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CONTROL OF ILL-CONDITIONED PLANTS

HIGH-PURITY DISTILLATION

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<u>Purpose</u>: Study general properties of ill-conditioned plants

<u>Not</u>: Distillation column control

ILL-CONDITIONED PLANT ($\gamma(G) \gg 1$)

The plant-gain is strongly dependent on input direction :

$$\bar{\sigma}(G) = \max_{m \neq 0} \frac{\|Gm\|_2}{\|m\|_2} \quad \text{"maximum gain"}$$

$$\underline{\sigma}(G) = \min_{m \neq 0} \frac{\|Gm\|_2}{\|m\|_2} \quad \text{"minimum gain"}$$

$$\gamma(G) = \frac{\bar{\sigma}(G)}{\underline{\sigma}(G)}$$

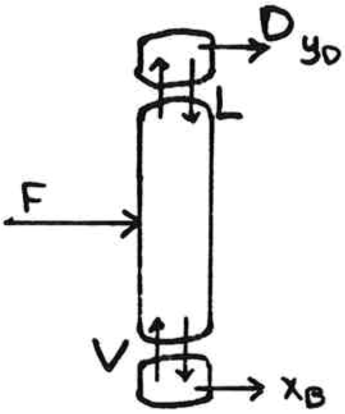
SVD : More specific information

$$G = U \Sigma V^H \quad , \quad \Sigma = \begin{pmatrix} \bar{\sigma}(G) & & \\ & \ddots & \\ & & \underline{\sigma}(G) \end{pmatrix}$$

Most effective input direction → $G\bar{v} = \bar{\sigma}(G)\,\bar{u}$ ← Most easily affected output direction

Least effective → $G\underline{v} = \underline{\sigma}(G)\,\underline{u}$ ← Least easily affected

Example: HIGH-PURITY DISTILLATION



$y_D = 0.99$
$x_B = 0.01$
$N = 40$
$\alpha = 1.5$
$z_F = 0.5$
$L/D = 5.4$

LV- configuration

$$\begin{pmatrix} \Delta y_D \\ \Delta x_B \end{pmatrix} = \overset{G_{LV}}{\begin{pmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{pmatrix}} \begin{pmatrix} \Delta L \\ \Delta V \end{pmatrix}$$

$$\gamma(G_{LV}) = 141.7$$

DV- configuration

$$\begin{pmatrix} \Delta y_D \\ \Delta x_B \end{pmatrix} = \overset{G_{DV}}{\begin{pmatrix} -0.878 & 0.014 \\ -1.082 & -0.014 \end{pmatrix}} \begin{pmatrix} \Delta D \\ \Delta V \end{pmatrix}$$

$$\gamma(G_{DV}) = 70.8$$

PHYSICAL INTERPRETATION OF SVD

$$G = U \Sigma V^H$$

LV-configuration:

$$\begin{pmatrix}\Delta y_D\\ \Delta x_B\end{pmatrix} = \begin{pmatrix}0.625 & 0.781\\ 0.781 & -0.625\end{pmatrix}\begin{pmatrix}1.972 & 0\\ 0 & 0.0139\end{pmatrix}\begin{pmatrix}0.707 & 0.708\\ -0.708 & 0.707\end{pmatrix}^H\begin{pmatrix}\Delta L\\ \Delta V\end{pmatrix}$$

$\bar{u}$ $\underline{u}$ $\bar{\sigma}$ $\underline{\sigma}$ $\bar{v}$ $\underline{v}$

output directions Input directions

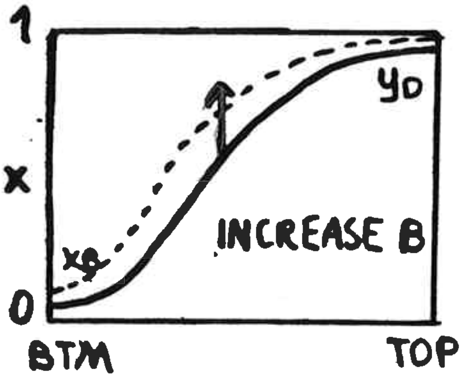
Look at max. gain direction

Input $\bar{v}$: $\Delta L \approx -\Delta V$

Physically: Maximizes changes in external flows $(\Delta B = -\Delta D = \Delta L - \Delta V)$

Output $\bar{u}$: (effect of $\bar{v}$)

Increase both compositions



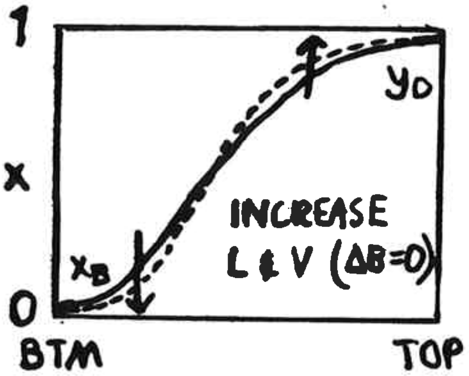
Look at min. gain direction

Input $\underline{v}$: $\Delta L \approx \Delta V$

Physically: Increase internal flows only $(\Delta B = \Delta D \approx 0)$

Output $\underline{u}$:

Increase y_D & decrease x_B ("both purer")



DV-configuration: Same conclusion

ARE ILL-CONDITIONED PLANTS INHERENTLY DIFFICULT TO CONTROL ?

NO. Depends of type of

1) disturbance
2) uncertainty
3) controller

BUT

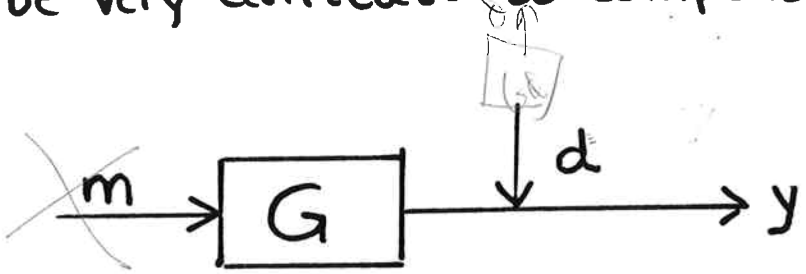
In general: Controller design difficult

(Look at each of these
three points

1) DISTURBANCE DIRECTIONS

Generally for

1 Ill-conditioned plants: SOME disturbances may be very difficult to compensate.



d : effect of disturbance on the output

$d = \underline{u}$: hard to control, $d = \bar{u}$: easy to control

↖ Recall $\underline{u}$: Direction of the output which is least easily effected

Disturbance condition number

$$\boxed{\gamma_d(G) = \frac{\|G^{-1}d\|_2}{\|d\|_2}\,\bar{\sigma}(G)} \qquad 1 \le \gamma_d(G) \le \gamma(G)$$

Distillation example:

"disturbance" (d)	$\gamma_d(G)$ LV-	DV-configuration
$d = \underline{u}$	141.7	70.8
→ Setpoint change in y_D	110.5	54.9
— " — x_B	88.5	44.6
Feed rate (F)	11.8	6.4
Feed composition (z_F)	1.5	1.5
$d = \bar{u}$	1.0	1.0

Implication: Control not that difficult if we do not care about setpoint changes

2) MODEL UNCERTAINTY

ILL-conditioned plants: SOMETIMES performance is greatly affected by model uncertainty.

For tight control:

- Apply LARGE input in direction with SMALL plant gain.
- Apply SMALL input — " — LARGE plant gain.

HOWEVER, Uncertainty may cause

- LARGE input in direction with LARGE plant gain.

⇛ Poor performance (or even instability)

INPUT UNCERTAINTY

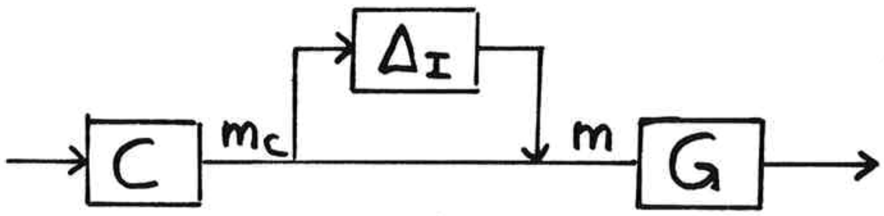
(uncertainty on manipulated flows):

Example: Desired: reflux L: 100 → 110 $(\Delta L = 10)$
Actual: reflux L: 100 → 111 $(\Delta L = 11)$

Uncertainty wrt. change of this input: 10%

IMPORTANT: This source of uncertainty is ALWAYS present.

Mathematically:



$$\Delta_I = \begin{pmatrix} \Delta_1 & & & 0 \\ & \Delta_2 & & \\ & & \ddots & \\ 0 & & & \Delta_n \end{pmatrix}$$

$$m_i = (1 + \Delta_i)\, m_{ci}$$

Δ_i = relative uncertainty on input i

Should always consider this first!

Example : DISTILLATION COLUMN

INVERSE-BASED controller : $C = \frac{0.7}{s} G^{-1}$

Both configurations : Decoupled nominal response :

$$y_D = \frac{1}{1.4s+1} y_{DS} \quad , \quad x_B = \frac{1}{1.4s+1} x_{BS}$$

Inverse-based controller

$\Rightarrow$ Apply large inputs in directions with small plant gain

$\Rightarrow$ Apply large changes in internal flows ($\Delta D = -\Delta B \approx 0$)

Lets try to find out what will happen when there is.

INPUT UNCERTAINTY

LV-configuration :
$$\Delta L = (1+\Delta_1)\Delta L_c \quad , \quad \Delta_1 = 0.2$$
$$\Delta V = (1+\Delta_2)\Delta V_c \quad , \quad \Delta_2 = -0.2$$

Difficult to apply large ΔL and ΔV without changing ΔB :

Assume $\Delta L_c = \Delta V_c \Rightarrow \Delta B = \Delta L - \Delta V = \underbrace{(\Delta_1 - \Delta_2)}_{0.4} \Delta L_c$

DV-configuration :
$$\Delta D = (1+\Delta_1)\Delta D_c \quad , \quad \Delta_1 = 0.2$$
$$\Delta V = (1+\Delta_2)\Delta V_c \quad , \quad \Delta_2 = -0.2$$

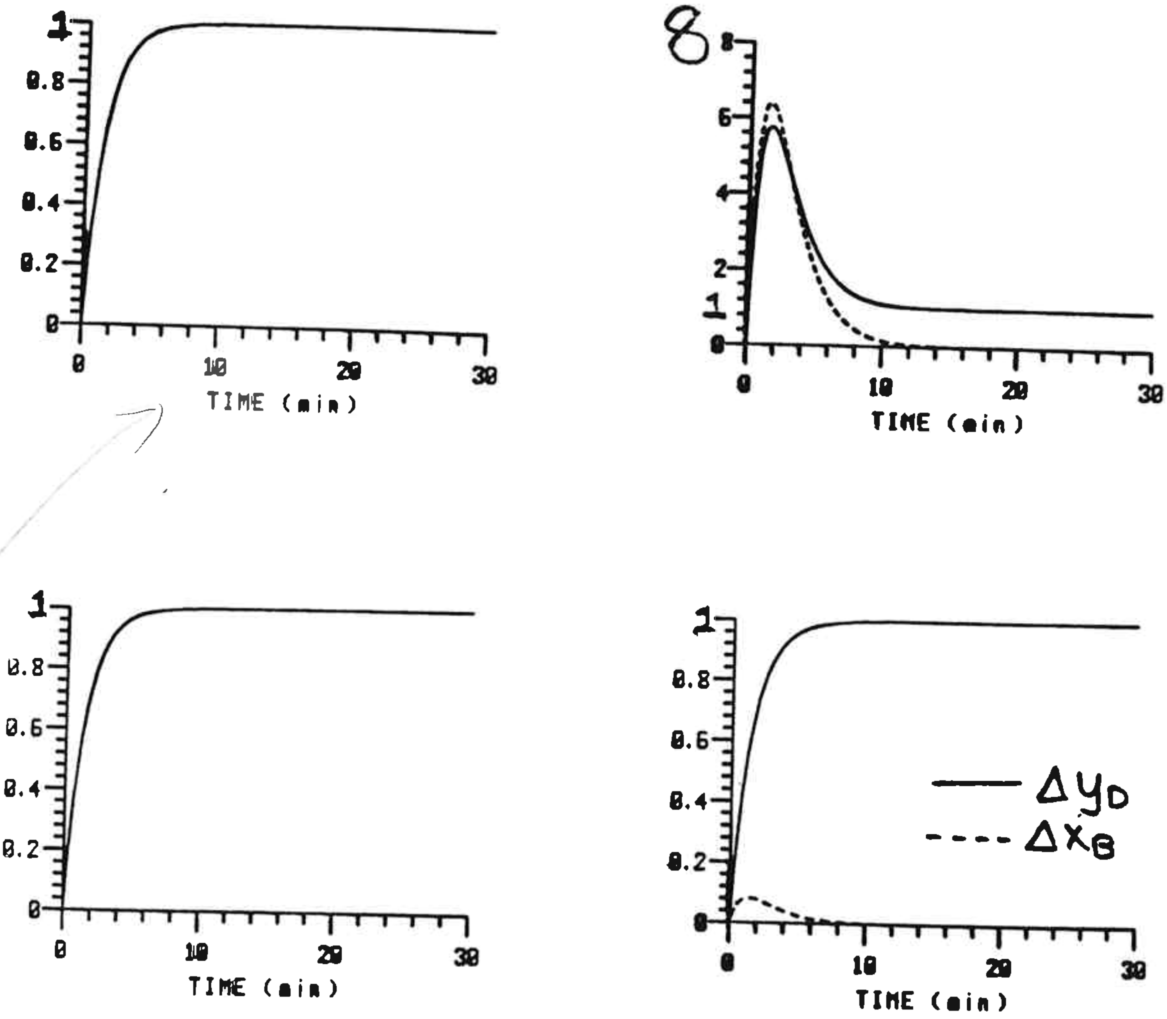
Easy to apply large ΔV (and ΔL) without changing $\Delta B = -\Delta D$.

- STEP-CHANGE IN $\underline{y_D}$.
- INVERSE-BASED CONTROLLERS.

$\Delta_1 = \Delta_2 = 0$ $\qquad$ $\Delta_1 = -\Delta_2 = 0.2$

LV-conf.
$\gamma = 141.7$
$\lambda_{11} = 35.1$

DV-conf.
$\gamma = 70.5$
$\lambda_{11} = 0.45$



- Performance MAY be greatly affeckd by uncertainty

- RGA (λ_{11}) is an excellent indicator of sensitivity to <u>input</u> uncertainty.

Two ill-cond. plants → One cons
→ One nat.

Nonlinear : Works OK

Nonlinear sim:
Very misleading

3) CONTROLLER STRUCTURE

Ill-conditioned plants: Sensitivity to uncertainty depends on controller structure.

LV-configuration :

INVERSE-based controller: VERY SENSITIVE

DIAGONAL controller : INSENSITIVE
BUT even nominal response may be bad for particular setpoints/disturbances.

Simulation with DIAGONAL controller

$$C = \frac{2.4}{s} I$$

Nominally: Time constant:

- Disturbance in "good" direction ($\bar{u}$) : $= \frac{1}{2.4 \cdot \bar{\sigma}(G)} = 0.2\,\text{min}$
- Disturbance in "bad" direction ($\underline{u}$): $= \frac{1}{2.4 \cdot \underline{\sigma}(G)} = 30\,\text{min}$
- Others : Linear combination of these extremes

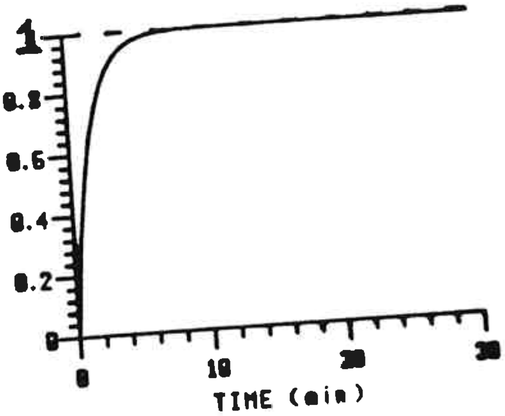
- LV-configuration
- Step-change in yo

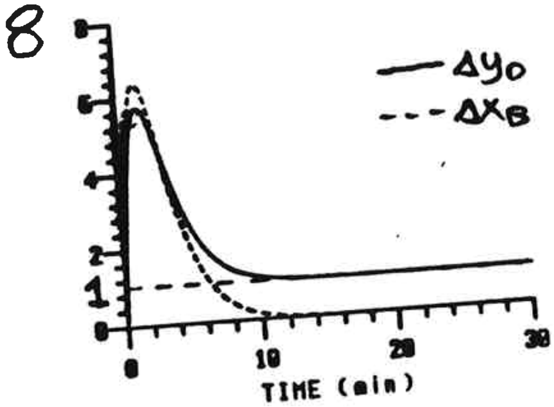
$\Delta_1 = \Delta_2 = 0$

$\Delta_1 = -\Delta_2 = 0.2$

NVERSE-
controller

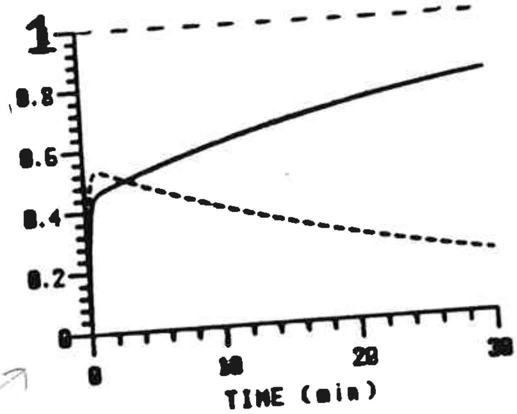
$C = \frac{0.7}{s} G^{-1}$

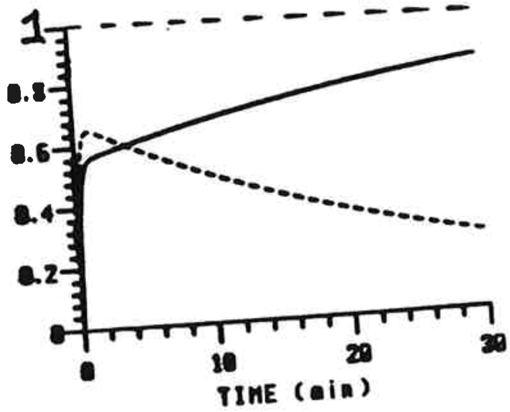




DIAGONAL
controller

$C = \frac{2.4}{s} I$





- Diagonal controller insensitive to input uncertaint
but (in this case) response is sluggish.

Fast initially
then sluggish

SIMULATIONS ~~SHOW~~ ILLUSTRATE :

Ill-conditioned plants CAN cause severe control problems
Depends on

- disturbances
- uncertainty
- controller structure

Available tools: $\gamma(G)$, $\gamma_d(G)$, RGA

In general: Conflicting, simplistic answers

Need more powerful tool:

μ: The STRUCTURED SINGULAR VALUE

(Doyle, 1982)

μ- analysis :

Given
- controller
- disturbances
- uncertainties

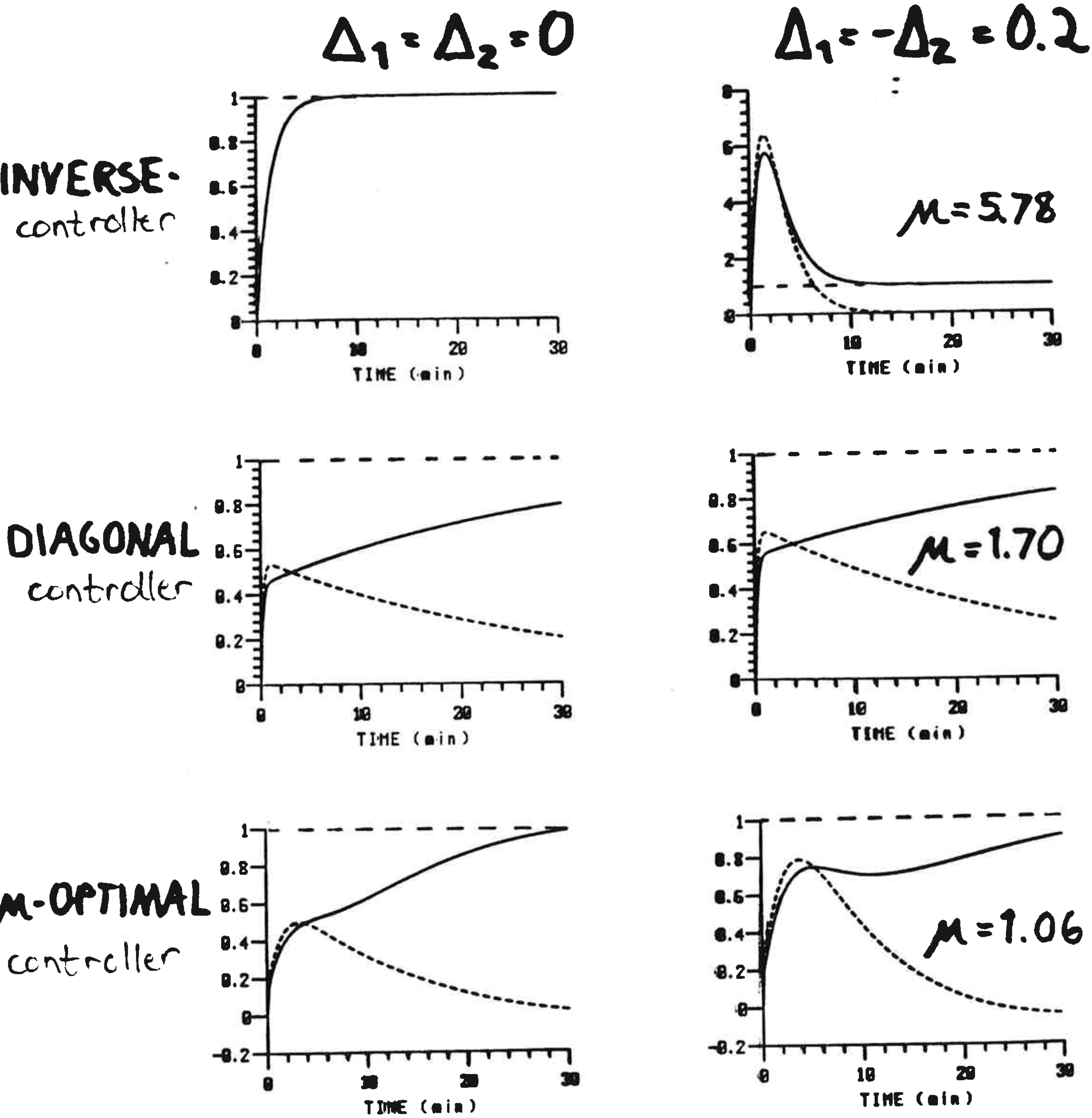
μ finds "worst-case" response
(Hopeless to find by trial & error simulation)

$\mu < 1$: Acceptable

μ-synthesis :

Find <u>controller</u> which makes "worst-case" as good as possible.

μ should be as small as possible



$\Delta_1 = \Delta_2 = 0$
$\Delta_1 = -\Delta_2 = 0.2$
INVERSE-controller
DIAGONAL controller
μ-OPTIMAL controller
μ=5.78
μ=1.70
μ=1.06
1
0.8
0.6
0.4
0.2
0
-0.2
8
6
4
2
0
10
20
30
TIME (min)

CONCLUSIONS

ILL-CONDITIONED PLANTS

- Inverse-based controller <u>often</u> very sensitive to input uncertainty (nominal response is great)

  Exception: Plant "decoupled at the input" (DV-distillation).

- Diagonal controllers <u>not</u> sensitive to uncertainty, but even nominal response ($\Delta=0$) may be poor.

EFFECT OF UNCERTAINTY

- μ finds "worst case"
- μ-optimal: Optimizes "worst case"

DISTILLATION

- Use input unc. in simulations