# CONTROLLABILITY ANALYSIS and PLANTWIDE CONTROL

**Sigurd Skogestad**

Department of Chemical Engineering

Norwegian University of Science and Technology

N-7034 Trondheim, Norway

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# OUTLINE

1. Introduction
2. What is Plantwide control and Controllability analysis?
3. Plantwide control - control structure design
4. Some tools – controllability analysis
5. An outline of a procedure
6. Some examples
   - Tennessee Eastman plantwide challenge problem
   - Reactor with recycle (“snowballing” ??)
   - Buffer tanks for pH-control
   - Distillation column control
   - Optimizing control of Petlyuk distillation
7. Conclusion



The Trondheim **Plantwide control group** :

- **Audun Faanes** (pH-neutralization; control structure design)
- **Marius Govatsmark** (Refinery; HDA plant)
- **Ivar Halvorsen** (Petlyuk distillation; self-optimizing control)
- **Truls Larsson** (methanol plants; control structure design)
- **Tore Lid** (Refinery)
- **Sigurd Skogestad**

associated (earlier) members

- John Morud (Sintef Kjemi)
- Kjetil Havre (ABB)
- Bjørn Glemmestad (Borealis)

plus

- Morten Hovd, *Inst. for teknisk kybertenikk*
- Stig Strand, *Statoil*

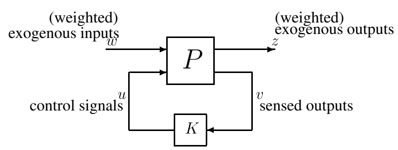


Written material:

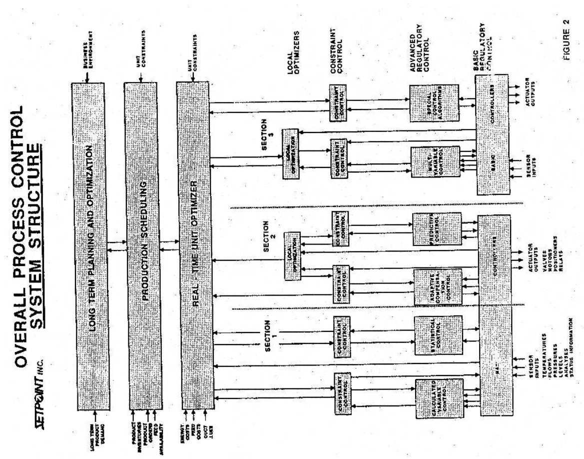
- *Multivariable Feedback Control*, S. Skogestad and I. Postlethwaite, Wiley, 1996
  - Chapter 5: Limations on performance in SISO systems (controllability)
  - Chapter 6: Limations on performance in MIMO systems (more controllability)
  - Chapter 10: Control structure design (plantwide control)
- *A review of plantwide control*, S. Skogestad and T. Larsson, (Internal report, Aug. 1998). Available at:
  `http://www.chembio.ntnu.no/users/skoge/publications/1998/plantwide review1.pdf`
- Ph.D. thesises of Morud (1995), Glemmestad (1997) and Havre (1998).
- *Plantwide control: The search for the self-optimizing control structure*, S. Skogestad (prepared for publication in J.Proc.Control). Avaialable at:
  `http://www.chembio.ntnu.no/users/skoge/publications/1999/self1.pdf`

# EXISTING CONTROL THEORY

**General controller design formulation (Ph.D.level)**

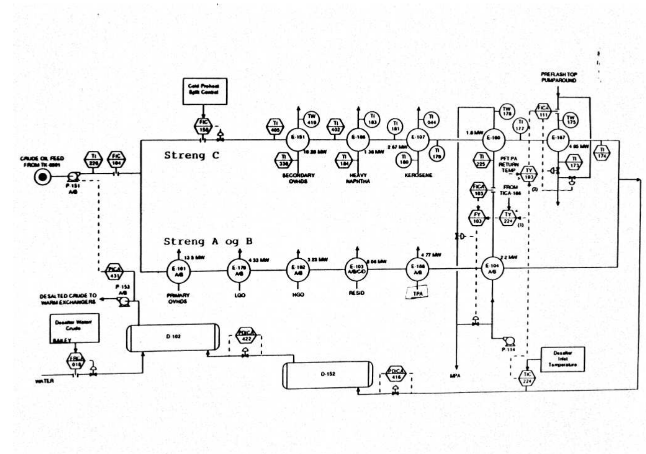



- $w$: Disturbances ($d$) and setpoints ($r$)
- $v$: Measurements ($y_m, d_m$) and setpoints ($r$)
- $u$: Manipulated inputs ($u$)
- $z$: Control error, $y - r$
- Find a controller $K$ which based on the information in $v$, generates a control signal $u$ which counteracts the influence of $w$ on $z$, thereby minimizing the closed-loop norm from $w$ to $z$.




PRACTICE
Typical control hierarchy






PRACTICE
Typical base level control structure

**Relevant questions in practice**

1. How should the plant be controlled (*plantwide control*) ?
2. Is the plant easy or difficult to control? (*controllability analysis*)?

**Controllability analysis**
(Is the plant easy or difficult to control?)

- Judge control properties without a detailed control system design
- Useful at the early DESIGN stage (compare designs)
- Useful for evaluating alternative control structures for plantwide control

*Need:* A dynamic model of the plant (from which we can obtain a linearized model)
Most of the tools: Based on a linear model

**Plantwide control**

- The strategy and STRUCTURE of the control system
- Which boxes should we have and which signals should be send between them?
- Useful at the later design stage (make P&ID)
- Useful for re-evaluation of control system at existing plant (Common!)



**Relationship between controllability analysis and planwtide control**

- **Plantwide control:** Controllability analysus is used for a quick evaluation of alternative control structures
- **Controllability analysis at early design stage:** Not done on the entire plant; must assume something about the plantwide control structre
- **Both controllability analysis and plantwide control**
  Two basic approaches:
  1. Systematic based on dynamic process model (theoretical)
  2. Process-oriented (insights)



## PLANTWIDE CONTROL

**The *control philosphy* for the overall plant with emphasis on the *structural decisions*:**

- Which "boxes" (controllers; decision makers) do we have and what information (signals) are send between them

**NOT:**

- The inside of the boxes (design and tuning of all the controllers)

The most important sub-problem: CONTROL STRUCTURE DESIGN



Alan Foss ("Critique of chemical process control theory", AIChE Journal, 1973):

*The central issue to be resolved ... is the determination of control system structure.*

*Which variables should be measured, which inputs should be manipulated and which links should be made between the two sets?*

*The gap is present indeed, but contrary to the views of many, it is the theoretician who must close it.*

# CONTROL STRUCTURE DESIGN

**Tasks:**

1. *Selection of controlled outputs* (a set of variables which are to be controlled to achieve a set of specific objectives)
2. *Selection of manipulations and measurements* (sets of variables which can be manipulated and measured for control purposes)
3. *Selection of* **control configuration** (a structure interconnecting measurements/commands and manipulated variables)
4. *Selection of controller type* (control law specification, e.g., PID, decoupler, LQG, etc.).

Note distinction between control *structure* (all tasks) and *configuration* (task 3).

Tasks 1 and 2 combined: **input/output selection**
Task 3 (configuration): **input/output pairing**

Shinskey (1967, 1988);Morari (1982); Stephanopoulos (1984); Balchen and Mumme (1988)

Nett (1989); van de Wal and de Jager (1995); Skogestad and Postlethwaite (1996)

**Approach Control structure design:**



- Top-down consideration of control objectives and available degrees of freedom to meet these (tasks 1 and 2)
- Bottom-up design of the control system, starting with stabilization (tasks 3, 4, 5).



# TASK 1: Selection of controlled outputs

*Controlled output $c$:* Measured output with reference ($r$, $c_s$)

Two distinct questions:

1. What should be the controlled variables $c$?
   (includes open-loop by selecting $c = u$)
2. What is their optimal values ($c_{\rm opt}$)?

**Second question:** A lot of theory.

**BUT First question:** Almost no theory. Decisions mostly made on experience and intuition.



- Why do we in a plant control a lot of internal variables, like temperatures, pressures and compositions, for which there are no control requirements?

ANSWER:

- We have degrees of freedom that need to be specified to achieve optimal operation
- But why do we select a particular set of controlled variables?

Example: A distillation column “inside” the plant. Why control the composition rather than specifying directly the reflux flow?

ANSWER:

- To reduce the sensitivity to uncertainty and achieve self-optimizing control

# Example 1: Cake baking

Given: Bake cake for 15 minutes

*Goal (purpose):* Well-baked inside and nice outside

Manipulated input (degree of freedom): Heat input $u = Q$

*Implementation*:

1. Open-loop implementation: Heat input $Q$
   - Problem: Sensitive to uncertainty and optimal value depends strongly on size of oven

2. Closed-loop implementation:
   $y =$ oven temperature
   "Optimizer": Cook book (look-up table)
   Used in practice. Insensitive to changes.



# Example 2: Distillation column

*Goal (purpose):* Operate the column such that overall operating costs of the entire plant are minimized (primarily determined by steady-state considerations)

*Degrees of freedom at steady state* (with given feed and pressure): 2
May for example be selected as

1. Split $D/F$ and reflux $L$

*Optimization* of the entire plant yields optimal value of these two variables
(along with the corresponding values for the other column operating variables, such as compositions, temperatures, etc.)

BUT how should the optimal solution be implemented?



There are many other possible pairs ($y$'s) which can be kept constant

2. Two product compositions $(x_D, x_B)$
3. Two temperatures $(T_{top}, T_{btm})$
4. Reflux and one temperature $(L, T_{top})$
5. Reflux ratio and one temperature $(L/D, T_{top})$
6. Two ratios (e.g. $L/D, V/B$)
7. Reflux and boilup $(L, V)$
8. ...

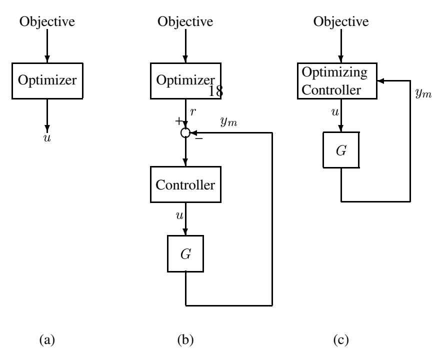
Question:

- Which of these pairs ($y$'s) should be kept at the given optimal setpoint?

Issues:

- Changes in operating point (disturbances)
- Accuracy of control (measurement noise)






(a) Open-loop optimization.
(b) HERE: Closed-loop implementation with separate control layer.
(c) Integrated optimization and control.

Hierarchical structuring:

- *optimization layer* — computes references $r$
- *control layer* — implements this in practice, $y \approx r$ $(c \approx c_s)$.

## SELF-OPTIMIZING CONTROL

- Find the set of controlled variables $c$, which when kept constant, result in close-to-optimal operation (in spite of disturbances and other changes)
- Steady-state analysis often sufficient



## Outline of the idea of self-optimizing control

- *Basis:* Have $N_u$ degrees of freedom at steady-state which should be used to optimize the operation
- *Assume:* Have an optimizer which for a given set of nominal disturbances computes all optimal values (of flows, temperatures etc.).
- *Question:* How should this optimal solution be included in practice? (Which $N_u$ variables should be specified?)
- *Initial Answer:* Does not matter (as long as they are independent)
- *But the real world is uncertain:*
  1. The data to the optimizer may be incorrect
  2. There are unknown disturbances entering during the time between each optimization
  3. The relative accuracy in the implementation ("signal to noise ratio") for each variable is different.

  Thus the initial answer is only true in a perfect world (with no uncertainty).



- *Better Answer:* Choose as specified variables (*controlled outputs*) the set which are least sensitive to uncertainty, i.e. which achieve self-optimizing control.
- *More precicely:* The selected set of variables $c$ should
  1. Have a small variation in optimal setpoints
  2. Be easy to control accurately (small implementation error)
  3. Should be sensitive to the manipulated inputs ($u$) (i.e. have a large range)
  4. Be independent (not closely correlated)

Today: Steady-state optimization is performed routinely

Thus: Have the main tools needed for a proper selection of controlled outputs:

- Steady-state model
- Information about operational constraints
- Well-defined economic objective (scalar cost function $J$ to be minimized)



## Procedure for selecting controlled outputs

**Step 1: Degree of freedom analysis.** Determine the number of degrees of freedom ($N_u$) available for optimization, and identify a base set ($u$) for the degrees of freedom.

**Step 2: Cost function and constraints.** Define the optimal operation problem by formulating a scalar cost function $J$ to be minimized for optimal operation, and specify the constraints that need to be satisfied.

**Step 3: Identify the most important disturbances (uncertainty).** These may be caused by

- Errors in the assumed (nominal) model used in the optimization (including the effect of incorrect values for the nominal disturbances $d_0$ used in the optimization)
- Disturbances $d - d_0$ (including parameter changes) that occur after the optimization
- Implementation errors ($d_c$) for the controlled variables $c$ (e.g. due to measurement error or poor control)

**Step 4: Optimization.**

Find the optimal steady-state operation for the various disturbances

**Step 5: Identify candidate controlled variables.** Typically, these are measured variables or simple combinations thereof. Look for variables which optimal setpoint is constant.

**Step 6: Evaluation of loss.** Compute the mean value of the loss for alternative sets of controlled variables $c$. This is done by evaluating the loss

$$L(u,d) = J(u,d) - J(u_{opt}(d),d); \quad u = f^{-1}(c_s + d_c, d) \tag{1}$$

with fixed setpoints $c_s$ for the defined disturbances $d \in \mathcal{D}$ and implementation errors $d_c \in \mathcal{D}_c$.

**Step 7: Further analysis and selection.**

- Select for further consideration the sets of controlled variables with acceptable loss.
- See if they are adequate with respect to other criteria that may be relevant, such like
  - region of feasibility
  - expected dynamic control performance (input-output controllability)

Let us next evaluate the losses. For this simple example the losses can be evaluated analytically, and we find for the three alternatives



$$L_1 = (10d_{c1})^2; \quad L_2 = (0.05d_{c2} - d)^2; \quad L_3 = (0.1d_{c3} - 0.5d)^2$$

(For example, for $c_3$ we have $u = (c_3 + 5d)/10$ and with $c_3 = c_{3s} + d_{c3} = d_{c3}$ we get $J = (u-d)^2 = (0.1d_{c3} + 0.5d - d)^2$). With $|d| = 1$ and $|d_c| = 1$ the worst-case values of the losses are

$$L_1 = 100; \quad L_2 = 1.05^2 = 1.1025; \quad L_3 = 0.6^2 = 0.36$$

and we find that *output $c_3$ is the best overall choice for self-optimizing control* (with the smallest loss), and $c_1$ is the worst. (This is the same conclusion that followed by considering the three requirements.) We see that with no implementation error ($d_c = 0$) $c_1$ would be the best, and with no disturbance ($d = 0$) $c_2$ would be the best.



#### Toy example

Let $J = (u-d)^2$ where nominally $d_0 = 0$. For this problem we always have $J_{opt}(d) = 0$ corresponding to $u_{opt}(d) = d$. Let is now consider three alternative choices for the controlled output (e.g. we can assume they are three alternative measurements)

$$c_1 = 0.1(u-d); \quad c_2 = 20u; \quad c_3 = 10u - 5d$$

For the nominal case with $d_0 = 0$ we have in all three cases that $c_{opt}(d_0) = 0$, so we select in all three cases $c_s = 0$. Since $u_{opt}(d) = d$, we have that the optimal value for the three alternative controlled outputs as a function of the disturbance are

$$c_{1opt}(d) = 0; \quad c_{2opt}(d) = 20d; \quad c_{3opt} = 5d$$

We assume that the variables have been scaled such that $|d| \le 1$ (disturbance) and $|d_{ci}| \le 1, i = 1,2,3$ (assume same magnitude of implementation error for all three variables).

Let us first evaluate how the three candidate variables meet the three requirements.

1. *Its optimal value is insensitive to disturbances.* From this point of view, the preferred controlled variable is $c_1$ (zero sensitivity), followed by $c_3$ (sensitivity 5) and $c_2$ (sensitivity 20).
2. *It is easy to control accurately.* There is no difference here since the implementation error $d_c$ is the same for the three variables.
3. *Its value is sensitive to changes in $u$.* This favors $c_2$ (gain 20), followed by $c_3$ (gain 10), whereas variable $c_1$ (gain 0.1) is very poor in this respect.

**Simple tool for selecting controlled outputs:**

- Let $c = Gu$ (linear model)



- Scale outputs $c$ (and $G$) such that $\|c - c_{\mathrm{opt}}(d)\| \approx 1$ (due to measurement errors and disturbances)
- Prefer a set of controlled outputs with large $\underline{\sigma}(G(0))$.

Note: $\bar{\sigma}(G^{-1}(0)) = 1/\underline{\sigma}(G(0))$.

## SUMMARY
## Rules for selecting controlled outputs $c$

Select the controlled outputs $c$ such that:

1. Optimal value $c_{\text{opt}}(d)$ is insensitive to disturbances (changes in the operating point)
2. Result insensitive to expected control error for $c$.
   (a) "Optimum is flat" and/or
   (b) Can achieve tight control of $c$ (need accurate measurement)
3. The outputs are weakly correlated

This is usually based on a steady-state analysis



## TASK 2: Selection of manipulations and measurements

Dynamics and *controllability* are more important here.

- Manipulations $u$ – usually fixed (the valves), but may not want to use all of them (see task 1) or may change their location.
- Measurements – may want to add secondary measurements $y_{2m}$ to
  1. Compensate for lack of measurements of primary output $y$
  2. Improve dynamic response – e.g. by adding a measurement of $y_2$ "close" to the manipulation $u$ the manipulation $u$

  Can perform **controllability analysis** of alternative combinations.



PROBLEM: Combinatorial growth

Possibilities with 1 to $M$ inputs and 1 to $L$ outputs (Nett, 1989):

$$\sum_{m=1}^{M}\sum_{l=1}^{L}\binom{L}{l}\binom{M}{m}$$

$M = L = 2$: 4+2+2+1=9 candidates

$M = L = 4$: 225 candidates, etc.

TOOLS THAT AVOID COMBINATORIAL GROWTH DESIRED.

RGA is one such tool.



## TASK 3: Selection of control configuration

Controller $K$ connects available measurements/commands ($v$) and manipulations ($u$):

$$u = Kv$$

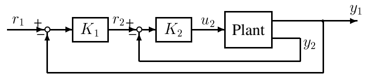
**Control configuration:** *The restrictions imposed on the structure of the overall controller $K$ by decomposing it into a set of local controllers (subcontrollers, units, elements, blocks) with predetermined links and with a possibly predetermined design sequence.*

Some elements used to build up configuration:

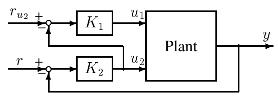
- Decentralized controllers ($K$ diagonal)
- Cascade controllers (with predetermined order for tuning)
- Feedforward elements
- Decoupling elements
- Selectors

⇒ Split the "big" controller into many smaller boxes.

## Cascaded controllers




(a) Extra measurements $y_2$ (conventional cascade control)




(b) Extra inputs $u_2$ (input resetting)



## Why use control configurations?

- Decomposed configurations often quite complex.
- Better performance: Optimization problem – resulting in a centralized multivariable controller.

So why use control configurations?

- Cost associated with obtaining good plant models (needed for centralized control).
- Cascade, decentralized, etc.: Controller is usually tuned **on-line** one at a time with little modelling effort.
- $\Rightarrow$ Rely on feedback rather than on models



Other advantages decentralized/cascade/hierarchical configurations:

- “Stabilize” the plant such that it is can be controlled by operators.
- Simple or even on-line tuning
- Tuning parameters have direct and “localized” effect
- Often easier to understand for operators
- Tend to be insensitive to uncertainty
- Allow simple models when designing higher layers
- Reduce the need for control links
- Allow for decentralized implementation
- Simpler implementation
- Reduced computation load
- Longer sampling intervals for the higher layers

*Comment.* The terms “stabilize” and “unstable” as used by operating people may not refer to a plant that is unstable in a mathematical sense, but rather to a plant that is *sensitive* to disturbances and which is difficult to control manually.



## THEORY FOR CONTROL CONFIGURATIONS

### Partial control

Close loop involving $u_2$ and $y_2$ using controller $K_2$:

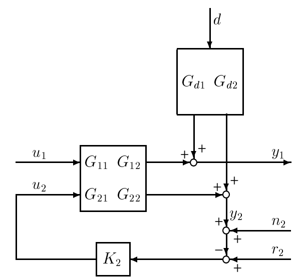



Figure .1: Block diagram of a partial control system

IMPORTANT

- Closing a loop does not imply a loss of degrees of freedom (DOFs) (since the setpoint $r_2$ replaces $u_2$ as a DOF), **BUT** we usually “use up” some of the dynamic range.

## Partial control

| | Meas./Control of $y_1$ ? | Control objective for $y_2$ ? |
|---|---|---|
| Sequential decentralized control | Yes | Yes |
| Sequential cascade control | Yes | No |
| "True" partial control | No | Yes |
| Indirect control | No | No |



Set $y_2 = r_2 - n_2$

$$y_1 = \underbrace{(G_{11} - G_{12}G_{22}^{-1}G_{21})}_{\triangleq\, P_u} u_1 + \underbrace{(G_{d1} - G_{12}G_{22}^{-1}G_{d2})}_{\triangleq\, P_d} d + \underbrace{G_{12}G_{22}^{-1}}_{\triangleq\, P_e}(r_2 - n_2)$$

*Some criteria for selecting $u_2$ and $y_2$ in lower-layer:*

1. Lower layer must quickly implement the setpoints from higher layers, i.e., controllability of subsystem $u_2$/ $y_2$ should be good. ($G_{22}$)
2. Provide for local disturbance rejection. (partial disturbanve gain $P_d$ should be small)
3. Impose no unnecessary control limitations on problem involving $u_1$ and/or $r_2$ to control $y_1$. ($P_u$ or $P_e$)
   - Avoid negative RGA for pairing $u_2/y_2$ – otherwise $P_u$ likely has RHP-zero

"Unnecessary": Limitations (RHP-zeros, ill-conditioning, etc.) not in original problem involving $u$ and $y$



## THEORY FOR CONTROL CONFIGURATIONS

### Stabilization

Tool: Pole directions
Example: Tennesse Eastman challenge problem



## Summary of procedure for plantwide control

The overall procedure consists of

**I. Top-down analysis** to identify degrees of freedom and control objectives

**II. Bottom-up design** of the contol structure

Iteration is required in this overall procedure!

## A plantwide control design procedure

| Step | Tools (in addition to insight) |
|---|---|
| **Top-down analysis:** | |
| 1. CONTROLLED VARIABLES: What is the control objective and which variables should be controlled? | *Steady-state model and operational objectives* <br>• Degree of freedom analysis <br>• Select condidate controlled variables <br>• Evaluate (economic) loss and look for "self-optimizing" control structure |
| 2. PRODUCTION RATE: Where should the throughput be set? This choice has a large effect on the remaining control system | Optimal choice follows from steady-state optimization, but may require continous reconfiguration (use MPC) If set one place: The throughput manipulator should have a strong and direct effect on the production rate. |

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## A plantwide control design procedure

| | |
|---|---|
| . **Bottom up design:** (With given controlled and manipualted variables) | **Controllability analysis** Compute zeros, poles, relative gain array, minimum singular value, etc. |
| 3. STABILIZATION. Selection of measurements and inputs for stabilization (including slowly drifting modes). Goal: enable manual operation of the plant | *Pole vectors* Give insight about which measurements and inputs can be used for each unstable mode. Select large elements: Small input energy needed and large noise tolerated. |
| 4. CASCADED CONTROLLERS. Selection of (extra) secondary measurements (and inputs) to improve dynamic control. Use of feedforward control (ratio controllers). | *Partially controlled plant* The relationships for a partially controlled plant tell how the plant looks from the above control layer. |
| 5. DECENTRALIZED CONTROL. If noninteracting process: design a decentralized control structure. | *Controllability analysis for decentralized control* Pair on relative gain array close to identity matrix at crossover frequency, provided not negative at stead-state. More information for tuning (required bandwidth): Closed loop disturbance gain (CLDG) performance gain array (PRGA). |
| 6. MULTIVARIABLE CONTROL <br>• Use for interacting process (coordination including feedforward control) <br>• MPC: Use for tracking active constraints (If steady-state optimization shows that the active constraints are changing with disturbances) | |
| 7. REAL TIME OPTIMIZATION | |
| **Validation:** | (Nonlinear) dynamic simulation |



## CONTROLLABILITY ANALYSIS

Before attempting controller design one should analyze the plant:

- Is it a difficult control problem?
- Does there exist a controller that meets the specs?
- How should the process be changed to improve control?



QUALITATIVE RULES from Seborg et al. (1989) (chapter on "The art of process control"):

1. *Control outputs that are not self-regulating*
2. *Control outputs that have favorable dynamic and static characteristics, i.e., there should exist an input with a significant, direct and rapid effect.*
3. *Select inputs that have large effects on the outputs.*
4. *Select inputs that rapidly effect the controlled variables*

- We have developed controllability tools which quantify these statements.
- Scale variables. Must then require

**1.** Self-regulation: $|G_d| < 1$ at all frequencies
**2.** Disturbance rejection: $\left|G_d\left(j\frac{1}{\theta}\right)\right| < 1$
**3.** Disturbance rejection: $|G| > |G_d|$ at frequencies where $|G_d| > 1$

## “PERFECT CONTROL” and plant inversion. (Morari, 1983)

$$y = G(s)\, u + G_d(s)\, d$$

Ideal feedforward control, $y = r$:

$$u = G^{-1}\, r - G^{-1} G_d\, d \tag{2}$$

Feedback control:

$$u = G^{-1} T\, r - g^{-1} T G_d\, d \tag{3}$$

For frequencies below the bandwidth $(\omega < \omega_B) : T \approx I$: Then (3) =(2).

Controllability is limited if $G^{-1}$ cannot be realized:

- Delay (Inverse yields prediction)
- Inverse response = RHP-zero (Inverse yields instability)
- Input constraints (Inverse yields saturation)
- Uncertainty (Inverse not correct)



## POOR CONTROLLABILITY CAN BE CAUSED BY:

1. Delay or inverse response in $G(s)$
2. or $G(s)$ is of “high order” (tanks-in-series) so that we have an “apparent delay”
3. Constraints in the plant inputs (a potential problem if the plant gain is small)
4. Large disturbance effects (which require “fast control” and/or large plant inputs to counteract)
5. Instability: Feedback with the active use of plant inputs is required. May be unable to react sufficiently fast if there is an effective delay in the loop. And: May have problems with input saturation if there is measurement noise or disturbances
6. With feedback: Delay/inverse response or infrequent or lacking measurement of $y$. May try
   (a) Local feedback (cascade) based on another measurement, e.g. temperature
   (b) Estimation of $y$ from other measurements



7. Nonlinearity or large variations in the operating point which make linear control difficult. May try
   (a) Local feedback (inner cascades)
   (b) Nonlinear transformations of the inputs or outputs, e.g. $\ln y$
   (c) Gain scheduling controllers (e.g. batch process)
   (d) Nonlinear controller
8. MIMO RHP-zeros: May have internal couplings resulting in multivariable RHP-zeros $\Rightarrow$ Fundamental problem in controlling some combination of outputs.
9. MIMO plant gain: May not be able to control all outputs independently (if the “worst case” plant gain $\underline{\sigma}(G)$ is small).
10. MIMO interactions: May have large RGA-elements (caused by strong two-way interactions between the outputs) which makes multivariable control difficult.
11. Feedforward control: Should be considered if feedback control is difficult (e.g. due to delays in the feedback loop or MIMO interactions) and an “early” measurement of the disturbance is possible.

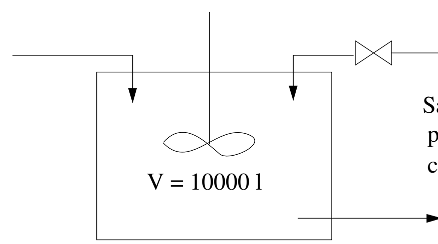
**There are tools available which quantify this.**



## EXAMPLE: pH-Neutralization process

Acid
pH = -1
$c_H = 10$ mol /l

Base
pH = 15
$c_{OH} = 10$ mol/l




Salt water (10 l/s)
pH = 7 $\pm$ 1
$c_H$ = 0.0000001 mol/l

$y$ = concentration of product (meas. delay $\theta$=10 s)
$u = \text{Flow}_{base}$
$d = \text{Flow}_{acid}$

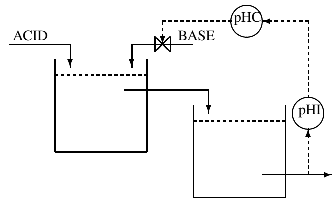
Introduce excess of acid $c = c_H - c_{OH}$ [mol/l].
In terms of $c$ the dynamic model is a simple mixing process !!

$$\frac{d}{dt}(Vc) = q_A c_A + q_B c_B - qc$$

EXTREMELY SENSITIVE TO DISTURBANCES.

# IMPROVE CONTROLLABILITY BY REDESIGN OF PROCESS

- Use several similar tanks in series with gradual adjustment



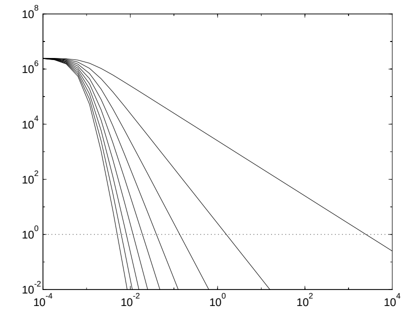

With $\theta = 10$ s the following designs have the same controllability:

| No. of tanks $n$ | Total volume $V_{tot}$ $[m^3]$ | Volume each tank $[m^3]$ |
|---|---|---|
| 1 | 250000 | 250000 |
| 2 | 316 | 158 |
| 3 | 40.7 | 13.6 |
| 4 | 15.9 | 3.98 |
| 5 | 9.51 | 1.90 |
| 6 | 6.96 | 1.16 |
| 7 | 5.70 | 0.81 |

Minimum total volume: 3.66 m$^3$ (18 tanks of 203 l each).
Economic optimum: 3 or 4 tanks.
Agrees with engineering rules.



With $n$ tanks: $G_d(s) = k_d/(1+\tau s)^n$.
$\tau$: residence time in each tank.




To reject disturbance must require

$$|G_d(j\frac{1}{\theta})| < 1$$

where $\theta$ is the measurement delay. Gives

$$\tau > \theta\sqrt{(k_d)^{2/n} - 1}$$

Total volume : $V_{tot} = n\tau q$ where $q = 0.01$ m$^3$/s.

# PLANTWIDE DYNAMICS

- Poles are affected by recycle of energy and mass and by interconnections
- Parallel paths may give zeros - possible control problems
- Recycle yields positive feedback and often large *open-loop* time constants
- This does *not* necessarily mean that *closed-loop* must be slow
- See MYTH on distillation contol where open-loop time constant for compositions is long because of positive feedback from reflux and boilup
- Luyben's "snowball effect" is mostly a steady-state design problem (do not feed more than the system can handle...)

# EXAMPLE: Recycle around reactor (snowball efect)

Simple example (Luyben, Yu):

- Reaction $A \rightarrow B$
- Recycle of unreacted A
- Product is pure B

At steady-state

$$\text{Feed of A} = \text{Generation of B in reactor} = \text{Production of B}$$

where Generation of B in reactor is

$$G_B = k(T)x_A V_R$$



$$G_B = k(T)x_A V_R$$

Three ways to increase $G_B$:

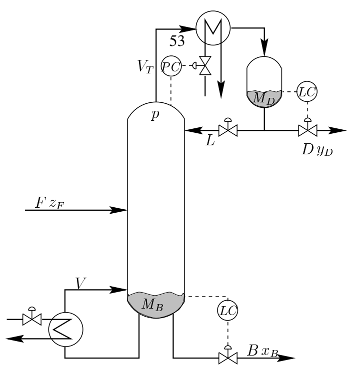
1. Increase reactor temperature $T$
2. Increase $x_A$ by increasing the recycle ratio $RR$

$$x_A = \frac{RR}{1+RR}$$

   (the "snowball effect" of Luyben is that $x_A \rightarrow 1$ as $RR \rightarrow \infty$ – occurs when the reactor is too small)
3. Increase the reactor volume $V_R$
   - BUT: Loose money by not operating at maximum volume (Possible trade-off between operating costs and controllability)
   - Gas phase reactor: Increasing the pressure has the same effect (larger inventory in reactor).



# DISTILLATION EXAMPLE




$$u = \begin{bmatrix} L \\ V \end{bmatrix}; \quad y = \begin{bmatrix} y_D \\ x_B \end{bmatrix} [\text{mol} - \%\ \text{light}]$$

Steady-state gains $y = Gu$ (LV-configuration)

$$G(0) = \begin{bmatrix} 87.8 & -86.4 \\ 108.2 & -109.6 \end{bmatrix}$$

RGA-value about 35 at steady-state $\Rightarrow$ Strong two-way interaction



**OVERALL DISTILLATION PROBLEM**

Typically, overall control problem has 5 inputs

$$u = [\, L \quad V \quad D \quad B \quad V_T \,]$$

(flows: reflux $L$, boilup $V$, distillate $D$, bottom flow $B$, overhead vapour $V_T$)
and 5 outputs

$$y = [\, y_D \quad x_B \quad M_D \quad M_B \quad p \,]$$

(compositions and inventories: top composition $y_D$, bottom composition $x_B$, condenser holdup $M_D$, reboiler holdup $M_B$, pressure $p$)

Without any control we have a $5 \times 5$ model

$$y = Gu + G_d d$$

(which generally has some large RGA-elements at steady-state)

**DISTILLATION CONFIGURATIONS**

There are usually three "unstable" outputs with no or little steady-state effect

$$y_2 = [\, M_D \quad M_B \quad p \,]$$

Remaining outputs

$$y_1 = [\, y_D \quad x_B \,]$$

Many possible choices for the three inputs for stabilization. For example, with

$$u_2 = [\, D \quad B \quad V_T \,]$$

we get the $LV$-configuration where

$$u_1 = [\, L \quad V \,]$$

are left for composition control.

Another configuration is the $DV$-configuration (has small RGA-elements) where

$$u_1 = [\, D \quad V \,]$$

After closing the stabilizing loops ($u_2 \leftrightarrow y_2$) we get a $2 \times 2$ model for the remaining "partially controlled" system

$$y_1 = G^{u_1} u_1 + G_d^{u_1} d$$

Which configurations is the best?



Analyze $G^{u_1}$ and $G_d^{u_1}$ with respect to

1. No composition control
   - Consider disturbance gain $G_d^{u_1}$ (e.g. effect of feedrate on compositions)
2. Close one composition loop ("one-point control")
   - Consider partial disturbance gain (e.g. effect of feedrate on $y_D$ with constant $x_B$)
3. Close two composition loops ("two-point control")
   - Consider interactions in terms of RGA
   - Consider "closed-loop disturbance gains" (CLDG) for single-loop control

Problem:

- No single best configuration
- Generally, get different conclusion on each of the three cases
- $\Rightarrow$ Stabilizing control is not necessarily a trivial issue



## A PARADOX

Distillation columns have large RGA-elements
$\Rightarrow$ Fundamental control problems (cannot have decoupling control)

**BUT: $DV$ configuration has small RGA-elements and we can decouple the compositions loops**
**How is this possible?**

Solution to paradox: $DV$ configuration has coupling between composition and level loops
(whereas $LV$ has decoupling between level and composition)



**CONTROLLABILITY ANALYSIS OF VARIOUS DISTILLATION CONFIGURATIONS**

- S. Skogestad, "Dynamics and control of distillation columns: A tutorial introduction", *Trans IChemE* (UK), **75**, Part A, 1997, 539 - 561.
- Available at: `http://www.chembio.ntnu.no/users/skoge/publications/1997/dist plenary.pdf`

# CONCLUSIONS / FUTURE WORK

1. Important problem - theory has been lacking
2. General procedure (not only process control)
3. Many theoretical tools are already there – still some effort left to get a unifying approach
4. Want to avoid “case study approach” (but the case studies are useful for understanding the issues)
5. Hope to make good progress in near future