 0 \\ 0 & 0 & c_D(s) & 0 & 0 \\ 0 & 0 & 0 & c_B(s) & 0 \\ 0 & 0 & 0 & 0 & c_V(s) \end{bmatrix} \begin{bmatrix} dy_D \\ dx_B \\ dM_D \\ dM_B \\ dM_V \end{bmatrix} \quad (1)$$

Ratios between L, V, D, and B as manipulated inputs

Of the possible nonlinear relationships among L, V, D, and B we will only consider ratios. These seem to be the only nonlinear combinations used in practice (Shinskey, 1984). The total number of independent ratios is six. They are L/V, L/D, L/B, V/D, V/B, and D/B. Including the four flows themselves, this results in $\binom{10}{2} = 45$ independent pairs of "manipulated" variables. Again, combinations of D, B, and D/B cannot be used for composition control. This eliminates three of these options, but still leaves us with 42 possible combinations.

Shinskey (1984) excludes the ratio D/B because it is not independent of D. However, even though configurations involving D, B, and D/B have the same value of the RGA, the resulting control systems are generally different. Shinskey also groups L/V, L/D, and V/D into a single manipulated variable, the separation factor S. He claims that this may be done because L/V and V/D uniquely determine L/D, and because L/D determines S uniquely. However, the relationships between the flows hold only at steady state and when the feed is liquid, and the relationship between L/D and S is only approximate. In practice, the three choices (L/V, L/D, V/D) can yield entirely different control systems.

If we look at the actual implementation there are even more than 42 options. Since the true manipulated variables are always L, V, D, and B, we have to determine how L/V, for example, is implemented as a "manipulated" variable. To increase L/V we may either increase L, decrease V, or change both at the same time. If the flow dynamics and level controls were immediate, these different implementations would not affect the composition response, but because they are not, it does make a difference. We adopt the following convention: Writing the ratio between L and V as L/V means that L is manipulated to change L/V, and writing V/L means that V is manipulated to change the ratio.

Ratio control systems have been used in industry for at least forty years (Rademaker et al., 1975, p. 445), yet almost no discussion is found in the literature on why such schemes may be beneficial. The simplest justification follows from steady-state considerations. To keep the compositions constant, the ratio L/V inside the column (slope of the operating line on the McCabe-Thiele diagram) should be constant. Intuitively, it seems that some disturbances may be counteracted by keeping this ratio constant (Ryskamp, 1980). However, these arguments do not explain what happens when ratios are used for closed-loop control of compositions. Furthermore, as will be shown, the effect of using a given ratio depends entirely on which second manipulated variable is chosen for composition control.

Clearly, using ratios as "manipulated" variables is a way of introducing a simple nonlinear control scheme. For example, the nonlinear implementation of L/D as a manipulated variable is (using the convention introduced above)

$$L = \left[\frac{L}{D}\right] D \tag{2}$$

As usual, when a linear approach is taken, we consider deviations (dL) from the nominal steady state, (for example, $L = L_0 + dL$ where L_0 is the steady state value). Then the linear implementation corresponding to Eq. 2 becomes

$$dL = Dd\left[\frac{L}{D}\right] + \frac{L}{D} dD \tag{3}$$

or equivalently

$$L = L_0 + D_0\Delta\left[\frac{L}{D}\right] + \frac{L_0}{D_0}\Delta D \tag{4}$$

The difference between Eq. 2 and Eq. 4 is important only if the flow rates D and L/D change significantly with operating conditions. Because this is usually not the case, there are only minor performance differences between the linear and nonlinear scheme. The nonlinear control system is often simpler to implement, however.

Consequently, for analysis in most cases it does not make much difference if we use linear combinations of L, V, D, and B as new manipulated variables instead of ratios. We would like to understand what kind of linear control system this corresponds to. To this end consider the following examples.

Example: (L/D)(V/B) Configuration. The $(L/D)(V/B)$ configuration is claimed by Shinskey (1984) to be applicable over the broadest range of cases; Rademaker et al. (1975, pp. 450, 463) also recommend using two-ratio control schemes. A small change in L/D and V/B is written

$$d\left[\frac{L}{D}\right] = \frac{1}{D}dL - \frac{L}{D^2}dD$$
$$d\left[\frac{V}{B}\right] = \frac{1}{B}dV - \frac{V}{B^2}dB \tag{5}$$

Note that the constant coefficients multiplying dD, dL, dV, and dB are determined at a chosen nominal steady state. The idea is to use $d[L/D]$ and $d[V/B]$ for composition control. According to the convention defined above dL and dV are manipulated to change the ratios L/D and V/B. Rearranging Eq. 5 yields:

$$dL = Dd\left[\frac{L}{D}\right] + \frac{L}{D}dD$$
$$dV = Bd\left[\frac{V}{B}\right] + \frac{V}{B}dB \tag{6}$$

Consequently, dL and dV depend on $d[L/D]$ and $d[V/B]$ (which are "manipulated" based on the compositions y_D and x_B) and on the flow rate changes dD and dB. We could measure dD and dB and use this in Eq. 6, but note that the values of dD and dB are determined by the level control system. Therefore, consider step 2 in the design procedure, which is the design of the level control system: Because L and V are manipulated for composition control, the most reasonable choices for the control of the condenser and reboiler levels are D and B, that is,

$$dD = c_D(s)dM_D$$
$$dB = c_B(s)dM_B \tag{7}$$

[The SISO controllers $c_D(s)$ and $c_B(s)$ are in many cases simple proportional controllers.] Using Eq. 7 to eliminate dD and dB, Eq. 6 yields

$$dL = Dd\left[\frac{L}{D}\right] + \frac{L}{D}c_D dM_D$$
$$dV = Bd\left[\frac{V}{B}\right] + \frac{V}{B}c_B dM_B \tag{8}$$

Let the composition controller (possibly multivariable) be

$$\begin{bmatrix} d(L/D) \\ d(V/B) \end{bmatrix} = \hat{K}\begin{bmatrix} dy_D \\ dx_B \end{bmatrix} \tag{9}$$

and define

$$K = \begin{bmatrix} D & 0 \\ 0 & B \end{bmatrix}\hat{K} \tag{10}$$

Then the overall controller becomes

$$\begin{bmatrix} dL \\ dV \\ dD \\ dB \end{bmatrix} = \begin{bmatrix} K & & (L/D)c_D & 0 \\ & & 0 & (V/B)c_B \\ 0 & 0 & c_D & 0 \\ 0 & 0 & 0 & c_B \end{bmatrix}\begin{bmatrix} dy_D \\ dx_B \\ dM_D \\ dM_B \end{bmatrix} \tag{11}$$

We see from Eq. 11 that the flow rates L and V are manipulated based both on the product compositions (y_D and x_B) and on the levels (M_D or M_B). Furthermore, the two SISO level controllers [$c_D(s)$ and $c_B(s)$] each manipulate two flow rates, and therefore appear at two places in the transfer matrix for the overall controller.

If the $(D/L)(B/V)$ configuration had been used instead, we would get a similar controller structure, but with K in the lower left corner. Also the $(L/D)(B/V)$ or $(D/L)(V/B)$ configurations would result in similar controller structures.

In summary, from a linear point of view the main feature of this ratio control system is to let the level be controlled by more than one flow; the controller changes both L and D in response to a change in M_D and both V and B in response to a change in M_B. Thus the use of ratios as manipulated variables introduces a multivariable control in an ad hoc system. In other cases it leads to a simplified MIMO controller for the composition control (but one that is tuned as two SISO controllers). This is illustrated by the following example.

Example D(L/D) Configuration. When L is manipulated in the $D(L/D)$ configuration, a linear analysis shows

$$dL = Dd\left[\frac{L}{D}\right] + \frac{L}{D}dD \tag{12}$$

Assume that SISO controllers (decentralized control) are used for composition control: D is manipulated based on y_D and L/D is manipulated based on x_B:

$$dD = k_1(s)dy_D$$
$$d\left(\frac{L}{D}\right) = k_2(s)dx_B \tag{13}$$

Combining Eqs. 12 and 13 yields (in this case the level control system influences B and V only, but not L and D):

$$\begin{bmatrix} dD \\ dL \end{bmatrix} = \begin{bmatrix} k_1(s) & 0 \\ (L/D)k_1(s) & Dk_2(s) \end{bmatrix} \begin{bmatrix} dy_D \\ dx_B \end{bmatrix} \tag{14}$$

Effectively, a MIMO (in this case triangular) composition controller results that is tuned like two SISO controllers.

Note that in this case the effect of using L/D as a manipulated variable, is entirely different from that found for the $(L/D)(V/B)$ configuration. In fact, the $D(L/D)$ configuration is not much different from the DL configuration, as seen from Eq. 14. On the other hand. the $(L/D)(V/B)$ configuration may behave significantly different from the $L(V/B)$ or (even more so) the LV configuration.

Example: (D/V)(V/B) Configuration. This example combines the features found in the previous two examples. Linearizing yields

$$dD = Vd\left[\frac{D}{V}\right] + \frac{D}{V}dV$$
$$dV = Bd\left[\frac{V}{B}\right] + \frac{V}{B}dB \tag{15}$$

Let the levels be controlled as follows

$$dL = c_D(s)dM_D$$
$$dB = c_B(s)dM_B \tag{16}$$

Combining Eqs. 15 and 16 yields

$$dD = Vd\left[\frac{D}{V}\right] + \frac{DB}{V}d\left[\frac{V}{B}\right] + \frac{D}{B}c_B dM_B$$
$$dV = Bd\left[\frac{V}{B}\right] + \frac{V}{B}c_B dM_B \tag{17}$$

corresponding to the control structure

$$\begin{bmatrix} dD \\ dV \\ dL \\ dB \end{bmatrix} = \begin{bmatrix} \multicolumn{2}{c}{\multirow{2}{*}{K}} & 0 & (D/B)c_B \\ & & 0 & (V/B)c_B \\ 0 & 0 & c_D & 0 \\ 0 & 0 & 0 & c_B \end{bmatrix} \begin{bmatrix} dy_D \\ dx_B \\ dM_D \\ dM_B \end{bmatrix} \tag{18}$$

where

$$K = \begin{bmatrix} V & DB/V \\ 0 & B \end{bmatrix}\hat{K}, \quad \begin{bmatrix} d(D/B) \\ d(V/B) \end{bmatrix} = \hat{K}\begin{bmatrix} dy_D \\ dx_B \end{bmatrix} \tag{19}$$

If $\hat{K}$ is diagonal, this results in a triangular K, but tuned as two SISO controllers. Also note that an increase in the reboiler level, M_B, will result in a simultaneous increase in D, V, and B.

Summary. Based on the three examples above let us state the following generalization: Assume that one of the "manipulated" variables for composition control is ℓ_1/ℓ_2 and assume that ℓ_1 is the flow that is manipulated to adjust ℓ_1/ℓ_2. Then the linear control system corresponding to ℓ_1/ℓ_2 has the following features compared to using the flow ℓ_1 alone for composition control.

1. If ℓ_2 is used for level control then this level is controlled both by ℓ_1 and ℓ_2 (but tuned as a single controller)
2. If ℓ_2 is used for composition control then this composition is controlled both by ℓ_1 and ℓ_2; that is, we get an effective MIMO controller using a SISO design.

In case 2 it makes little sense to use a MIMO controller to "manipulate" ℓ_1/ℓ_2, since the same result may be obtained by using ℓ_1 alone. Consider Eq. 19 in the last example. If $\hat{K}$ is a full 2×2 matrix, then the tuning is not simpler than when K is designed directly. Consequently, if a multivariable controller is used we do not expect much difference between, for example, the $(D/V)(V/B)$ and $D(V/B)$-configurations, or between the $(L/V)V$ and LV configurations, or between the $(D/V)V$ and DV configurations.

In most cases the major effect of using ratios for composition control is captured by the linear analysis summarized in (1) and (2) above. Ratios do not tend to correct the nonlinear behavior of distillation columns because the manipulated inputs vary only moderately with operating conditions (neglecting start-up). On the other hand, the product compositions do often vary significantly with operating conditions, and a significant linearization effect may be obtained, for example, by using $\ln(1 - y_D)$ and $\ln x_B$ as redefined controlled outputs. This is discussed in another paper (Skogestad and Morari, 1987a).

Differences Between Control Configurations

Assuming immediate flow responses, perfect level control, and constant molar flows, we have in the absence of feed disturbances

$$dV = dL + dD \tag{20}$$

$$dL = dV + dB \tag{21}$$

These two equations suggest that any pair of input variables has the same effect. Changing L and V, for example, is equivalent to changing V and D or V and B.

Consequently, we might expect to get good and almost identical control performance for any choice of control configuration. However, there are at least eight reasons why the choice of control configuration can make a significant difference:

1. "Uncertainty"
2. Disturbances vs. set points
3. Dynamic considerations
4. Rejection of flow disturbances
5. One-point (manual) composition control
6. Changes between manual and automatic

7. Constraints
8. Level control

In many cases conflicting conclusions arise from these considerations, and the engineer has to perform a more detailed analysis or use his judgment in making the final choice. Before looking into these eight points, we will consider some general characteristics of distillation columns that are used in the subsequent discussions.

Model characteristics of distillation columns. From a control point of view the most important characteristic of distillation columns appears to be that for high-purity separations (x_B and $1 - y_D$ are small) the 2 × 2 system considered for composition control is always ill-conditioned regardless of what control configuration is used (Skogestad and Morari, 1987a). By ill-conditioned we mean that the plant gain in certain directions is much larger than in others. Irrespective of the control configuration, the two operating variables corresponding to the high and low plant gain are respectively the external flows (product flow rates, D and B) and the internal flows (which are changed by changing the reflux L and boilup V while keeping D and B constant) (Rosenbrock, 1962). As an illustration of a change in external flows, consider the column in Table 2 with $z_F = 0.5$, $y_D = 0.99$, $x_B = 0.01$, and $D = B = 0.5$ kmol/min. Assume that the distillate flow D is increased by 5% to 0.525 kmol/min. Since there is only 0.5 kmol/min of light component in the feed, at least 0.025 kmol/min of this has to be heavy component. The best attainable value for the top composition, even with total reflux, is then $y_D = 0.5/0.525 = 0.952$. This is far from the desired $y_D = 0.99$.

More generally, the effect of the external flows on the product compositions is found using

$$\frac{D}{B} = \frac{z_F - x_B}{y_D - z_F} \tag{22}$$

This exact expression can be derived from an overall material balance for the light component. It implies that the ratio D/B should be kept constant for any flow disturbance. Furthermore, for high-purity columns, the relative changes in y_D and x_B are extremely sensitive to changes in D/B. For example, with y_D constant, differentiation of Eq. 22 yields

$$\frac{dx_B}{x_B} = -\frac{y_D - z_F}{x_B} d\left(\frac{D}{B}\right) \tag{23}$$

Table 2. Data for Distillation Column Example (Skogestad and Morari, 1986a, 1987c)

Binary Separation, Constant Molar Flows, Feed Liquid
- Relative volatility, $\alpha = 1.5$
- No. of theoretical trays, $N = 40$
- Feed tray location, $N_F = 21$ (1 = reboiler)
- Feed rate and composition, $F = 1$ kmol/min, $z_F = 0.5$
- Product compositions, $y_D = 0.99$, $x_B = 0.01$
- Product rates, $D = B = 0.5$ kmol/min
- Reflux rate, $L = 2.71$ kmol/min (1.39 L_{min})

Linearized Steady-State Gains, LV Configuration

$$\begin{bmatrix} dy_D \\ dx_B \end{bmatrix} = \begin{bmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{bmatrix} \begin{bmatrix} dL \\ dV \end{bmatrix} + \begin{bmatrix} 0.394 \\ 0.586 \end{bmatrix} dF + \begin{bmatrix} 0.881 \\ 1.119 \end{bmatrix} dz_F$$

1-1 Element in the RGA for Various Configurations

	LV	$(L/D)(V/B)$	$(L/D)V$	$(L/D)D$	DV	LD
λ_{11}	35.1	3.22	5.85	0.60	0.45	0.56

The factor multiplying $d(D/B)$ approaches infinity when $x_B \rightarrow 0$.

For a more quantitative analysis, a singular value decomposition (SVD) can be performed on the 2 × 2 transfer function model for each configuration (Skogestad and Morari, 1986a). For high-purity columns, the singular values are always found to be very different in magnitude (ill-conditioned system). The singular vectors confirm that the large plant gain is associated with a change in the external flows while changes in the internal flows have a much smaller effect on the compositions. As we will show, the observed advantages of certain control configurations can be explained from these basic characteristics.

The Relative Gain Array (Bristol, 1966). The RGA is determined by the plant transfer matrix G. For 2 × 2 plants:

$$\text{RGA} = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} = \begin{bmatrix} \lambda_{11} & 1 - \lambda_{11} \\ 1 - \lambda_{11} & \lambda_{11} \end{bmatrix}, \quad \lambda_{11} = \frac{1}{1 - (g_{12}g_{21}/g_{11}g_{22})} \tag{24}$$

The RGA is used extensively by Shinskey (1984) to compare control configurations. From his book the reader is led to believe that the RGA is useful because it provides a measure of interactions when using a decentralized controller. His rule (although he does not express it explicitly) is to choose a configuration with λ_{11} in the range of about 0.9 to 4 (Shinskey, 1984, Table 5.2). If λ_{11} were used only as an interaction measure this recommendation would not make any sense; in this case λ_{11} should be chosen to be as close to one as possible and $\lambda_{11} = 0.67$ would be almost equivalent to $\lambda_{11} = 2$ (both have $|g_{12}g_{21}/g_{11}g_{22}| = 0.5$). Consequently, Shinskey's use of the RGA is a way of categorizing his experience on distillation columns, rather than expressing the effect of interactions. In fact, his rules also apply when a multivariable controller is used. His recommendations regarding the RGA should therefore only be used for distillation columns. One objective of this section is to provide some justification for Shinskey's rules.

Uncertainty

Since we are considering different choices of manipulated inputs, the uncertainty associated with these manipulated inputs may cause different control behavior. These issues have been discussed in detail by Skogestad and Morari (1986a,b) and the main result is summarized below.

RGA and Input Uncertainty. The RGA is a good indicator of plant sensitivity to input uncertainty (Skogestad and Morari, 1986b). In general, a plant with large elements in the RGA is difficult to control in the presence of input uncertainty and, in particular, inverse-based controllers should be avoided.

Let Δ_1 and Δ_2 represent the magnitude of the relative uncertainty on each manipulated input. Then the actual (perturbed) plant can be written in terms of the model G and this uncertainty:

$$G_p = G(I + \Delta_I), \quad \Delta_I = \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix} \tag{25}$$

For good closed-loop performance, an inverse-based controller is desirable, for example, $C(s) = c(s)G(s)^{-1}$ where $c(s)$ is a scalar. The loop transfer function in this case becomes

$$G_pC = GC(I + G\Delta_I G^{-1}) \tag{26}$$

If the error term $G\Delta_I G^{-1}$ is large, the actual loop transfer function G_pC will be significantly different from the desired (nominal) loop transfer function GC, and the closed-loop response is expected to be poor or even unstable. The diagonal elements of $G\Delta_I G^{-1}$ are a function of the RGA only

$$G\Delta_I G^{-1} = \begin{bmatrix} \lambda_{11}\Delta_1 + \lambda_{12}\Delta_2 & -\lambda_{11}(g_{12}/g_{22})(\Delta_1 - \Delta_2) \\ \lambda_{11}(g_{21}/g_{11})(\Delta_1 - \Delta_2) & \lambda_{21}\Delta_1 + \lambda_{22}\Delta_2 \end{bmatrix} \tag{27}$$

Equation 27 clearly shows that the closed-loop response for plants with large RGA elements is extremely sensitive to input uncertainty if a tight (inverse-based) controller is chosen. Note that it is the value of the RGA around the crossover frequency that is of main interest. Using the steady-state value may be misleading (yield too large values). This is generally the case for columns with both products of equal purity, for example, the column in Table 2.

The result, Eq. 27, explains in a quantitative way why configurations with large RGA elements should be avoided. However, through the following discussion we want to give the reader a more intuitive feeling for why some configurations are sensitive to input uncertainty and others are not.

A Physical Interpretation of the Effect of Flow (Input) Uncertainty. From Eq. 20, which applies to the case with perfect level control, it seems that a change in distillate flow dD may be achieved in two equivalent ways:

1. Manipulate D directly
2. Manipulate L and V such that $dV - dL = dD$

Similar arguments apply to other flows. However, such arguments only hold in the absence of input uncertainty. In practice, the actual flows are not the same as those demanded by the controller (the controller may try to increase a particular flow by 1 kmol/min, but the actual increase may be only 0.9 kmol/min, corresponding to 10% uncertainty with respect to the change). This input uncertainty may lead to an enormous difference in the behavior of the various configurations, since for tight control it is often desirable to make dL and dV large while keeping dD small. This is almost impossible if, for example, the LV configuration is used since we cannot in practice control differences ($dL - dV$) between two large flows accurately.

Example. Consider the column in Table 2 with $D = B = 0.5$ kmol/min, $L/D = 5.4$, $V/B = 6.4$, and assume that we want to increase the internal flows (desired: $dL = dV = 1$ kmol/min) without changing the external flows (desired: $dD = dB = 0$). Let us see how three different configurations would perform under these assumptions in the presence of uncertainty.

LV Configuration

Assume that there is 10% uncertainty about the flow rate change, that is,

$$dL = 1 \pm 0.1 \text{ kmol/min}, \quad dV = 1 \pm 0.1 \text{ kmol/min}$$

(in practice the uncertainty in the boilup V is probably larger than that for the reflux L). With the LV configuration the distillate D and bottom B flows will feel the full effect of this uncertainty

$$dB = -dD = dL - dV = 0 \pm 0.2 \text{ kmol/min} \tag{28}$$

This is highly undesirable because of the strong sensitivity of the compositions to changes in the external flows. The high value of the RGA ($\lambda_{11} = 35.1$) for this configuration predicts the sensitivity.

DV Configuration

For the same flow uncertainty we get

$$dL = dV = 1 \pm 0.1 \text{ kmol/min}$$

However, since D is manipulated directly, these have to change by the same amount and do not result in any change in D.

$$dB = -dD = 0 \text{ kmol/min} \tag{29}$$

Not surprisingly, the RGA elements are generally less than one for this configuration ($\lambda_{11} = 0.45$ for this example).

(L/D)(V/B) Configuration

If initially $dL \neq dV$ (because of uncertainty) then changes in the top and bottom accumulator levels occur. As is apparent from Eq. 8, these changes lead to adjustments of dL and dV, which will counteract the initial imbalance.

Assume now that the 10% uncertainty on L and V initially (before the level loops take action) results in $dL_1 = 1.1$ and $dV_1 = 0.9$ kmol/min. Let the subsequent flow adjustments made by the level control system be denoted as dL_2 and dV_2. Then the final steady-state flows are

$$dL = dL_1 + dL_2, \quad dV = dV_1 + dV_2$$

Furthermore, we must have perfect level control at steady state

$$dV = dL + dD, \quad dL = dV + dB$$

and according to Eq. 11 the levels are adjusted such that

$$dL_2 = \frac{L}{D}dD, \quad dV_2 = \frac{V}{B}dB$$

Solving these equations gives

$$dB = (dL_1 - dV_1)\Big/\left(1 + \frac{L}{D} + \frac{V}{B}\right) = 0.2/12.8 = 0.015 \text{ kmol/min} \tag{30}$$

The resulting error in B and D due to uncertainty in L and V is therefore reduced by a factor of $(1 + L/D + V/B)$ compared to the LV configuration. Interestingly, Skogestad and Morari (1987c) have shown that the elements in the RGA are also reduced by a factor of about $(1 + L/D + V/B)$ compared to the LV configuration. (The exact value is $\lambda_{11} = 3.22$ for this example.) However, we may still have control problems for very high purity columns because of the extreme sensitivity to changes in D and B. In this case the RGA should be computed to get a reliable indication of whether or not input uncertainty will cause problems (recall Eq. 27).

Summary. The presence of input uncertainty favors using configurations with small elements in the RGA (Skogestad and Morari, 1986b). In general, all configurations involving D or B have $|\lambda_{11}| < 1$, while all others have $|\lambda_{11}| > 1$ (Shinskey, 1984, p. 146). The LV configuration generally has the largest RGA elements. Any configuration that uses D or B is therefore insensitive to input uncertainty, but the ratios L/D, V/B, L/B, or V/D (or their inverses) may also be a good choice for columns with high reflux.

Disturbances vs. set points

Although we just concluded that plants with large RGA elements should be avoided, it is really large RGA elements in the controller that cause control problems (Skogestad and Morari, 1986b). However, in most cases (in particular, if good set-point tracking is desired), it is desirable to use an inverse-based controller to get good performance, and in this case the controller has large RGA elements whenever the plant does.

For distillation columns, if we do not care too much about set-point tracking, it may not be necessary to use an inverse-based controller to achieve good control performance. A diagonal controller—that is, a set of single loops—always has $\lambda_{11}(C) = 1$ and is therefore not sensitive to input uncertainty, but often it does not yield adequate control performance. For distillation columns, however, the disturbances are often aligned with the plant and may be counteracted with a diagonal controller. An accurate measure of how a disturbance d (which has the effect g_d on the outputs y) is aligned with the plant G is provided by the disturbance condition number (Skogestad and Morari, 1987b):

$$\gamma_d(G) = \frac{\|G^{-1}g_d\|_2}{\|g_d\|_2}\bar{\sigma}(G)$$

where $\bar{\sigma}(G)$ denotes the maximum singular value of G, and $\|\cdot\|_2$ denotes the Euclidean norm. Depending on the direction of g_d, $\gamma_d(G)$ ranges in magnitude between 1 and $\gamma(G)$ (the condition number of G). For distillation columns the values of $\gamma_d(G)$ for the disturbances are usually significantly smaller than $\gamma(G)$. For example, consider the distillation column in Table 2, which has $\gamma(G) = 141.7$ for the LV configuration. Disturbances in $d = z_F$, F, q_F, L, and V yield $\gamma_d(G) = 1.48, 11.75, 1.09, 1.41$, and 1.41, and a diagonal controller may give acceptable response (Skogestad and Morari, 1987b).

Summary. Configurations with large RGA elements (e.g., the LV configuration) are not sensitive to input uncertainty if a diagonal controller is used. A diagonal controller may be acceptable if the disturbance condition number is small for all expected disturbances (and tight set-point tracking is not required). This means that the LV configuration may be acceptable in some cases even when it yields large RGA values.

Dynamic considerations

These issues are addressed in detail in the literature (Rademaker et al., 1985, Shinskey, 1984), and only a short summary is given here.

The flow rates L and V (or V_T) are the only ones that influence compositions directly. The direct effect of changing B or D is to change M_B and M_D, which has no effect on compositions. The effect on composition is caused by the level loops, which change L, V, or V_T in response to the change in B and D. However, even a 1 or 2 min lag caused by the level loops may make it difficult to counteract a large disturbance, which may change the product composition considerably in a matter of minutes. (The speed of the level loops is limited by noise on the level measurements, but is otherwise independent of the amount of holdup). These considerations are even more important for packed columns where the holdup inside the column is smaller. A possible solution (Shinskey, 1984, p. 128) is to let the composition loop also influence the flow used for level control (i.e., L or V) (This effect is only temporary, but will improve the dynamic response.)

Other issues that should be considered are:

- Even L has only a delayed effect on x_B.
- An increase in boilup V may in some cases initially push liquid off the trays and result in a temporary increase in liquid flow in the column ($\lambda \geq 0.5$ in Table 1). The effect is a possible inverse response for V's effect on M_B and x_B (the K_2 effect; Rijnsdorp, 1965).
- Large overshoots in the open-loop response are often encountered with the material balance configurations (using D or B). For example, for the DV configuration, an increase in V will first cause x_B to fall. However, since D is constant, the increase in V will eventually produce an equal increase in L, which brings more light component back to the bottom, and cause x_B to return almost to its original value. This large overshoot in the response corresponds to a left half-plane zero close to the origin. Shinskey (1984, p. 157) claims that this LHP zero causes control problems. This may be the case if a PID controller is used (which cannot easily counteract the effect of the zero), but should not cause problems in general.

Summary. L and V should be manipulated directly for composition control to get a fast initial response. This is probably one of the main reasons for the popularity of the LV configuration. The $(L/D)(V/B)$ configuration also has this feature. Use of D or B for composition control is generally not recommended if a fast initial response is desired.

Rejection of flow disturbances

The major flow disturbances are in the

- Feed rate F
- Feed enthalpy q_F
- Boilup V
- Condenser vapor rate V_T
- Reflux temperature

The fraction liquid in the feed, q_F, is used as a measure of feed enthalpy. The result of a decrease in reflux temperature (possibly caused by subcooling the reflux) is equivalent to a simultaneous increase in L and a decrease in V_T. There will also be disturbances in L, D, and B (e.g., due to measurement noise), but those are usually of less importance. Three ways of handling flow disturbances are:

1. By feedforward control
2. Through their effect on composition
3. Through their effect on levels and pressure

The first option is possible only if the disturbance can be measured. The level and pressure loops are usually much faster than the composition loops, and intuitively it seems preferable to try to reject the flow disturbances with the level loops (option 3). However, since any flow disturbance that is not rejected by the level loops will result in a composition upset, one may argue that the composition control system may as well take care of all disturbances (option 2). The problem is that it may not be possible to tune the composition loops sufficiently fast to get acceptable response for large disturbances. This is the case in particular if L and V are not manipulated directly for composition control (see Dynamic Considerations, above). Furthermore, by using option 3 we retain some disturbance rejection capability in the case the composition loops are in manual.

Rejecting Flow Disturbances with the Level Loops (Option 3). The effect of flow disturbances on compositions in this case is found by assuming that the inputs used for composition control (u_1 and u_2) are constant. The effect depends strongly on the chosen control configuration:

- If there is a disturbance directly on a flow that is manipulated for pressure or level control alone, it will be corrected almost immediately by the level loop.
- On the other hand, if there is a disturbance on a flow used for composition control alone then the flow itself is not corrected. However, corrections on other flows may counteract the effect of the disturbance on the compositions.

As an example, consider the DV configuration and assume that there are disturbances on the boilup V. Then disturbances on V are not corrected and affect the operation. However, the effect on the compositions is small because the disturbance on V causes L to increase and the product flow rates (D and B) do not change. Therefore the steady-state values of the compositions are almost unaffected.

The Disturbance Gain Matrix (G_d). The effect of disturbances on the product compositions is expressed mathematically by the disturbance gain matrix. (The steady-state matrix may be used since the level and pressure loops are much faster than the composition loops.) Assume that all the gains (including $[\partial y_D/\partial d]_{L,V}$) are known for L and V as manipulated inputs. (This is the most natural choice, as seen from Table 1.) We can then express $(\partial y_D/\partial d)$ for any other set of manipulated inputs as follows

$$\left[\frac{\partial y_D}{\partial d}\right]_{u_1,u_2} = \left[\frac{\partial y_D}{\partial L}\right]_V \left[\frac{\partial L}{\partial d}\right]_{u_1,u_2} + \left[\frac{\partial y_D}{\partial V}\right]_L \left[\frac{\partial V}{\partial d}\right]_{u_1,u_2} + \left[\frac{\partial y_D}{\partial d}\right]_{L,V} \quad (31)$$

The terms $(\partial L/\partial d)_{u_1,u_2}$ and $(\partial V/\partial d)_{u_1,u_2}$ are easy to evaluate if constant molar flows are assumed. Clearly, it is advantageous to choose configurations that have small values of $(\partial y_D/\partial d)_{u_1,u_2}$ for all disturbances. We will return with a more detailed discussion on how to evaluate the disturbance gains in a future paper.

Effect of Flow Disturbances on D/B. The described procedure, Eq. 31, is exact but does not give much insight. Since the product compositions are most sensitive to changes in the external flows (or equivalently D/B, see Eq. 22), an alternative approach is to consider the effect of flow disturbances on D/B. Configurations for which the effect is large should be avoided. It can be shown that an important feature of some of the ratio control schemes is that they have a good built-in rejection of flow disturbances.

Example. Assume that the feed is liquid and consider a feed flow disturbance. If the LV or DV configuration is used, this disturbance will immediately give an increase in bottoms flow rate B, leading to a large upset in x_B and y_D. However, if the $(L/D)(V/B)$ configuration is used, all flows are adjusted proportionally, and the effect on compositions is zero at steady state. The increased feed flow rate initially brings light components down the column, which would increase x_B. However, it also leads to an increase in reboiler level. From Eq. 11 we see that this leads to a simultaneous increase in B and V (while the LV and DV configurations keep V constant). The increased boilup V returns light components to the column and counteracts the initial effect the increased feed flow had on compositions. Furthermore, the increase in V leads to an increase in distillate flow D. The feed flow disturbance is therefore distributed to both products, and D/B is kept unchanged.

Table 3 summarizes the effect of some flow disturbances on D/B. Note that disturbances in V, L, and $-q_F$ all increase the net flow from the reboiler to the condenser, and have the same effect on D/B. The results in Table 3 seem to be new and provide a simple explanation why, for example, the $(L/D)(V/B)$ configuration is less sensitive to flow disturbances than the LV configuration.

Note from Table 3 that the $(L/V)V$ and LV configurations have identical open-loop properties with respect to disturbances (this confirms the findings on ratios as inputs). Also, all configurations using D as one of the manipulated variables are identical in this respect, at least at steady state.

Table 3 can be used to explain results from the literature in a simple way. Take for example the simulation study by McCune and Gallier (1973). They found the DV configuration to be better than the LV configuration for rejecting disturbances in the subcooling of the reflux (their controller for the LV configuration is obviously very poorly tuned, but this issue is beyond the scope of the paper). Note that increased subcooling corresponds to a simultaneous decrease in V_T and increase in L (the cold reflux condenses some of the vapor in the top of the column). Disturbances in V_T are taken care of by the pressure control system. However, the disturbance in L will affect the two control systems entirely differently. For the DV configuration, L is used to control the condenser holdup, and the disturbance in L is corrected by the level control system (giving the entry 0 in Table 3). For the LV configuration, no adjustment is made and the steady-state effect on D/B is $-F/B^2dL$, as shown in Table 3.

McCune and Gallier also found similar differences for a disturbance in feed enthalpy q_F; this again follows directly from Table 3. For a disturbance in feed rate, we see from Table 3 that the effect on D/B is equal for the two configurations if the feed is liquid $q_F = 1$. This also agrees with the simulation results of McCune and Gallier.

Summary. It is preferable to use the level control system to reject flow disturbances. V_T is usually used for pressure control, and disturbances in condenser duty are rejected perfectly (at least at steady state). However, no configuration can reject all flow disturbances using the level control system: The commonly used LV configuration does not reject disturbances in F, V, L, and q_F. Configurations using D or B as one of the manipulated variables for composition control are insensitive to disturbances in V, L, and q_F, but do not reject disturbances in F. (However, F is often measured and a feedforward control scheme may be used.) The $(L/D)(V/B)$ configuration is insensitive to disturbances in F and rejects other flow disturbances as well, provided the reflux is large.

One-point (manual) composition control

Very few distillation columns are actually operated with a two-point control system. In most cases one of the compositions is controlled manually, at least part of the time. Since the operators do not monitor the compositions continually and manipulate

Table 3. Linearized Effect of Flow Disturbances on D/B When Both Composition Loops Are Open*

Configuration (u_1, u_2)	Disturbance, d			
	dF	$dV_d - dL_d - Fdq_F$	dD_d	dB_d
$LV, \frac{L}{V}V$	$k(1 - q_F - D/F)$	k	0	0
$\frac{L}{D}\frac{V}{B}$	0	$\frac{k}{1 + L/D + V/B}$	$\frac{kL/D}{1 + L/D + V/B}$	$\frac{-kV/B}{1 + L/D + V/B}$
$\frac{L}{D}V$	$-k\frac{V/F}{1 + L/D}$	$\frac{k}{1 + L/D}$	$\frac{kL/D}{1 + L/D}$	0
$L\frac{V}{B}$	$k\frac{L/F}{1 + V/B}$	$\frac{k}{1 + V/B}$	0	$-\frac{kV/B}{1 + V/B}$
DX	$-kD/F$	0	k	0
BX	kB/F	0	0	$-k$

$^*\left(\frac{\partial D/B}{\partial d}\right)_{u_1,u_2}$

It is assumed that V_T is not used for composition control and that disturbances in V_T are rejected by the pressure and level control system. Applies to steady state and constant molar flows.

For derivation of table, see Appendix.

q_F, fraction of liquid in feed

X, any other manipulated input (L, V, L/D, etc.) except D, B, and D/B

$k = (1 + D/B)/B = F/B^2$

Subscript d denotes an additive disturbance on this flow

the inputs accordingly, it is important that the effect of expected disturbances on the manually controlled ("uncontrolled") composition is as small as possible.

Both Composition Loops Open. This issue was discussed above for the case of flow disturbances, and the $(L/D)(V/B)$ configuration was found to give good disturbance rejection. However, a feed composition z_F disturbance has no direct effect on the flows at least for columns with constant molar flows. Consequently, if both composition loops are open, the effect of a feed composition disturbance will be the same for all configurations. Furthermore, the effect will usually be large because a change in feed composition requires a change in D/B (Eq. 22), and if this correction is not made, large changes in y_D and x_B will result for high-purity separations. As an example, assume that initially $z_F = 0.5$, $x_B = 1 - y_D = 0.01$ and $D/B = 0.5$. A feed composition disturbance results in $z_F = 0.6$, but $D/B = 0.5$ remains constant. Then, according to Eq. 22, x_B has to increase at least to $x_B = 0.20$ (corresponding to $y_D = 1.0$). This is clearly not acceptable. Therefore, at least one of the compositions has to be controlled carefully, either by a feedback controller or by the operator.

One-point Composition Control (One Composition Loop Open). Assume that we have closed one loop, and are using u_2 to control y_2. The output y_1 is not controlled and the manipulated input u_1 is constant. What is the effect of a disturbance d on the uncontrolled output y_1? First consider the steady state, where we have perfect control of y_2. The disturbance d has the effect $[g_{1d} \quad g_{2d}]$ on the outputs when the inputs u_1 and u_2 are constant. Using deviation variables we have

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = G \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} g_{1d} \\ g_{2d} \end{bmatrix} d \tag{32}$$

Solving for $y_2 = 0$ and $u_1 = 0$ gives

$$\frac{y_1}{d} = -\frac{g_{12}}{g_{22}} g_{2d} + g_{1d} \tag{33}$$

Consequently, the disturbance will not affect the uncontrolled output y_1 if

$$\frac{g_{12}}{g_{22}} = \frac{g_{1d}}{g_{2d}} \tag{34}$$

This result should be obvious. If the disturbance has the same relative effect (g_{1d}/g_{2d}) on the outputs as input $u_2 (g_{12}/g_{22})$, then we can get perfect disturbance rejection by using only this input.

Example. Consider again the column in Table 2. For a feed composition z_F disturbance all configurations have $g_{1d}/g_{2d} = 0.787$. The ratio g_{1i}/g_{2i} to g_{1d}/g_{2d} (denoted r_i) is given in Table 4 for various configurations. If this ratio is close to one, then perfect disturbance rejection is achieved with manipulated input i alone (the other input being constant) (see also Rademaker et al., 1975, p. 461). D (or B) should obviously never be held constant. Configurations that keep L or V constant come out favorably. For feed flow disturbances the same configurations are preferred. The reason is that in both cases the major effect of the disturbances may be counteracted by changing the product flow

Table 4. Effect of Feed Composition Disturbance ($d = z_f$) when One Composition Loop is in Manual for Column in Table 1

Configuration (u_1, u_2)	$r_1 = \frac{g_{11}/g_{21}}{g_{1d}/g_{2d}}$ (u_2 constant)	$r_2 = \frac{g_{12}/g_{22}}{g_{1d}/g_{2d}}$ (u_1 constant)
LV	1.03	1.00
$(L/V)(D/B)$	1.24	0.85
$(L/D)V$	1.03	0.85
$(L/D)D$	−1.27	0.85
DV	1.03	−1.27
LD	−1.27	1.00

If $r_1(r_2)$ is close to one, then good composition control is maintained over the "uncontrolled" composition $x_B(y_D)$ when $u_2(u_1)$ is constant.

rates (adjusting D/B to satisfy Eq. 22), which is easily accomplished using the LV configuration.

However, we have not considered the dynamic effects. For the LV configuration, flow disturbances are very poorly rejected by the level loops and large changes in the uncontrolled composition may occur. Assume that the top composition y_D is controlled with L, and x_B is left uncontrolled (i.e., V is constant). If the feed is liquid, a feed flow disturbance will reach the reboiler very fast and lead to a large change in bottom composition in a matter of minutes. Because in many cases it will take time before the feed flow disturbance is noticed in the top composition, and because of the time delay between a change in liquid flow L at the top and its effect on liquid flow in the bottom, the bottom composition may experience a large deviation before returning to its desired value. The $(L/D)(V/B)$ configuration may be preferable from a dynamic viewpoint since the level loops will counteract the feed flow disturbance directly (without having to wait for the compositions to change).

Summary. Operating both composition loops open is not acceptable because no correction can be made for feed composition disturbances. When one-point composition control is used, reasonably good control of the "uncontrolled" composition is maintained with most configurations, provided D or B are not kept constant. The LV configuration (keeping L or V constant) comes out favorably when only steady-state considerations are taken into account, but it may be preferable to use one of the ratio control schemes [e.g., $(L/V)(V/B)$] in order to obtain better dynamic rejection of flow disturbances. One advantage of controlling only one composition is that tuning is simple and very tight control can be maintained for this composition.

Changes between manual and automatic control

Changing one of the composition loops between manual and automatic control is frequently done when controlling distillation columns, for example, due to stability problems, or constraints or failures in measurements or actuators. It is clearly desirable to be able to do this without upsetting the rest of the system or having to retune the controllers.

From its definition, we might expect the RGA to give a reliable measure of how the system is affected by changing loops from manual to automatic. Each element in the RGA is defined as the open-loop gain (all the other loops in manual) divided by the gain between the same two variables when all the other loops are under "perfect" control (in automatic) (Bristol, 1966). For example, for 2 × 2 plants

$$\lambda_{11} = \frac{(\partial y_1/\partial u_1)_{u_2}}{(\partial y_1/\partial u_1)_{y_2}} = \frac{\text{Gain all other loops open}}{\text{Gain all other loops closed}} \quad (35)$$

However, the RGA is actually of very limited usefulness in this respect because it does not take into account the effect of disturbances, as illustrated below for the DV and LV configurations.

DV Configuration. Assume that a decentralized control system is used (D controls y_D, and V controls x_B). This control system will provide acceptable control of both compositions in many cases. However, if the loop involving D is put in manual (i.e., D is constant) then the material balance is locked and the response of y_D will be very poor when there are disturbances in the feed conditions (Rademaker et al., 1975, p. 461; Ryskamp, 1980). This was discussed in the preceding section, Table 4, and is even more transparent from the following exact expression

$$\left[\frac{\partial y_{DH}/y_{DH}}{\partial F/F}\right]_{D,x_B} = -\frac{z_F - x_B}{y_{DH}}\frac{F}{D} \quad (36)$$

Here $y_{DH} = 1 - y_D$ represents the mole fraction of heavy component in D. The relative change in y_{DH} is seen to be extremely sensitive to changes in F if the distillate is of high purity ($y_{DH} \rightarrow 0$).

LV Configuration. A decentralized control system may in some cases give reasonable control of both compositions when there are feed disturbances, as discussed previously. Furthermore, if the loop involving L or V is put in manual, we still get reasonably good control of the uncontrolled composition.

In general, the LV configuration yields large RGA elements while the DV configuration yields small elements (for the column in Table 1, λ_{11} is 35.1 and 0.45 for the two cases). Yet, when the loop involving y_D is put in manual, the response of this uncontrolled composition is still acceptable for the LV configuration, but poor for the DV configuration. The RGA is therefore not a reliable indicator of changes in performance when changes from automatic to manual are made.

Summary. Configurations that use D or B may give very poor response for the uncontrolled composition when the loop involving D or B is put in manual. (This is the opposite of what one might expect from the RGA, since one can always choose pairings such that $0.5 < \lambda_{11} < 1$ in this case.) The LV and $(L/D)(V/B)$ configurations, which are preferable for one-point composition control, are also most easily changed between manual and automatic (although the response for the controlled composition may deteriorate when the other loop is closed).

Constraints

Avoiding Constraints. Constraints on flow rates or on holdups (level and pressure) may also be important when choosing the best configuration. Whenever a manipulated input hits a constraint, it is no longer useful for control purposes. Since level and pressure control always has to be maintained, this means that one of the product compositions can no longer be controlled. If a constraint on a flow used for composition control is reached and two-point composition control is still maintained, then the constraint is akin to an input uncertainty. Therefore, constraints are an additional reason for not using controllers with large RGA elements (for example, avoid a decoupler for the LV configuration).

Flows used for level control will usually have the largest variations in magnitude, and are most likely to hit constraint. This leads to the following conclusions.

- A flow that may easily reach its constraint should not be used to control holdup. In particular this statement will generally imply the following:
- A very small flow should not be used to control level. One example documented in the literature (McNeill and Sacks, 1969) is the use of distillate D to control M_D in a high reflux column with $L/D = 70$. This is clearly next to impossible. Any imbalance in the large flows L and V_T will result in wild variations in D, and because of constraints on D the reflux drum is likely to run empty or to overflow.

The possibility of meeting constraints makes it necessary to have some override control system (e.g., the operator) that is able to identify constraints and change the control configuration if the constrained flow rate is used for inventory control.

Operating at Constraints. Many industrial columns are operated at their capacity limit, usually with respect to the boilup V, the reflux L, or the condensation rate V_T. This is another reason why many columns are operated with only one composition being controlled. Fortunately, as pointed out in the discussion of one-point composition control, keeping L, V, or V_T constant will also result in reasonably small variations in the uncontrolled product, at least at steady state. Since the active constraint may vary with operating conditions, an override control system is also needed in this case.

Level control

This paper has dealt with the choice of manipulated variables for composition control and we have only briefly discussed how the level loops should be paired. Nevertheless, considerations regarding the level loops restrict in most cases the possible options for composition control (McCune and Gallier, 1973).

For example, it is generally not desirable to use reflux L or distillate flow D to control reboiler level M_B because of the effective delay of liquid flow from the top to the bottom of the column; this may exclude, for example, the VB configuration as a viable option [but it does not exclude, e.g., the $D(V/B)$-configuration.] Also, as mentioned previously, some columns have an inverse response in the response of reboiler level to changes in V. Therefore, except for cases where the bottoms flow rate is extremely small, most installations control reboiler holdup M_B by manipulating bottoms flow B (Shinskey, 1984).

It is not recommended to control condenser level M_D by manipulating bottoms flow rate B (McCune and Gallier, 1973). This follows since changing B has no direct effect on M_D; a change in B results in a change in M_B, which subsequently, through the action of the bottom level loop, will result in a change in flow that affects the condenser level.

Choice of control configuration: Conclusion

The $(L/D)(V/B)$ configuration comes out very favorably when all the points mentioned above are considered as a whole. This is also in accordance with the recommendation given by Shinskey (1984), and our analysis provides added justification for his claim. The main exception is very high purity columns or columns with low reflux (i.e., large relative volatility), which may result in large elements in the RGA and give a system that is sensitive to input uncertainty and flow disturbances. For these columns a configuration using D or B for composition control should be considered [e.g., the $D(V/B)$ configuration]. These configurations have all RGA elements less than one and are always insensitive to input uncertainty.

Conclusions

The primary goal of this paper has been to present in a systematic manner the main issues that must be addressed when designing a composition control system. In order to avoid excessive length, a number of important issues have been addressed only qualitatively. More quantitative results on specific issues will follow. These include relationships for computing steady-state gains for various configurations, and simple dynamic models. Nevertheless, it is clear that a number of generally conflicting considerations have to be taken into account.

Two-Point Composition Control. The RGA is a useful tool for addressing the issue of input uncertainty. Configurations with large values of λ_{11} should be avoided. For distillation columns all material balance configurations (using D or B) have $\lambda_{11} < 1$. However, these configurations often result in a poor dynamic response and give very poor disturbance rejection if the loop involving D or B is taken out of service. This is probably the reason that Shinskey (1984) recommends avoiding configurations with $\lambda_{11} < 1$ (provided λ_{11} is not too large). (These considerations only hold for distillation column control; for other processes there is no reason to try to avoid $\lambda_{11} < 1$.)

One-point Composition Control (One Loop in Manual). Most industrial columns have closed-loop control of only one composition. This may seem suboptimal, but is in many cases reasonable, since one product is usually much more important than the other. Furthermore, if the column is operating at its capacity limit (which is often the case), it is impossible to control more than one composition. Uncertainty does not pose any particular problem when only one composition is controlled. Reasonably good control of the uncontrolled composition is maintained provided D or B is not kept constant. The LV and $(L/D)(V/B)$ configurations will generally both perform satisfactory. The $(L/D)(V/B)$ configuration is preferable because it has a better built-in rejection of flow disturbances, which leads to less variations in the uncontrolled composition. One case in which it may be worthwhile to use D or B as the manipulated input for one-point composition control is for columns with very large reflux ($L/D \gg 1$ or $V/B \gg 1$) where level control using D or B may be almost impossible.

Acknowledgment

Support from the National Science Foundation and Norsk Hydro is gratefully acknowledged.

Appendix: Derivation of Table 3

Assuming constant molar flows, the following exact steady-state relationships apply

$$dD = (1 - q_F)dF - Fdq_F + dV - dL \tag{A1}$$

$$dD = dF - dB \tag{A2}$$

Furthermore

$$d(D/B) = \frac{1}{B}dD - \frac{D}{B^2}dB \tag{A3}$$

combining Eqs. A2 and A3:

$$d(D/B) = kdD - kD/FdF, \quad k = F/B^2 \tag{A4}$$

We consider disturbances in F and q_F. In addition, each manipulated flow may have an additive disturbance. For example, the distillate flow D can be expressed

$$D = D_d + D_m \tag{A5}$$

Here D_m represents the "manipulated" part of the distillate (which is what D is "believed" to be), while D_d represents the disturbance. We want to find the effect of the disturbances on

D/B when the composition loops are open, that is,

$$du_{1m} = du_{2m} = 0 \quad \text{(A6)}$$

To derive Table 3 these equations are combined to express $d(D/B)$ in Eq. A4 as a function of the disturbances only (i.e., express dD as a function of dD_d, dV_d, dL_d, dF, etc.).

Example: LV configuration

With

$$dL_m = dV_m = 0$$

Eq. A1 becomes

$$dD = (1 - q_F)dF - Fdq_F + dV_d - dL_d$$

which upon insertion in Eq. A4 yields

$$d(D/B) = k(1 - q_F F - D/F)dF + k(dV_d - dL_d - Fdq_F) \quad \text{(A7)}$$

Example: DX configuration

With

$$dD_m = 0$$

Eq. A5 becomes

$$d(D/B) = kdD_d - dD/FdF \quad \text{(A8)}$$

Example: (L/D)(V/B) configuration

With

$$dL_m = L/DdD_m \quad dV_m = V/BdB_m$$

Eq. A1 becomes

$$dD = (1 - q_F)dF - Fdq_F + dV_d - dL_d + V/BdB_m - L/DdD_m \quad \text{(A9)}$$

Here

$$dD_m = dD - dD_d$$
$$dB_m = dB - dB_d = dF - dD - dB_d$$

which upon insertion in Eqs. A9 yields

$$dD\left(1 + \frac{L}{D} + \frac{V}{B}\right) = (1 - q_F)dF - Fdq_F + dV_d - dL_d + \frac{V}{B}dF - \frac{V}{B}dB_d + \frac{L}{D}dD_d$$

The term involving dF drops out when this is substituted into Eq. A4 [using $(1 - q_F)F + V = D + L$], and we derive the expression given in Table 3:

$$d\left(\frac{D}{B}\right)\left(1 + \frac{L}{D} + \frac{V}{B}\right) = k\left(dV_d - dL_d - Fdq_F + \frac{L}{D}dD_d - \frac{V}{B}dB_d\right) \quad \text{(A10)}$$

The fact that a change in F does not affect D/B when L/D and V/B are constant is expected, since a feed flow change is counteracted by keeping all flow ratios constant.

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Manuscript received Jan. 21, 1987, and revision received May 6, 1987.