UKACC International Conference on Control 2012
Cardiff, UK, 3-5 September 2012

An Improved On-Line Neuro-Identification Scheme

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Abstract— **In this paper, an on-line identification scheme is proposed to enhance the residual state error performance in face of disturbances. The proposed scheme is based on an e_1-modification adaptive law for the weights to approximate the unknown nonlinearities with bounded error. Besides, an identification model with feedback is introduced to improve the state error performance. The feedback is based on a bounding function to estimate an upper bound for the disturbances. Via an adaptive bounding technique and Lyapunov methods, it is proved that the residual state error performance is practically immune to disturbances. To validate the theoretical results, the identification of a four-order generalized Lü hyperchaotic system is performed.**

Identification; neural networks; uncertain systems; Lyapunov methods; chaotic systems.

I. INTRODUCTION

The use of neural networks (NNs) paradigm as a powerful tool for identification of uncertain nonlinear systems has encouraged, starting from the 90s, several heuristic and theoretical studies, see for instance [1]-[8] and the references therein. This interest is motivated by the capability of the NNs to learn complex input-output mappings, since they are universal approximators, and by the inevitable presence of uncertainties in modeling problems, due to the simplification imposed by the mathematical modeling, unexpected faults, changes in operation conditions, aging of equipment, and so on. On the other hand, system neuro-identification is important not only to predict the behavior of the system, but also for providing an appealing system parameterization, which can later be used in the synthesis of control algorithms, since mathematical characterization is often a prerequisite to controller design.

Neural identification models usually employed are the dynamic ones, being their weights mainly adjusted by using gradient and backpropagation algorithms or their robust modifications [1], [3]-[8]. Most used robust modifications in neuro-identification are the σ, switching- σ, ε_1, parameter projection, and dead zone [3]-[8] which avoid the parameter drift. Nevertheless, to the best of our knowledge, at present most of learning algorithms for neuro-identification ensure that the residual state error is related directly to upper bounds for the approximation error, ideal weight and disturbances.

For instance in [3], the identification of a general class of uncertain continuous-time dynamical systems was proposed, and a σ -modification adaptive law for the weights of recurrent high-order neural networks (RHONNs) was chosen to ensure that the state error converges to the neighborhood of the origin, whose radius depends directly of the approximation error and disturbances. In [4], dynamic NNs based on two-layer neural networks were used to identify a general class of uncertain nonlinear systems. It was shown that in the presence of disturbances the state error is uniformly ultimately bounded where the ultimate bound is directly proportional to an upper bound of the disturbance. In [5], the identification of delayed nonlinear system was investigated. By using identification models based on delayed neural networks with learning laws for the weights designed using a Lyapunov-Krasovskii approach, it was shown that the state error is upper bounded by a constant which depends directly of the disturbance. More recently, also others relevant works, such as [6]-[7], shown that discrete high-order neural networks and dynamic neural networks with two different time scales, respectively, can be used to identify nonlinear systems with bounded errors, which are straightforwardly related to the disturbances.

From the discussion above, observe that most of neuro-identification schemes ensure a state error performance that is directly related to the disturbance. In practice, uncertainties are inevitable, hence it is desirable to propose identification schemes with improved state error performance in face of disturbances. This is the main motivation for this paper.

Hence, in this paper we propose a neuro-identification algorithm in which the residual state error is inversely correlated with the disturbances to make the residual state error performance practically immune to disturbances. To this end, based on an adaptive bounding technique [9] and Lyapunov methods [10], a neural identification model with explicit feedback based on a bounding function is proposed. The aim is to approximate an upper bound for the disturbances, which is used in the stability proof, to make the Lyapunov derivative ($\dot{V}$) negative semi-definite practically in all error space, since bounding functions can be used to dominate positive terms in $\dot{V}$ and hence improve the performance.

II. LINEARLY PARAMETERIZED NEURAL NETWORKS

Linearly parameterized neural networks (LPNNs) can be expressed mathematically as

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$$\rho_{nn}(W,\zeta)=W\pi(\zeta) \quad (1)$$

where $W\in\Re^{n\times L_\rho}$, $\zeta\in\Re^{L_\zeta}$, $\pi:\Re^{L_\zeta}\mapsto\Re^{L_\rho}$ is the so-called basis function vector, which can be considered as a nonlinear vector function whose arguments are preprocessed by a scalar function $s(\cdot)$, and n, L_ρ, L_ζ are integers strictly positive. Commonly used scalar functions $s(\cdot)$ include sigmoid, tanh, gaussian, Hardy's, inverse Hardy's multiquadratic, etc [8]. However, here we are only interested in the class of LPNNs for which $\pi(\cdot)$ is bounded, since in this case we have,

$$\|\pi(\zeta)\|\le\pi_0 \quad (2)$$

being π_0 a strictly positive constant.

The class of LPNNs considered in this work includes HONN [3], RBF networks [8], wavelet networks [11], and also others linearly parameterized approximators as Takagi-Sugeno fuzzy systems [12]. Universal approximation results in [8], [11]-[12] indicate that:

Property 1: Given a constant $\varepsilon_0>0$ and a continuous function $f:\Omega\mapsto\Re^n$, where $\Omega\subset\Re^{L_\zeta}$ is a compact set, there exists a weight matrix $W=W^*$ such that the output of the neural network architecture (where L_ρ may depend on ε_0 and f) satisfies

$$\sup_{\zeta\in\Omega}\left|f(\zeta)-W^*\pi(\zeta)\right|\le\varepsilon_0 \quad (3)$$

where $|\cdot|$ denotes the absolute value if the argument is a scalar. If the argument is a vector function in $\Re^n$ then $|\cdot|$ denotes any norm in $\Re^n$.

III. PROBLEM FORMULATION

Consider the following nonlinear differential equation

$$\dot{x}=F(x,u,v,t),\qquad x(0)=x_0 \quad (4)$$

where $x\in X$ is the n-dimensional state vector, $u\in U$ is a m-dimensional admissible input vector, $v\in V\subset\Re^q$ is a vector of time varying uncertain variables and $F:X\times U\times V\times[0,\infty)\mapsto\Re^n$ is a continuous map. In order to have a well-posed problem, we assume that X,U,V are compact sets and F is locally Lipschitzian with respect to x in $X\times U\times V\times[0,\infty)$, such that (4) has a unique solution.

We assume that the following can be established

Assumption 1: On a region $X\times U\times V\times[0,\infty)$

$$\|h(x,u,v,t)\|\le h_0 \quad (5)$$

where

$$h(x,u,v,t)=F(x,u,v,t)-f(x,u) \quad (6)$$

f is an unknown map, h are internal or external disturbances, and $\bar{h}_0$, such that $\bar{h}_0>h_0\ge 0$, is a known constant.

Hence, except for the Assumption 1, we say that $F(x,u,v,t)$ is an unknown map and our aim is to design a NNs-based identifier for (4) to ensure that the residual state error is ultimately bounded with ultimate bound which is, practically, not affected by the disturbances.

IV. IDENTIFICATION MODEL AND STATE ERROR EQUATION

We start by presenting the identification model and the definition of the relevant errors associated with the problem.

Let $\bar{f}$ be the best known approximation of f, $P\in\Re^{n\times n}$ a scaling matrix defined as $P=P^T>0$, $\bar{g}=P^{-1}g$, and $g(x,u)=f(x,u)-\bar{f}(x,u)$. Then, by adding and subtracting $\bar{f}(x,u)$, (4) can be rewritten as

$$\dot{x}=\bar{f}(x,u)+P\bar{g}(x,u)+h(x,u,v,t) \quad (7)$$

Remark 1: It should be noted that if the designer has no previous knowledge of f, then $\bar{f}$ is simply assumed as being the zero vector.

From (7), by using LPNNs, the nonlinear mapping $\bar{g}(x,u)$ can be replaced by $W^*\pi(x,u)$ plus an approximation error term $\varepsilon(x,u)$. More exactly, (7) becomes

$$\dot{x}=\bar{f}(x,u)+PW^*\pi(x,u)+P\varepsilon(x,u)+h(x,u,v,t) \quad (8)$$

where $W^*\in\Re^{nxL}$ is an "optimal" or ideal matrix, which can be defined as

$$W^*:=\arg\min_{\hat{W}\in\Gamma}\left\{\sup_{\substack{x\in X,\\ u\in U}}\left|\bar{g}(x,u)-\hat{W}\pi(x,u)\right|\right\} \quad (9)$$

with $\Gamma=\left\{\hat{W}\ \middle|\ \|\hat{W}\|\le\alpha_{\hat{W}}\right\}$, $\alpha_{\hat{W}}$ is a strictly positive constant, $\hat{W}$ is an estimate of W^*, and $\varepsilon(x,u)$ is an approximation error term, corresponding to W^*, which can be defined as

$$\varepsilon(x,u):=\bar{g}(x,u)-W^*\pi(x,u) \quad (10)$$

The approximation, reconstruction, or modeling error ε in (10) is a quantity that arises due to the incapacity of LPNNs to match the unknown map $\bar{g}(x,u)$. Since X, U are compact sets and from (2), the following can be established

Assumption 2: The Frobenius matrix norm $\left\|W^*-W_0\right\|_F$, where $W_0\in\Re^{nxL}$ is upper bounded by a known positive constant $\bar{\beta}$, such that

$$\left\| W^* - W_0 \right\|_F \le \bar{\beta} \tag{11}$$

Assumption 3: On a region $X \times U$, the approximation error is upper bounded by

$$\left\| \varepsilon(x,u) \right\| \le \varepsilon_0 \tag{12}$$

where $\bar{\varepsilon}_0$, such that $\bar{\varepsilon}_0 > \varepsilon_0 \ge 0$, is an known constant.

Remark 2: Assumption 1 is usual in identification. Assumption 2 is quite natural since $\bar{g}$ is continuous and their arguments evolve on compact sets.

Remark 3: Note that any $\bar{\pi}_0 > \pi_0$, $\bar{h}_0 > h_0$, and $\bar{\varepsilon}_0 > \varepsilon_0$ also satisfy (2), (5), and (12). Hence, to avoid confusion, we define π_0, h_0, and ε_0 to be the smallest constants such that (2), (5), and (12) are satisfied.

Remark 4: It should be noted that W^* and $\varepsilon(x,u)$ might be nonunique. However, the uniqueness of $\left\| \varepsilon(x,u) \right\|$ is ensured by (9).

Remark 5: It should be noted that W^* was defined as being the value of $\hat{W}$ that minimizes the L_∞ - norm difference between $\bar{g}(x,u)$ and $\hat{W}\pi(x,u)$. The scaling matrix *P* from (7) is introduced to manipulate the magnitude of uncertainties and hence the magnitude of the approximation error. This procedure improves the performance of the identification process.

Remark 6: Notice that the proposed neuro-identification scheme is a black-box methodology, hence the external disturbances and approximation error are related. Based on the system input and state measurements, the uncertain system (including the disturbances) is parameterized by a neural network model plus an approximation error term. However, the parameterization (8) is motivated by the fact that neural networks are not adequate for approximating external disturbances, since the basis function depends on the input and states, whereas the disturbances depend on the time and external variables. The aim for presenting the uncertain system in the form (8), where the disturbance *h* is explicitly considered, is also to highlight that the proposed scheme is in addition valid in the presence of unexpected changes in the systems dynamics that can emerge, for instance, due to environment change, aging of equipment or faults.

Based on structure (8) and to ensure improved state error performance, the identification model is chosen as

$$\dot{\hat{x}} = \bar{f}(x,u) + P\hat{W}\pi(x,u) - \left(l_0\hat{\psi} + \psi_0^2 P/4\right)(\hat{x} - x) \tag{13}$$

where $\hat{x}$ is the estimated state, $\hat{\psi}$ is a bounding scalar function, l_0 and ψ_0 are positive constants. It will be demonstrated that the identification model (13) used in conjunction with a convenient adjustment laws for $\hat{W}$ and $\hat{\psi}$, to be proposed in the next section, improve the residual state error performance in the presence of disturbances.

Remark 7: It should be noted that in our formulation, the LPNN is only required to approximate $P^{-1}\left[f(x,u) - \bar{f}(x,u)\right]$ (whose magnitude is often small) instead of the entire function $P^{-1}\left[f(x,u)\right]$. Hence, standard identification methods (to obtain some previous $\bar{f}$) can be used together with the proposed algorithm to improve performance.

By defining the state estimation error as $\tilde{x} := \hat{x} - x$, from (8) and (13), we obtain the state estimation error equation

$$\dot{\tilde{x}} = P\tilde{W}\pi(x,u) - \left(l_0\hat{\psi} + \psi_0^2 P/4\right)\tilde{x} - P\varepsilon(x,u) - h(x,u,v,t) \tag{14}$$

where $\tilde{W} := \hat{W} - W^*$.

V. ADAPTIVE LAWS AND STABILITY

Before presenting the main theorem, we state a fact, which will be used in the stability analysis.

Fact 1: Let $W^*, W_0, \hat{W}, \tilde{W} \in \Re^{n \times L_p}$. Then, with the definition of $\tilde{W} = \hat{W} - W^*$, the following equalities are true:

$$\begin{aligned} 2tr\left[\tilde{W}^T\left(\hat{W} - W_0\right)\right] &= \left\|\tilde{W}\right\|_F^2 \\ &+ \left\|\left(\hat{W} - W_0\right)\right\|_F^2 - \left\|\left(W^* - W_0\right)\right\|_F^2 \end{aligned} \tag{15}$$

We now state and prove the main theorem of the paper.

Theorem 1: Consider the class of nonlinear systems described by (4) and the Assumptions 1-3. Let the identification model be given by (13) with

$$\dot{\hat{W}} = -\gamma_W [\gamma_0 (\hat{W} - W_0) \left\|\tilde{x}\right\| + 2\tilde{x}\pi^T(x,u)] \tag{16}$$

and

$$\dot{\hat{\psi}} = -\gamma_\psi \left[\psi_0\hat{\psi}\left\|\tilde{x}\right\| - \psi_0\left\|\tilde{x}\right\| - \psi_2 l_0 \left\|\tilde{x}\right\|^2\right], \hat{\psi}(0) > 0 \tag{17}$$

where

$$\gamma_\psi > 0, \quad \psi_2 = 2\lambda_{\min}\left(P^{-1}\right), \tag{18}$$

$$\psi^* = 1 - \frac{1}{\psi_0}\left[\psi_0^2 - 2\psi_0\left(2\bar{\varepsilon}_0 + 2\bar{h}_0\left\|P^{-1}\right\|_F + \gamma_0\bar{\beta}^2/2\right)\right]^{\frac{1}{2}}$$

Then, the signal errors $\tilde{W},\tilde{\psi}$ are uniformly bounded and the state error $\tilde{x}$ is uniformly ultimately bounded.

Proof: Consider the Lyapunov function candidate

$$V = \tilde{x}^T P^{-1}\tilde{x} + tr\left(\tilde{W}^T \gamma_W^{-1}\tilde{W}\right)/2 + \gamma_\psi^{-1}\tilde{\psi}^2/2 \qquad (19)$$

where $\tilde{W} = \hat{W} - W^*$ and $\tilde{\psi} = \hat{\psi} - \psi^*$.

By evaluating (19) along the trajectories of (14), (16) and (17), we obtain

$$\begin{aligned}\dot{V} &= 2\tilde{x}^T\tilde{W}\pi - 2l_0\hat{\psi}\tilde{x}^T P^{-1}\tilde{x} - 2\tilde{x}^T\varepsilon - 2\tilde{x}^T P^{-1}h \\ &- \gamma_0 \|\tilde{x}\| tr[\tilde{W}^T(\hat{W} - W_0)] - 2tr(\tilde{W}^T\tilde{x}\pi^T) \\ &- \psi_0\|\tilde{x}\|\tilde{\psi}\hat{\psi} + \psi_0\tilde{\psi}\|\tilde{x}\| + \psi_2 l_0\tilde{\psi}\|\tilde{x}\|^2 - \psi_0^2\|\tilde{x}\|/2\end{aligned} \qquad (20)$$

Furthermore, by using the representations $tr\left(\tilde{W}^T\tilde{x}\pi^T\right) = \tilde{x}^T\tilde{W}\pi$ and $2\tilde{\psi}\hat{\psi} = \tilde{\psi}^2 + \hat{\psi}^2 - \psi^{*2}$, the fact 1 and the definition (18), (20) can be upper bounded as

$$\begin{aligned}\dot{V} \le &- l_0\psi_2\hat{\psi}\|\tilde{x}\|^2 + (2\bar{\varepsilon}_0 + 2\bar{h}_0\|P^{-1}\|_F)\|\tilde{x}\| \\ &- \gamma_0\|\tilde{x}\|\|\tilde{W}\|_F^2/2 + \gamma_0|W^* - W_0\|_F^2\|\tilde{x}\|/2 \\ &- \psi_0\|\tilde{x}\|(\tilde{\psi}^2 + \hat{\psi}^2 - \psi^{*2})/2 + \psi_0\tilde{\psi}\|\tilde{x}\| \\ &+ \psi_2 l_0\tilde{\psi}\|\tilde{x}\|^2 - \psi_0^2\|\tilde{x}\|^2/2\end{aligned} \qquad (21)$$

By employing the definitions of ψ_2 and ψ^*, see (18), and recalling that $\tilde{\psi} = \hat{\psi} - \psi^*$, (21) implies

$$\dot{V} \le -\psi_2 l_0\psi^*\|\tilde{x}\|^2 + \psi_0\hat{\psi}\|\tilde{x}\| - \psi_0\|\tilde{x}\|\hat{\psi}^2/2 - \psi_0^2\|\tilde{x}\|^2/2 \qquad (22)$$

Since $\hat{\psi} \le \hat{\psi}^2/2 + 1/2$, we arrive at

$$\dot{V} \le -(\psi_2 l_0\psi^* + \frac{\psi_0^2}{2})\|\tilde{x}\|^2 + \psi_0\|\tilde{x}\|/2 \qquad (23)$$

Hence, $\dot{V} < 0$ as long as

$$\|\tilde{x}\| > \frac{\psi_0}{2\psi_2 l_0\psi^* + \psi_0^2} := \alpha \qquad (24)$$

Thus, since α is constant, by using Lyapunov arguments [10], we concluded that $\tilde{x}$ are uniformly ultimately bounded, with ultimate bound α. Based on (16) and (17) $\tilde{W},\tilde{\psi}$ are also bounded. Note that if, by any reason, $\|\tilde{x}\|$ escapes of the residual set Ω, where $\Omega = \{\ \tilde{x} \mid \|\tilde{x}\| \quad \alpha\ \}$, $\dot{V}$ becomes negative definite again, and force the convergence of the state error to the ball of radius α.

□

Remark 8: The existence of ψ^* is guaranteed as long as $\psi_0 \ge 2\bar{\varepsilon}_0 + 2\bar{h}_0\left\|P^{-1}\right\|_F + \gamma_0\bar{\beta}/2$. However, it is a mild condition, since any increase of ψ_0 has only a positive impact on the residual state error, as can be seen in (24).

Remark 9: Since the ultimate bound α is inversely proportional to ψ^*, which depends on an upper bound for the disturbances, see (18), the performance of the proposed method cannot be adversely affected by the increase of disturbances.

Remark 10: It should be noted that ψ^* might be nonunique. In fact,

$$\psi^* = 1 + \frac{1}{\psi_0}\left[\psi_0^2 - 2\psi_0\left(2\bar{\varepsilon}_0 + 2\bar{h}_0\left\|P^{-1}\right\|_F + \gamma_0\bar{\beta}^2/2\right)\right]^{\frac{1}{2}} \qquad (25)$$

also satisfy (22). However, note from (24) that the ultimate bound for the residual state error is practically of order $1/\psi_0$ for large ψ_0. Hence, the residual state error is, practically, not affected by disturbances, as long as the design constants are adequately selected. The above mentioned peculiarity, to the best of our knowledge, is the main advantage of the proposed scheme in comparison with the literature.

IV. Simulations

To illustrate the application of the proposed scheme, we consider a generalized Lü hyperchaotic system described by [13], [14]

$$\begin{aligned}\dot{\bar{x}} &= a(y - \bar{x}) + u_{\bar{x}} + d_{\bar{x}} \\ \dot{y} &= b\bar{x} - k\bar{x}z + \omega + u_y + d_y \\ \dot{z} &= -cz + l\bar{x}^2 + u_z + d_z \\ \dot{\omega} &= -d\bar{x} + u_\omega + d_\omega\end{aligned} \qquad (26)$$

where *a*, *b*, *c*, *d*, *l* and *k* are constant parameters, $u_{\bar{x}}, u_y, u_z$ and u_ω are control inputs, and $d_{\bar{x}}, d_y, d_z$ and d_ω are unknown disturbances. It was considered that *a*=10, *b*=40, *c*=2.5, *d*=10.6, *k*=1 and *l*=4. Notice that system (26) satisfies Assumptions 1-3, since the state variables evolved on compact sets.

To identify the uncertain system (26) the proposed identification model (13) and the adaptive laws (16) and (17) were implemented. The initial conditions for the hyperchaotic system and the identification model were $\bar{x}(0)=-4,\ y(0)=-8,\ z(0)=-6,\quad \omega(0)=12$ and $\hat{x}(0)=0,$ in order to evaluate the performance of the proposed algorithm under adverse initial conditions.

The others design parameter were chosen as u=0,

$$\gamma_W=9,\ \gamma_0=2.5, \psi_0=50, \psi_1=10,$$
$$lo=10,\ s(\cdot)=10/[1+\exp-0.5(\cdot)],$$

$$\pi=\left[s(\bar{x}); s(y); s(z); s(\omega); s^2(\bar{x}); s^2(y); s^2(z); s^2(\omega)\right],$$

$$P=\begin{bmatrix}100 & 0 & 0 & 0\\ 0 & 50 & 0 & 0\\ 0 & 0 & 100 & 0\\ 0 & 0 & 0 & 100\end{bmatrix},$$

and $P=\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\end{bmatrix}.$

By keeping all design parameters as before, we introduced disturbances at $t=0.1$ in order to check the robustness for the proposed method. Two cases are considered:

a) $h_1(x,t)=\eta\left[e^{0.1t}\quad 2\sin(5t)\quad e^{0.1t}\cos(t)\quad 0.1\log(10+2\eta)\right]^T$

b) $h_2(x,t)=\eta[\sin(t)\quad 1.2\sin(2t)\quad \cos(4t)\quad 1.5\sin(t)]^T,$

where $\eta=0.5\|x\|$ and $x=[\bar{x}\quad y\quad z\quad \omega]^T$.

It should be noted that the last disturbance h_1 is unbounded as $t\rightarrow\infty$. However, it was considered here in order to evaluate the residual state error performance in the presence of severe disturbances.

The performances in the estimation of the states $\bar{x}, y, z$ and ω when disturbance h_1 is present are shown in Figures 1-5, and when disturbance h_2 is present is shown in Figure 6. We can see that the simulations confirm the theoretical results, that is, the algorithm is stable and the residual state error was, practically, not affected by the disturbance in $t=0.1\,\mathrm{s}$.

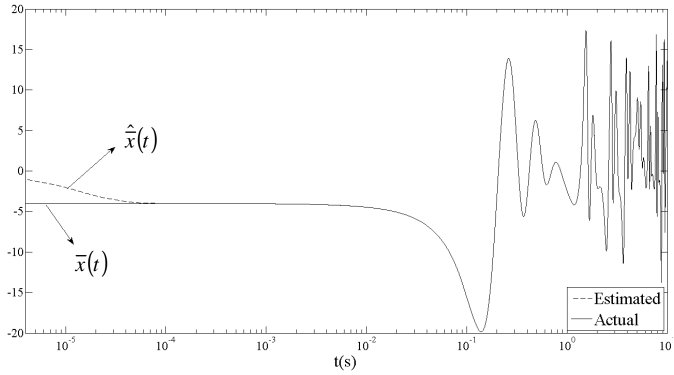

Figure 1. Actual and estimated state $\bar{x}$.

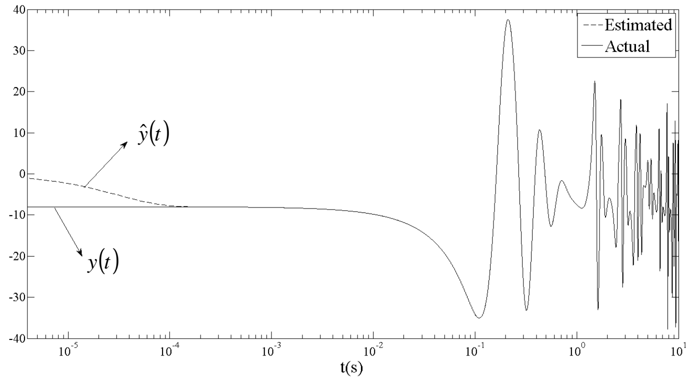

Figure 2. Actual and estimated state y.

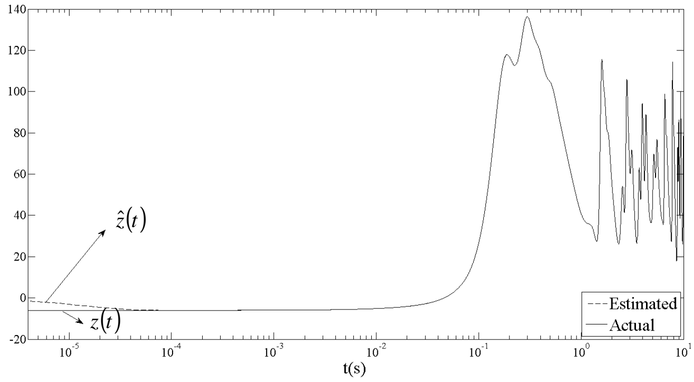

Figure 3. Actual and estimated state z.

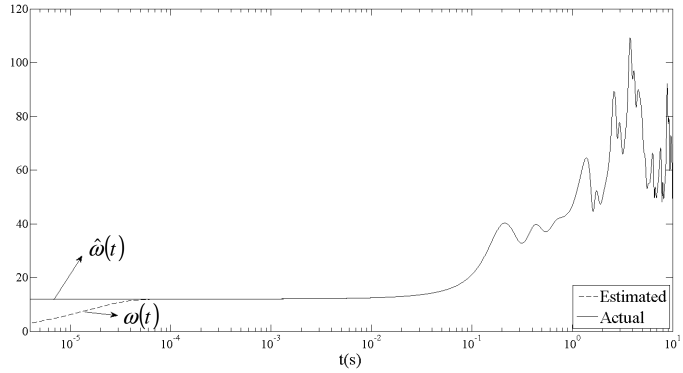

Figure 4. Actual and estimated state ω.

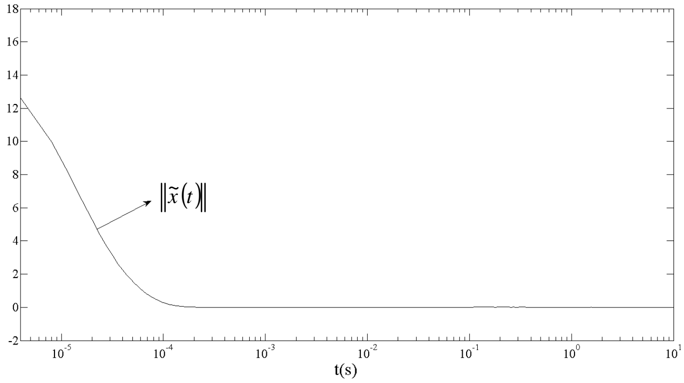

Figure 5. State error norm performance for disturbance h_1.

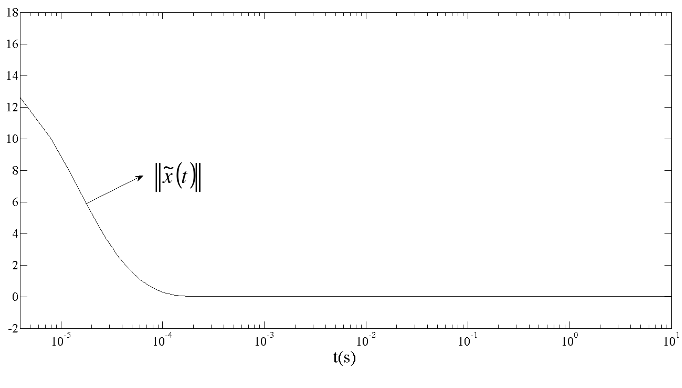

Figure 6. State error norm performance for disturbance h_2.

V. Conclusions

In this work, by using Lyapunov analysis and an adaptive bounding technique, we have proposed an on-line identification scheme which presents an improved tolerance in face of disturbances. The proposed algorithm is based on a ε_1-modification and uses an explicit feedback on the identification model to improve residual state error performance. A simulation example showed the effectiveness of the proposed method.

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