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Consensusability of Discrete-Time Linear Networked Multi-Agent Systems

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***Abstract*—The consensusability problem of discrete-time linear networked multi-agent systems with a communication delay is investigated in this paper. Based on the networked predictive control scheme and dynamic output feedback control, a novel protocol is proposed to compensate for communication delay actively. For discrete-time linear networked multi-agent systems with a directed topology and a constant network delay, necessary and/or sufficient conditions of consensusability with respect to a set of admissible consensus protocols are given. A simulation result demonstrates the effectiveness of theoretical results.**

***Index Terms*—Consensusability, networked multi-agent systems, networked predictive control.**

I. Introduction

Recently, consensus problem has received significant attention as a fundamental research topic in decentralized control of networks of dynamic agents, due to its broad applications in cooperative control of unmanned aerial vehicles, scheduling of automated highway systems, formation control of satellite clusters, distributed optimization of multiple robotic systems, etc.

The most of existing works [1] about consensus problems focused on how consensus protocols are designed to achieve good performances. However, the consensusability problem of networked multi-agent systems is lack of enough attention, which is concerned about the existence of consensus protocols, and important in both synthesis and implementation of the protocols [2], [3].

Since the communication among agents in the networked multi-agent systems (NMASs) is achieved by a network, it is inevitable that the network-induced delay will occur while exchanging data among devices connected to the shared network, due to the limited bandwidth of the communication channels and the finite transmission speed. However, time delay can degrade the performance of control systems and even destabilize the system [4], [5]. Besides, due to economic costs or constraints on measurement in practice, it is often difficult or even unavailable to get the information of all the agents' states. Therefore, the consensusability problem of discrete-time linear networked multi-agent systems with a communication delay is investigated in this paper. By exploiting the networked predictive control scheme proposed by Liu [6], [7], a novel distributed protocol is put forward to overcome the effect of network delay actively rather than passively. For the NMAS consisting of uniform discrete-time linear time-invariant dynamical nodes with a directed topology and a constant network delay, delay-independent necessary and/or sufficient conditions of consensusability with respect to a set of admissible consensus protocols are established.

The paper is organized as follows. Some preliminaries of graph theory are briefly reviewed in Section II. Main results are given in Section III. To illustrate the theoretical results, a numerical simulation is provided in Section IV. Finally, Section V concludes the paper.

II. Preliminaries

In this context, some necessary notations are introduced to make readers to easily understand. Let $\mathbb{R}$ and $\mathbb{C}$ be real and complex number fields, respectively. $M_{m,n}(\mathbb{F})$ denotes the set of all m-by-n matrices over a field $\mathbb{F}$, and $M_{n,n}(\mathbb{F})$ is abbreviated to $M_n(\mathbb{F})$. For $A \in M_{m,n}(\mathbb{C})$, A^{T} denotes the transpose of A. Specially $m = n$, A is said to be Schur if $\sigma(A) \subseteq U_0$, where $\sigma(A)$ represents the spectrum of matrix A, and U_0 denotes an open unit disk centered at the origin. $\otimes$ stands for the Kronecker product of matrices. $\mathbf{1}_{N}$ denotes a N-dimension column vector with all entries equal to one. 0 represents zero matrix with an appropriate dimension. $\|\cdot\|$ represents l^2 norm on vectors or its induced norm on matrices. "With respect to" is short for w.r.t..

First of all, some basic concepts and notations in graph theory are briefly introduce, which is very important and helpful in the analysis of NMASs. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted digraph of order N, where the set of nodes $\mathcal{V} = \{1, 2, \cdots, N\}$, set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a nonnegative weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in M_{N}(\mathbb{R})$. An edge from i to j is denoted by $e_{ij} = (i, j)$ and adjacency element a_{ji} associated with edge e_{ij} is positive, i.e., $e_{ij} \in \mathcal{E} \Leftrightarrow a_{ji} > 0$. Moreover, assume that $a_{ii} = 0$ for all $i \in \mathcal{V}$. The set of neighbors of the node i is denoted by $N_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$. The set of all reachable nodes to node i is denoted by N_i^*. The Laplacian matrix $\mathcal{L} = [l_{ij}]_{n \times n}$ of weighted digraph $\mathcal{G}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \mathrm{diag}(d_{in}(1), d_{in}(2), \cdots, d_{in}(N))$ and $d_{in}(i) = \sum_{j=1, j \neq i}^{N} a_{ij}$, $i = 1, 2, \cdots, N$. Obviously, all the row-sums of $\mathcal{L}$ are zero, which implies that $\mathcal{L}$ has always an

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eigenvalue zero corresponding the right eigenvector $\mathbf{1}_N$. For a comprehensive restatement of the graph theory, the reader is referred to [8].

III. Consensusability Based on the Networked Predictive Control Scheme

Consider an NMAS composed of N agents, where the dynamics of agent i are described by a discrete-time linear time-invariant system as follows:

$$\begin{aligned} x_i(t+1) = & \ Ax_i(t) + B_i u_i(t), \\ y_i(t) = & \ Cx_i(t), \\ i = & \ 1, 2, \cdots, N, \ t = 0, 1, 2, \cdots, \end{aligned} \tag{1}$$

where $x_i \in M_{n,1}(\mathbb{R})$, $u_i \in M_{m,1}(\mathbb{R})$ and $y_i \in M_{l,1}(\mathbb{R})$ are the state, control input and measured output of the agent i, respectively. $A \in M_n(\mathbb{R})$, $B_i \in M_{n,m}(\mathbb{R})$, $C \in M_{l,n}(\mathbb{R})$ are constant matrices.

Regarding the above N agents as nodes of a graph, the communication relationship among agents can be conveniently represented by a weighted digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with the set of nodes $\mathcal{V} = \{1, 2, \cdots, N\}$, set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a nonnegative weighted adjacency matrix $\mathcal{A}$. The directed edge $e_{ij} \in \mathcal{E}$ means that agent j can receive the information from agent i.

In this paper, it is considered that the information exchanged among all agents is achieved by a network with a time delay τ. And the following assumptions can reasonably be made:

Assumption 1: (A1) Network delay τ is a constant and known positive integer.

(A2) States of all agents are not available but the outputs of them can be measured.

(A3) Each agent can receive information from itself and all reachable nodes to it, i.e. for $\forall j \in \{i\} \cup N_i^*$, agent i can receive information from agent j, where $i \in \mathcal{V}$.

Because agent i receives the information from agent j ($j \in \{i\} \cup N_i^*$) with time delay τ, in order to overcome the effect of the network delay, based on the output data of agent j up to time $t-\tau$, the state predictions of agent j from time $t-\tau$ to t are constructed as:

$$\begin{aligned} \hat{x}_j(t-\tau+1|t-\tau) = & \ A\hat{x}_j(t-\tau|t-\tau-1) \\ & + B_j u_j(t-\tau) + G_j\left[y_j(t-\tau)\right. \\ & \left. - C\hat{x}_j(t-\tau|t-\tau-1)\right], \end{aligned} \tag{2a}$$

$$\begin{aligned} \hat{x}_j(t-\tau+2|t-\tau) = & \ A\hat{x}_j(t-\tau+1|t-\tau) \\ & + B_j u_j(t-\tau+1), \\ \hat{x}_j(t-\tau+3|t-\tau) = & \ A\hat{x}_j(t-\tau+2|t-\tau) \\ & + B_j u_j(t-\tau+2), \\ & \vdots \\ \hat{x}_j(t|t-\tau) = & \ A\hat{x}_j(t-1|t-\tau) + B_j u_j(t-1), \\ & \ j \in \{i\} \cup N_i^*, \end{aligned} \tag{2b}$$

where $\hat{x}_j(t-\tau+1|t-\tau) \in M_{n,1}(\mathbb{R})$ and $u_j(t-\tau) \in M_{m,1}(\mathbb{R})$ are the one-step ahead state prediction and the input of the observer at time $t-\tau$, respectively, and $G_j \in M_{n,l}(\mathbb{R})$ can be designed using observer design approaches, $\hat{x}_j(t-\tau+d|t-\tau) \in M_{n,1}(\mathbb{R})$ is a state prediction of agent j at time $t-\tau+d$ on the basis of information up to time $t-\tau$, and $u_j(t-\tau+d-1) \in M_{m,1}(\mathbb{R})$ is the input at time $t-\tau+d-1$, $d = 2, 3, \cdots, \tau$, $j \in \{i\} \cup N_i^*$.

For agent i of NMAS (1), the following protocol based on the dynamic output feedback is designed:

$$\begin{aligned} z_i(t+1) &= \hat{A}_i z_i(t) + \hat{H}_i \zeta_i(t|t-\tau), \\ u_i(t) &= \hat{C}_i z_i(t) + \hat{F}_i \zeta_i(t|t-\tau), \\ i &= 1, 2, \cdots, N, \ t = 0, 1, 2, \cdots, \end{aligned} \tag{3}$$

where $z_i \in M_{\tilde{n},1}(\mathbb{R})$ is the protocol state, and

$$\zeta_i(t|t-\tau) = \sum_{j \in N_i} a_{ij}(\hat{y}_j(t|t-\tau) - \hat{y}_i(t|t-\tau)) \tag{4}$$

is the weighted sum of output prediction differences between agent i and its neighboring ones, $\hat{y}_i(t|t-\tau) = C\hat{x}_i(t|t-\tau)$ is the output prediction of agent i at time t based on the output data of agent i up to time $t-\tau$. $\mathcal{A} = [a_{ij}] \in M_N(\mathbb{R})$ is the weighted adjacency matrix of digraph $\mathcal{G}$. $\hat{A}_i$, $\hat{C}_i$, $\hat{H}_i$, $\hat{F}_i$ are matrices to be designed.

Let $u(t) = \begin{bmatrix} u_1^{\mathrm{T}}(t) & u_2^{\mathrm{T}}(t) & \cdots & u_N^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$. The following admissible control set is considered:

$$\begin{aligned} \mathscr{U}^* = & \ \{u(t) : [0, +\infty) \to M_{Nm,1}(\mathbb{R}) \,|\, u_i(t) \text{ satisfies} \\ & (3), i = 1, 2, \cdots, N\}. \end{aligned} \tag{5}$$

Definition 1: NMAS (1) is said to be consensusable w.r.t. $\mathscr{U}^*$, if there exists a $u(t) \in \mathscr{U}^*$ such that for any initial value $x_i(0)$, $z_i(0)$ and $e_i(t)$, $t = -\tau, -\tau+1, \cdots, -1, 0$, $i \in \mathcal{V}$, the following conditions hold:

(1) $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \ \forall i, j \in \mathcal{V}$,

(2) $\lim_{t\to\infty} z_i(t) = 0, \ \forall i \in \mathcal{V}$,

(3) $\lim_{t\to\infty} e_i(t) = 0, \ \forall i \in \mathcal{V}$,

where $e_i(t) = \hat{x}_i(t|t-1) - x_i(t)$ is the estimate error satisfying

$$\begin{aligned} e_i(t+1) = (A - G_i C)e_i(t), \ i \in \mathcal{V}, \\ t = -\tau, -\tau+1, \cdots, -1, 0, 1, \cdots. \end{aligned} \tag{6}$$

Let

$$\begin{aligned} \delta_i(t) &= x_1(t) - x_i(t), \ i = 1, 2, \cdots, N, \\ \delta(t) &= \begin{bmatrix} \delta_2^{\mathrm{T}}(t) & \delta_3^{\mathrm{T}}(t) & \cdots & \delta_N^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}, \\ x(t) &= \begin{bmatrix} x_1^{\mathrm{T}}(t) & x_2^{\mathrm{T}}(t) & \cdots & x_N^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}, \\ z(t) &= \begin{bmatrix} z_1^{\mathrm{T}}(t) & z_2^{\mathrm{T}}(t) & \cdots & z_N^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}, \\ e(t) &= \begin{bmatrix} e_1^{\mathrm{T}}(t) & e_2^{\mathrm{T}}(t) & \cdots & e_N^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}. \end{aligned}$$

For NMAS (1) with a fixed and directed topology, along with a constant network delay, a necessary and sufficient condition of consensusability of NMAS (1) w.r.t. $\mathscr{U}^*$ will be presented as follows.

Theorem 1: Consider NMAS (1) with a directed topology $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ and a network delay τ. NMAS (1) is consensusable w.r.t. $\mathscr{U}^*$ if and only if

$$(A, C) \text{ is detectable,} \tag{7}$$

and there exist $\hat{A}_i \in M_{\tilde{n}}$, $\hat{C}_i \in M_{m,\tilde{n}}$, $\hat{H}_i \in M_{\tilde{n},l}$ and $\hat{F}_i \in M_{m,l}$, $i = 1, 2, \cdots, N$, such that

$$\Theta \triangleq \begin{bmatrix} I_{N-1} \otimes A + RB_D\hat{F}_D(\mathcal{L}_2 \otimes C) & RB_D\hat{C}_D \\ \hat{H}_D(\mathcal{L}_2 \otimes C) & \hat{A}_D \end{bmatrix} \text{ is Schur,} \tag{8}$$

where $R = \begin{bmatrix} \mathbf{1}_{N-1} & -I_{N-1} \end{bmatrix} \otimes I_n$, $\mathcal{L}_2 = \mathcal{L} \begin{bmatrix} 0 & I_{N-1} \end{bmatrix}^{\mathrm{T}}$ and $\mathcal{L}$ is the Laplacian matrix of the graph $\mathcal{G}$.

Proof: From (2),

$$\begin{aligned} \zeta_i(t|t-\tau) = \; & C(\tilde{l}_i \otimes I_n)\delta(t) \\ & - CA^{\tau-1}(l_i \otimes I_n)e(t-\tau+1), \end{aligned} \tag{9}$$

where

$$\tilde{l}_i = \begin{bmatrix} l_{i2} & l_{i3} & \cdots & l_{iN} \end{bmatrix}$$

and

$$l_i = \begin{bmatrix} l_{i1} & \tilde{l}_i \end{bmatrix}, \; i = 1, 2, \cdots, N.$$

Substituting (9) into (3) derives

$$\begin{aligned} u_i(t) &= \hat{C}_i z_i(t) + \hat{F}_i\zeta_i(t) \\ &= \hat{C}_i z_i(t) + \hat{F}_i C(\tilde{l}_i \otimes I_n)\delta(t) \\ &\quad - \hat{F}_i CA^{\tau-1}(l_i \otimes I_n)e(t-\tau+1), \\ i &= 1, 2, \cdots, N. \end{aligned} \tag{10}$$

Hence, the closed-loop systems subjected to protocol (3) have the following forms:

$$\begin{aligned} x_i(t+1) &= Ax_i(t) + B_i u_i(t) \\ &= Ax_i(t) + B_i\hat{C}_i z_i(t) + B_i\hat{F}_i C(\tilde{l}_i \otimes I_n)\delta(t) \\ &\quad - B_i\hat{F}_i CA^{\tau-1}(l_i \otimes I_n)e(t-\tau+1) \end{aligned}$$

and

$$\begin{aligned} z_i(t+1) &= \hat{A}_i z_i(t) + \hat{H}_i\zeta_i(t) \\ &= \hat{A}_i z_i(t) + \hat{H}_i C(\tilde{l}_i \otimes I_n)\delta(t) \\ &\quad - \hat{H}_i CA^{\tau-1}(l_i \otimes I_n)e(t-\tau+1), \\ i &= 1, 2, \cdots, N. \end{aligned}$$

Then the following compact form can be presented:

$$\begin{aligned} x(t+1) &= (I_N \otimes A)x(t) + B_D\hat{C}_D z(t) \\ &\quad + B_D\hat{F}_D(\mathcal{L}_2 \otimes C)\delta(t) \\ &\quad - B_D\hat{F}_D[\mathcal{L} \otimes (CA^{\tau-1})]e(t-\tau+1) \end{aligned}$$

and

$$\begin{aligned} z(t+1) &= \hat{A}_D z(t) + \hat{H}_D(\mathcal{L}_2 \otimes C)\delta(t) \\ &\quad - \hat{H}_D[\mathcal{L} \otimes (CA^{\tau-1})]e(t-\tau+1). \end{aligned}$$

It should be noted that

$$\delta(t) = Rx(t)$$

and

$$e_i(t) = (A - G_iC)e_i(t-1), \; i = 1, 2, \cdots, N.$$

Therefore, the generalized closed-loop system can be described as

$$\xi(t+1) = \Omega\xi(t), \tag{11}$$

where

$$\begin{aligned} \xi(t) &= \begin{bmatrix} \delta^{\mathrm{T}}(t) & z^{\mathrm{T}}(t) & e^{\mathrm{T}}(t-\tau+1) \end{bmatrix}^{\mathrm{T}}, \\ \Omega &= \begin{bmatrix} \Omega_1 & RB_D\hat{C}_D & \Omega_2 \\ \hat{H}_D(\mathcal{L}_2 \otimes C) & \hat{A}_D & \Omega_3 \\ 0 & 0 & \Omega_4 \end{bmatrix}, \\ \Omega_1 &= I_{N-1} \otimes A + RB_D\hat{F}_D(\mathcal{L}_2 \otimes C), \\ \Omega_2 &= -RB_D\hat{F}_D[\mathcal{L} \otimes (CA^{\tau-1})], \\ \Omega_3 &= -\hat{H}_D[\mathcal{L} \otimes (CA^{\tau-1})], \\ \Omega_4 &= I_N \otimes A - G_D(I_N \otimes C). \end{aligned}$$

From Definition 1, NMAS (1) is consensusable w.r.t. $\mathscr{U}^*$ if and only if system (11) is asymptotically stable or, equivalently, Ω is Schur. So it follows from (7) and (8) that the conclusion holds. ∎

Theorem 1 provides a necessary and sufficient condition of the consensusability of NMAS (1) w.r.t. $\mathscr{U}^*$. Based on it, a sufficient condition of the consensusability will be presented.

Corollary 1: Consider NMAS (1) with a directed topology $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ and a network delay τ. If there exist $\hat{A}_i \in M_{\tilde{n}}$ and $\hat{F}_i \in M_{m,l}$, $i = 1, 2, \cdots, N$, satisfying that the following conditions (i) – (iii) hold, then NMAS (1) is consensusable w.r.t. $\mathscr{U}^*$.

(i) (A, C) is detectable.
(ii) $\hat{A}_i$ is Schur, $i = 1, 2, \cdots, N$.
(iii) $I_{N-1} \otimes A + RB_D\hat{F}_D(\mathcal{L}_2 \otimes C)$ is Schur,

where R and $\mathcal{L}_2$ are defined in Theorem 1.

Proof: By choosing $\hat{C}_i$ to satisfy $B_i\hat{C}_i = 0$, or choosing $\hat{H}_i$ to satisfy $\hat{H}_iC = 0$, $i = 1, 2, \cdots, N$, it is sufficient that $\hat{A}_D$ and $I_{N-1} \otimes A + RB_D\hat{F}_D(\mathcal{L}_2 \otimes C)$ are Schur. Hence, from Theorem 1, NMAS (1) is consensusable w.r.t. $\mathscr{U}^*$. The proof is completed. ∎

IV. Simulation

In this section, a numerical simulation is presented to illustrate the effectiveness of the proposed theoretical results.

Example 1: Consider an NMAS with a network delay $\tau = 3$ and four agents indexed by 1, 2, 3 and 4, respectively. The dynamics of agent i ($i = 1, 2, 3, 4$) are described by (1), where

$$A = \begin{bmatrix} 1 & 1.5 & 0.6 \\ -0.5 & -0.8 & 0.5 \\ 0 & 0.65 & 0.05 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix},$$

$$B_1 = B_2 = B_3 = B_4 = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 1 & 0 \end{bmatrix}.$$

The interconnection among four agents is described by $\mathcal{G}$ in Fig. 1 with the adjacent matrix

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

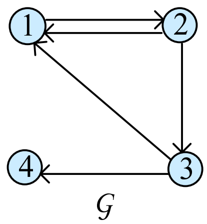

Fig. 1. Fixed topology.

It is obvious that (A, C) is detectable. By choosing $Q = 6I_3$ and using MATLAB, a solution of Riccati equation

$$APA^{\mathrm{T}} - P - APC^{\mathrm{T}}(I + CPC^{\mathrm{T}})^{-1}CPA^{\mathrm{T}} + Q = 0$$

can be obtained as:

$$P = \begin{bmatrix} 7.1235 & -0.2844 & 0.4230 \\ -0.2844 & 6.2245 & -0.1377 \\ 0.4230 & -0.1377 & 6.2029 \end{bmatrix}.$$

Then a gain matrix of the observer (2a) can be got

$$G = \begin{bmatrix} -0.2752 & 0.1567 & 0.9671 \\ 0.2981 & -0.3275 & -0.1394 \\ -0.2913 & -0.0057 & 0.3157 \end{bmatrix}.$$

Take

$$\hat{A}_1 = \begin{bmatrix} -0.2 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \ \hat{A}_2 = \begin{bmatrix} -0.3 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.1 \end{bmatrix},$$

$$\hat{A}_3 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}, \ \hat{A}_4 = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.85 \end{bmatrix},$$

$$\hat{C}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 4 & 1 \end{bmatrix}, \ \hat{C}_2 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 2 \end{bmatrix},$$

$$\hat{C}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}, \ \hat{C}_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0.5 & -2 & 1 \end{bmatrix},$$

$$\hat{H}_1 = \begin{bmatrix} -1.4980 & -1.2360 & 2.3740 \\ -3.2956 & -2.7192 & 5.2228 \\ 1.4980 & 1.2360 & -2.3740 \end{bmatrix},$$

$$\hat{H}_2 = \begin{bmatrix} 1.4980 & 1.2360 & -2.3740 \\ -2.3519 & -1.9405 & 3.7272 \\ -4.4940 & -3.7080 & 7.1220 \end{bmatrix},$$

$$\hat{H}_3 = \begin{bmatrix} -0.4494 & -0.3708 & 0.7122 \\ 0 & 0 & 0 \\ -1.4980 & -1.2360 & 2.3740 \end{bmatrix},$$

$$\hat{H}_4 = \begin{bmatrix} 0 & 0 & 0 \\ 3.5203 & 2.9046 & -5.5789 \\ -1.4980 & -1.2360 & 2.3740 \end{bmatrix},$$

$$\hat{F}_1 = \begin{bmatrix} -0.0997 & 0.0449 & -0.1473 \\ -0.1898 & 0.0896 & 0.0370 \end{bmatrix},$$

$$\hat{F}_2 = \begin{bmatrix} -0.0806 & 0.0453 & -0.1508 \\ -0.2257 & 0.1075 & 0.0389 \end{bmatrix},$$

$$\hat{F}_3 = \begin{bmatrix} -0.0935 & 0.0532 & -0.1715 \\ -0.0332 & 0.0140 & 0.2052 \end{bmatrix},$$

$$\hat{F}_4 = \begin{bmatrix} -0.3333 & 0.0667 & 0.4000 \\ 0.8333 & 0.3333 & -0.5000 \end{bmatrix}.$$

It is easy to verify that

$$\sigma(\Theta) = \{0.2, 0.6, 0.5, 0.3, 0.8, 0.1, 0.1, 0.2, 0.3, 0.85, 0.3803, 0.2861, 0.0705, 0.6245, 0.6245, 0.8272, 0.8272, 0.7703, 0.7703, 0, 0.5\} \subseteq U_0.$$

Hence, NMAS (1) is consensusable w.r.t. $\mathscr{U}^*$ by Theorem 1.

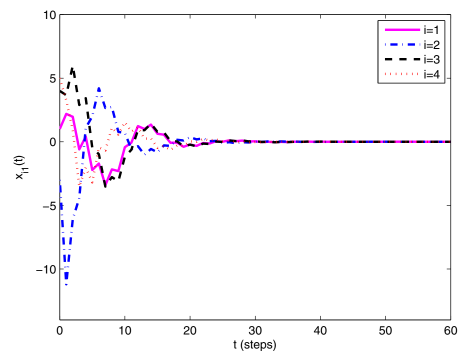

Fig. 2. Trajectories of the first state variables.

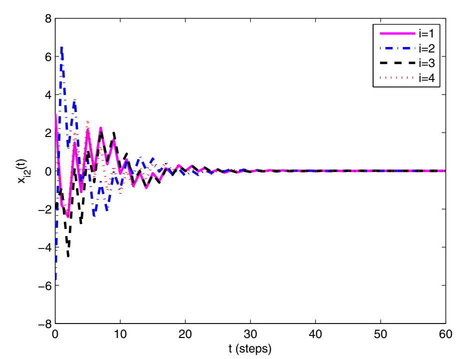

Fig. 3. Trajectories of the second state variables.

Initial conditions of NMAS (1), protocol (3) are chosen as

$$x_1(0) = \begin{bmatrix} 1 & 3 & -2 \end{bmatrix}^{\mathrm{T}}, \quad x_2(0) = \begin{bmatrix} -3 & -6 & 1 \end{bmatrix}^{\mathrm{T}},$$
$$x_3(0) = \begin{bmatrix} 4 & -2 & 2 \end{bmatrix}^{\mathrm{T}}, \quad x_4(0) = \begin{bmatrix} 5 & 1 & -1 \end{bmatrix}^{\mathrm{T}},$$
$$z_1(0) = \begin{bmatrix} -1 & 1 & 2 \end{bmatrix}^{\mathrm{T}}, \quad z_2(0) = \begin{bmatrix} 1 & 4 & 1 \end{bmatrix}^{\mathrm{T}},$$
$$z_3(0) = \begin{bmatrix} 4 & 6 & 2 \end{bmatrix}^{\mathrm{T}}, \quad z_4(0) = \begin{bmatrix} -10 & 5 & -1 \end{bmatrix}^{\mathrm{T}},$$

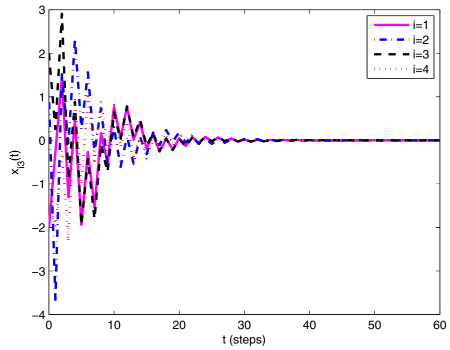

Fig. 4. Trajectories of the third state variables.

Initial conditions of estimate error (6) are chosen as

$$\begin{aligned}
e_1(0) &= \left[\begin{array}{ccc} 0.1 & -0.1 & -0.1 \end{array}\right]^{\mathrm{T}}, \\
e_2(0) &= \left[\begin{array}{ccc} 0.1 & 0.3 & 0.5 \end{array}\right]^{\mathrm{T}}, \\
e_3(0) &= \left[\begin{array}{ccc} 1 & 0 & -0.5 \end{array}\right]^{\mathrm{T}}, \\
e_4(0) &= \left[\begin{array}{ccc} 0.6 & 0 & -0.8 \end{array}\right]^{\mathrm{T}}, \\
e_1(-1) &= -\left[\begin{array}{ccc} 0.1 & 0.1 & 0.1 \end{array}\right]^{\mathrm{T}}, \\
e_2(-1) &= -\left[\begin{array}{ccc} 0.5 & 0.1 & 0.1 \end{array}\right]^{\mathrm{T}}, \\
e_3(-1) &= -\left[\begin{array}{ccc} 1 & 0.1 & 0.1 \end{array}\right]^{\mathrm{T}}, \\
e_4(-1) &= -\left[\begin{array}{ccc} 0.6 & 1 & 0.1 \end{array}\right]^{\mathrm{T}}, \\
e_1(-2) &= \left[\begin{array}{ccc} 0.1 & 0.3 & -0.1 \end{array}\right]^{\mathrm{T}}, \\
e_2(-2) &= \left[\begin{array}{ccc} -0.1 & 0.2 & 0.4 \end{array}\right]^{\mathrm{T}}, \\
e_3(-2) &= \left[\begin{array}{ccc} -0.6 & 0.5 & 0.1 \end{array}\right]^{\mathrm{T}}, \\
e_4(-2) &= \left[\begin{array}{ccc} 0.2 & 0.1 & -0.6 \end{array}\right]^{\mathrm{T}}.
\end{aligned}$$

The state trajectories of NMAS (1) are shown in Figures 2–4, respectively, which demonstrates that states of NMAS (1) achieve consensus under protocol (3).

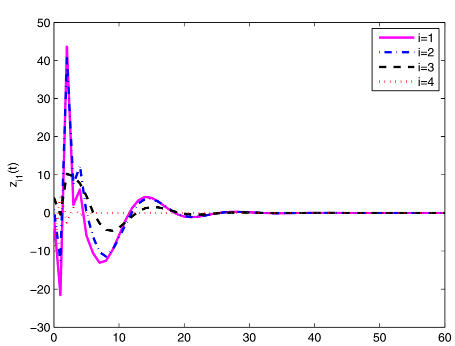

Fig. 5. Trajectories of the first protocol state variables.

The state trajectories of protocol (3) are shown in Figures

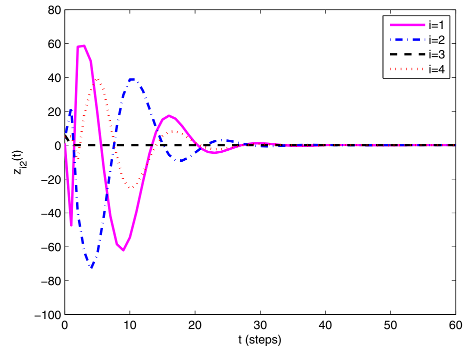

Fig. 6. Trajectories of the second protocol state variables.

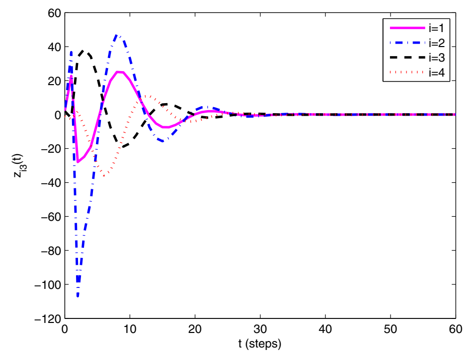

Fig. 7. Trajectories of the third protocol state variables.

5–7, respectively. It is thus clear that protocol states asymptotically converge to zero.

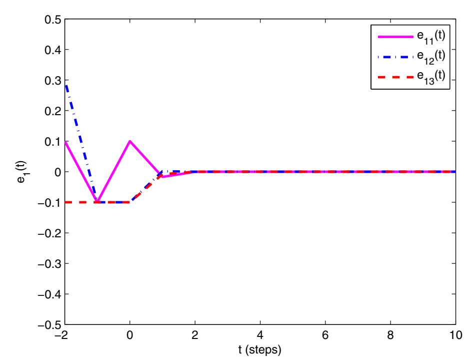

Fig. 8. The error trajectory $e_1(t)$.

The estimate error trajectories of observers are shown in Figures 8–11, respectively. It is obvious that states of observers track ones of NMAS (1).

V. Conclusion

The consensusability problem of discrete-time linear NMASs with uniform dynamical agents and a communication delay has been investigated. A new distributed protocol is proposed by using the networked predictive control scheme. For discrete-time linear NMASs with a directed topology and

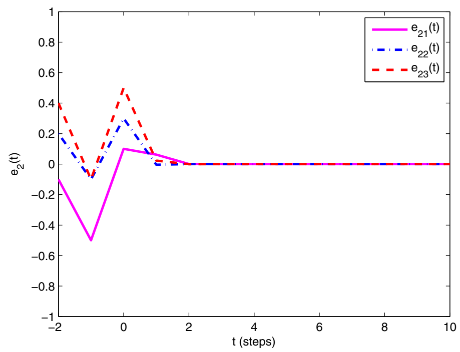

Fig. 9. The error trajectory $e_2(t)$.

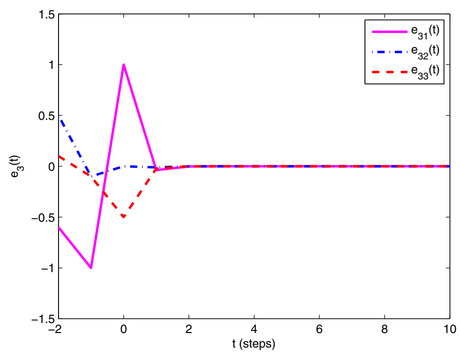

Fig. 10. The error trajectory $e_3(t)$.

a network delay, delay-independent necessary and/or sufficient criteria of consensusability have been obtained. A numerical example is provided to demonstrate the effectiveness of theoretical results.

REFERENCES

[1] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, 2004.
[2] C. Q. Ma and J. F. Zhang, "Necessary and sufficient conditions for consensusability of linear multi-agent systems," *IEEE Trans. Autom. Control*, vol. 55, no. 5, pp. 1263–1268, 2010.
[3] K. Y. You and L. H. Xie, "Network topology and communication data rate for consensusability of discrete-time multi-agent systems," *IEEE Trans. Autom. Control*, vol. 56, no. 10, pp. 2262–2275, 2011.

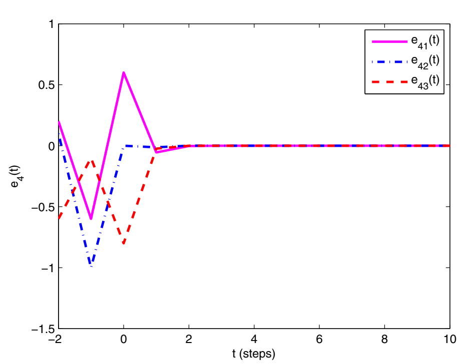

Fig. 11. The error trajectory $e_4(t)$.

[4] X. W. Liu, W. L. Lu, and T. P. Chen, "Consensus of multi-agent systems with unbounded time-varying delays," *IEEE Trans. Autom. Control*, vol. 55, no. 10, pp. 2396–2401, 2010.
[5] J. H. Qin, H. J. Gao, and W. X. Zheng, "Second-order consensus for multi-agent systems with switching topology and communication delay," *Syst. Control Lett.*, vol. 60, no. 6, pp. 390–397, 2011.
[6] G. P. Liu, Y. Q. Xia, D. Rees, and W. S. Hu, "Design and stability criteria of networked predictive control systems with random network delay in the feedback channel," *IEEE Trans. Syst. Man Cybern. Part C-Appl. Rev.*, vol. 37, no. 2, pp. 173–184, 2007.
[7] G. P. Liu, Y. Q. Xia, J. Chen, D. Rees, and W. S. Hu, "Networked predictive control of systems with random network delays in both forward and feedback channels," *IEEE Trans. Ind. Electron.*, vol. 54, no. 3, pp. 136–140, 2007.
[8] C. Godsil and G. Royle, *Algebraic Graph Theory*, ser. Graduate Texts in Mathematics. New York: Springer-Verlag, 2001, vol. 207.