UKACC International Conference on Control 2012
Cardiff, UK, 3-5 September 2012

Robust predictor for uncertain dead time systems

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Abstract—**This paper presents a method to stabilize uncertain dead time system based on new robust state predictor. The proposed predictor consists of a state delayed observer. Controller gain and predictor parameters are calculated by solving a nonlinear matrix inequality. More importantly, this method is extended to dead time system with a long time delay or significant uncertainty using sequential sub-predictors (SSP). This predictor composed of collection of sub predictors that each of them predicts the state for a small part of a long time delay. The number of predictors can increase attending the unstability, delay value or uncertainty where the stability condition is satisfied. Examples illustrate the capability of this method.**

Keywords- Dead time systems, Robust Sequential sub-predictor, Robust control.

I. INTRODUCTION

Prediction of states or output plays a fundamental role in the control of dead time systems. This is because of delay in input impedes of stabilizing closed loop system via classical controller. To overcome this challenge, many efforts are devoted on presenting a dead time compensator (DTC) or predictor.

The famous model based predictor was presented by Smith [1] in 1957 for stable systems. Smith predictor is then modified for unstable system by Watanabe [2] (MSP) and for mixed unstable and stable systems (USP) in [3]. Moreover, [4] and [5] are applied it for stable systems with an integrator and long time delay. However, MSP may lead to unstable pole-zero cancelation for unstable systems. Therefore good approximate of distributed delay term in MSP is unavoidable to achieve the stability [6]. In addition, MSP is very sensitive to parameters and delay uncertainty. In recent years, some researchers try to improve the robustness of this method [6-12]. Also there exist a few examples that used this predictor for H_∞ control of dead time systems [13-15].

Another family of predictor is classified as Finite-Spectrum Assignment [16] (FSA). The main idea of some state predictor like as Artstein reduction model [17] is much closed to FSA. [18] has been proposed a different version of FSA attending the pole-assignment methods of delay free systems. Moreover, the FSA and MSP scheme can lead to equivalent stabilize method for single input delay systems [19]. So, FSA is also sensitive to the method of distributed delay approximation [6]. This challenge will be deeper for unstable system with long time delay and uncertainty.

The main problem in these method roots in inflexibility of FSA and MSP due to uncertainty of model. This is because of the delay term of dead time systems is eliminated directly by them and they have significant challenge when face to uncertainty in system model.

To address this challenge, this paper presents a new robust state predictor that is based on state delayed observer. This predictor forecasts the state of system asymptotically. The parameters of proposed predictor can be set attending the weight of uncertainty and time delay. The state feedback is then designed applying the prediction state and the robust stability of closed loop system is proven. More importantly, proposed state predictor is extended to sequential sub-predictors (SSP) for unstable systems with a long time delay. SSP is founded on a collection of sub-predictors. The state of system is successively forecasted by each sub-predictor for a small part of delay, such that totally, SSP predicts the state for whole time delay. Consequently, the state feedback is calculated using SSP. Examples illustrate the ability of this method to stabilize dead time systems with long time delay and uncertainty.

II. PROBLEM STATEMENT AND PRELIMINARIES

Consider linear input-delay uncertain system described by

$$\begin{cases} \dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t-d) \\ x(t) = \varphi(t) \quad \forall t \in [-d, 0] \end{cases} \tag{1}$$

where $x \in \Re^n$, $u \in \Re^m$ and $d > 0$. The matrices A and B are known and time-varying bonded matrices ΔA and ΔB are described the uncertainty of this system where

$$[\Delta A \; \Delta B] = DJ(t)\,[E \;\; E_b] \tag{2}$$

$$J(t)^T J(t) \le I \tag{3}$$

The delay in the input prevents achieving stability for unstable systems with long time delay or significant uncertainty. Therefore, it is suggested to predict the state of this system to eliminate the delay in state feedback. In the other words, if $x(t+d) \approx x_p(t)$, then the delay in the controllable input can be compensated by using the predicted state instead of real state, i.e. $u(t-d) = Kx_p(t-d) \approx Kx(t)$.

The target goal in this paper is to suggest a robust control method to stabilize dead time system based on new robust predictor. Section 3 presents the simple form of this predictor and also investigates the calculation of the predictor parameters and controller gain. This method is extended to sequential sub-predictor for unstable systems with long time delay in Section 4. The predictor parameters and controller gain are also designed in this section. Examples show the capability of this method to stabilize dead time systems in Section 5. First, necessary lemma is presented as follows.

Lemma 1: [20] Given matrices Ω, Γ and Σ of appropriate dimensions and with Ω symmetrical, then

$$\Omega + \Gamma J(\tau)\Sigma + \Sigma^T J(\tau)^T \Gamma^T < 0 \tag{4}$$

For all $F(\tau)$ satisfying $J(\tau)^T J(\tau) \le I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$\Omega + \varepsilon\Gamma\Gamma^T + \varepsilon^{-1}\Sigma^T\Sigma < 0 \tag{5}$$

III. STATE PREDICTOR

In this section, the initial format of robust state predictor is presented as

$$\dot{\bar{x}}(t) = A\bar{x}(t) + Bu(t) + L(\bar{x}(t-d) - x(t)) \tag{6}$$

where $\bar{x} \in \Re^n$ is the predicted state that will forecast x for d seconds. Error is described by

$$e(t) = \bar{x}(t-d) - x(t) \tag{7}$$

The state of predictor forecasts the state of system if error converges to zero, i.e.

$$\bar{x}(t) \to x(t+d) \tag{8}$$

Now, the error dynamic equation can be calculated as

$$\dot{e}(t) = Ae(t) - \Delta Ax(t) + Le(t-d) - \Delta Bu(t-d) \tag{9}$$

The predictor matrix L must be chosen such that the error converges to zero asymptotically. Following Theorem investigates the design of prediction parameter L such that error equation converges to zero and calculation of the controller gain, K, to achieve robust closed loop stability.

Theorem 1: Consider system (1) with following control law.

$$\begin{cases} \dot{\bar{x}}(t) = A\bar{x}(t) + Bu(t) + L(\bar{x}(t-d) - x(t)) \\ u(t) = K\bar{x}(t) \end{cases} \tag{10}$$

Assume that (A, B) is controllable. The closed-loop system is robust asymptotically stable and $\bar{x}$ predicts x for d second if there exist symmetric matrices $\bar{P}_0 > 0$, $\bar{P}_1 > 0$, $\bar{Q} > 0$, $\bar{S} > 0$, matrices Y, U, M_1, F_0 of appropriate dimensions, and scalar ε such that the following inequality holds.

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & -\tilde{d}\bar{Y} & \tilde{d}\bar{P}\tilde{A}^T & \bar{P}\tilde{E}^T + F^T E_b{}^T \\ * & \Omega_{22} & -\tilde{d}\bar{U} & \tilde{d}M^T & 0 \\ * & * & -\tilde{d}\bar{P}\bar{S}^{-1}\bar{P} & 0 & 0 \\ * & * & * & -\tilde{d}\bar{S} + \varepsilon\tilde{D}\tilde{D}^T & 0 \\ * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{11}$$

where

$$\Omega_{11} = \bar{P}\tilde{A}^T + \tilde{A}\bar{P} + \bar{Y} + \bar{Y}^T + \bar{Q} + \varepsilon\tilde{D}\tilde{D}^T$$

$$\Omega_{12} = M - \bar{Y} + \bar{U}^T$$

$$\Omega_{22} = -\bar{Q} - \bar{U} - \bar{U}^T$$

And

$$\bar{P} = diag\{\bar{P}_0, \bar{P}_1\},\ \tilde{A} = diag\{A, A\},\ F = [F_0 \quad 0],\ \tilde{d} = d,$$

$$\tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix},\ M = \begin{bmatrix} 0 & M_1 \\ 0 & M_1 \end{bmatrix},\ \tilde{D} = \begin{bmatrix} 0 \\ -D \end{bmatrix},\ \tilde{E} = [E \quad -E], \tag{12}$$

Moreover, L and K are given as:

$$K = P_0^{-1}F, \quad L = P_1^{-1}M_1 \tag{13}$$

Proof: By considering (7), the closed-loop system (1) and (10) can be rewritten as:

$$\dot{\tilde{x}}(t) = (\tilde{A} + \Delta\tilde{A} + \tilde{B}\tilde{K} + \Delta\tilde{B}\tilde{K})\tilde{x}(t) + \tilde{A}_h\tilde{x}(t-\tilde{d}) \tag{14}$$

where

$$\tilde{x} = \begin{bmatrix} \tilde{x}_1 = x + e \\ \tilde{x}_2 = e \end{bmatrix},\ \tilde{A}_h = \begin{bmatrix} 0 & L \\ 0 & L \end{bmatrix},\ \Delta\tilde{A} = \begin{bmatrix} 0 & 0 \\ -\Delta A & \Delta A \end{bmatrix},$$

$$\Delta\tilde{B} = \begin{bmatrix} 0 \\ -\Delta B \end{bmatrix},\ \tilde{K} = [K \quad 0], \tag{15}$$

and $\tilde{A}$ and $\tilde{B}$ appear in (12). The stability of (14) is equivalent to closed loop system (1) and (10) and $\bar{x}$ approaches $x(t+d)$ if $\tilde{x}_2 = e(t)$ converge to zero asymptotically. A Lyapunov function is candidate to investigate the stability of (14) as follows.

$$V(\tilde{x}) = V_1(\tilde{x}) + V_2(\tilde{x}) + V_3(\tilde{x}) \tag{16}$$

where

$$V_1(\tilde{x}) = \tilde{x}(t)^T P\tilde{x}(t)$$

$$V_2(\tilde{x}) = \int_{-\tilde{d}}^{0}\int_{t+\beta}^{t} \dot{\tilde{x}}(\alpha)^T S\dot{\tilde{x}}(\alpha)\, d\alpha\, d\beta$$

$$V_3(\tilde{x}) = \int_{t-\tilde{d}}^{t} \tilde{x}(\alpha)^T Q\tilde{x}(\alpha) d\alpha \tag{17}$$

By using Newton-Leibniz formula and free-weighting matrix (FWM), similar to the proof of Theorem 1 in [21], the derivative of $V(t)$ can be written as

$$\dot{V}(t) = \frac{1}{\tilde{d}}\int_{t-d}^{t} \xi(t,\alpha)^T \Lambda(\tilde{d})\xi(t,\alpha)d\alpha \tag{18}$$

where

$$\xi(t,\alpha) = \begin{bmatrix} x(t)^T & x(t-\tilde{d})^T & x(\alpha)^T \end{bmatrix}^T$$

$$\Lambda(\tilde{d}) = \begin{bmatrix} \Xi_{11} & \Xi_{12} & -\tilde{d}Y \\ * & -Q-U-U^T+\tilde{d}\tilde{A}_h{}^T S\tilde{A}_h & -\tilde{d}U \\ * & * & -\tilde{d}S \end{bmatrix} \tag{19}$$

where

$$\begin{aligned}\Xi_{11} &= (\tilde{A}+\Delta\tilde{A}+\tilde{B}\tilde{K}+\Delta\tilde{B}\tilde{K})^T P + P(\tilde{A}+\Delta\tilde{A}+\tilde{B}\tilde{K}+\Delta\tilde{B}\tilde{K}) \\ &\quad + d(\tilde{A}+\Delta\tilde{A}+\tilde{B}\tilde{K}+\Delta\tilde{B}\tilde{K})^T S(\tilde{A}+\Delta\tilde{A}+\tilde{B}\tilde{K}+\Delta\tilde{B}\tilde{K}) \\ &\quad + Y + Y^T + Q\end{aligned}$$

$$\Xi_{12} = P\tilde{A}_h - Y + U^T + d(\tilde{A}+\Delta\tilde{A}+\tilde{B}\tilde{K}+\Delta\tilde{B}\tilde{K})^T S\tilde{A}_h$$

Using Schur complement and Lemma 1, it is possible to show that $\Lambda(\tilde{d})$ is negative definite (i.e. $\dot{V}(t) < 0$) if following inequality holds.

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & -\tilde{d}Y & \tilde{d}\tilde{A}^T S & E^T + K^T E_b{}^T \\ * & \Psi_{22} & -\tilde{d}U & \tilde{d}\tilde{A}^T S & 0 \\ * & * & -\tilde{d}S & 0 & 0 \\ * & * & * & -\tilde{d}S + \varepsilon S\tilde{D}\tilde{D}^T S & 0 \\ * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{20}$$

where

$$\Psi_{11} = \tilde{A}^T P + P\tilde{A} + Y + Y^T + Q + \varepsilon P\tilde{D}\tilde{D}^T P$$

$$\Psi_{12} = P\tilde{A}_h - Y + U^T$$

$$\Psi_{22} = -Q - U - U^T$$

Taking $\overline{P} = P^{-1} = diag\{\overline{P}_0, \overline{P}_1\}$, $\overline{S} = S^{-1}$, $\overline{Q} = P^{-1}QP^{-1}$, $\overline{Y} = P^{-1}YP^{-1}$, $\overline{U} = P^{-1}UP^{-1}$, pre- and post multiplying both sides of LMI (20) by $diag\{\overline{P},\overline{P},\overline{S},\overline{P},I\}$ and considering (13), it is possible to rewrite (20) as (11). □

Note that (11) is not linear due to $\overline{P}\,\overline{S}^{-1}\overline{P}$ term. The best idea to solve it without any limitation of degree of freedoms is reducing the original non-convex problem to an LMI-based nonlinear minimization problem. Then a modified cone complementarity linearization (CCL) algorithm [22] can be used to obtain a solution. This method is described in next section.

Theorem 1 may not be useful for unstable dead systems with a long time delay and weighty uncertainty (see examples in Section 5). To overcome this challenge, next section presents an extended form of this predictor called sequential sun-predictors. In this method, a series of sub-predictor is employed to each of them forecasts the state of system for small part of time delay.

IV. SEQUENTIAL SUB -PREDICTORS

Long time delay may impede stabilizing of unstable systems with uncertainty using proposed predictor in Section 3 and it may exist no L such that (11) is feasible. For this case, sequential sub-predictor (SSP) is suggested in this section. In SSP, time delay is divided to R small part, and then a collection of successive sub-predictors are used to forecast the state for each small part of delay, $\overline{d}$, where

$$\overline{d} = \frac{d}{R}, \qquad R \in Z^+ \tag{21}$$

The SSP is described by

$$\begin{cases} \dot{\overline{x}}_1(t) = A\overline{x}_1(t) + Bu(t-\overline{d}) \\ \qquad\qquad + L_1(\overline{x}_1(t-\overline{d}) - \overline{x}_2(t)) \\ \vdots \\ \dot{\overline{x}}_{R-1}(t) = A\overline{x}_{R-1}(t) + Bu(t-(R-2)\overline{d}) \\ \qquad\qquad + L_{R-1}(\overline{x}_{R-1}(t-\overline{d}) - \overline{x}_R(t)) \\ \dot{\overline{x}}_R(t) = A\overline{x}_R(t) + Bu(t-(R-1)\overline{d}) \\ \qquad\qquad + L_R(\overline{x}_R(t-\overline{d}) - x(t)) \end{cases} \tag{22}$$

where $\overline{x}_i \in \Re^n, i = 1,\ldots,R$, Defining the prediction error as

$$\begin{cases} e_1(t) = \overline{x}_1(t-R\overline{d}) - \overline{x}_2(t-(R-1)\overline{d}) \\ \vdots \\ e_{R-1}(t) = \overline{x}_{R-1}(t-2\overline{d}) - \overline{x}_R(t-\overline{d}) \\ e_R(t) = \overline{x}_R(t-\overline{d}) - x(t) \end{cases} \tag{23}$$

Note that $\overline{x}_1(t)$ predicts the $x(t+d)$, if all error equation converge to zero i.e.

$$e_t(t) = \overline{x}_1(t-R\overline{d}) - x(t) = e_1(t) + \cdots + e_R(t) \tag{24}$$

The error dynamics are

$$\begin{cases} \dot{e}_1(t) = Ae_1(t) + L_1e_1(t-\overline{d}) - L_2e_2(t-\overline{d}) \\ \vdots \\ \dot{e}_{R-1}(t) = Ae_{R-1}(t) + L_{R-1}e_{R-1}(t-\overline{d}) - L_Re_R(t-\overline{d}) \\ \dot{e}_R(t) = Ae_R(t) - \Delta Ax(t) + L_Re_R(t-\overline{d}) - \Delta Bu(t-d) \end{cases} \tag{25}$$

Following theorem investigates stability of closed loop system based on SSP. In this theorem, predictor parameters, L_i, and controller gain, K, will be calculated.

Theorem 2: Consider system (1) with following control law.

$$\begin{cases} \dot{\bar{x}}_1(t) = A\bar{x}_1(t) + Bu(t-\bar{d}) \\ \qquad\qquad + L_1(\bar{x}_1(t-\bar{d}) - \bar{x}_2(t)) \\ \vdots \\ \dot{\bar{x}}_R(t) = A\bar{x}_R(t) + Bu(t-(R-1)\bar{d}) \\ \qquad\qquad + L_R(\bar{x}_R(t-\bar{d}) - x(t)) \\ u(t) = K\bar{x}_1(t) \end{cases} \tag{26}$$

Assume that (A, B) is controllable. The closed-loop system is robust asymptotically stable and $\bar{x}_1$ predicts x for $d = R\bar{d}$ second if there exist symmetric matrices $\bar{P}_i > 0,\ i = 0,1,\cdots,R,$ $\bar{Q} > 0,\ \ \bar{S} > 0$, matrices $Y,\ U,\ M_j,\ \ j = 1,\cdots,R,\ F_0$ of appropriate dimensions, and scalar ε such that (11) holds, substituted

$$\bar{P} = diag\{\bar{P}_0, \bar{P}_1, \cdots, \bar{P}_R\},\ \tilde{A} = diag\{A, A, \cdots, A\},\ \tilde{d} = d/R,$$

$$\tilde{B} = \begin{bmatrix} B \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix},\ M = \begin{bmatrix} 0 & M_2 & 0 & \cdots & 0 \\ 0 & M_2 & -M_2 & \ddots & 0 \\ 0 & 0 & M_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & -M_R \\ 0 & 0 & 0 & \cdots & M_R \end{bmatrix},\ \tilde{D} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -D \end{bmatrix},$$

$$F = \begin{bmatrix} F_0 & 0 & 0 & \cdots & 0 \end{bmatrix},\ \tilde{E} = \begin{bmatrix} -E & E & E & \cdots & E \end{bmatrix}. \tag{27}$$

Moreover, L and K are given as:

$$K = P_0^{-1}F,\quad L = P_j^{-1}M_j,\ j = 1,\cdots,R \tag{28}$$

Proof: By considering (23), the closed-loop system (1) and (26) can be rewritten as:

$$\dot{\tilde{x}}(t) = (\tilde{A} + \Delta\tilde{A} + \tilde{B}\tilde{K} + \Delta\tilde{B}\tilde{K})\tilde{x}(t) + \tilde{A}_h\tilde{x}(t-\tilde{d}) \tag{29}$$

where

$$\tilde{x} = \begin{bmatrix} \tilde{x}_1 = x + e_1 + \cdots + e_R \\ \tilde{x}_2 = e_1 \\ \vdots \\ \tilde{x}_R = e_R \end{bmatrix},\ \Delta\tilde{B} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -\Delta B \end{bmatrix},$$

$$\tilde{A}_h = \begin{bmatrix} 0 & L_2 & 0 & \cdots & 0 \\ 0 & L_2 & -L_2 & \ddots & 0 \\ 0 & 0 & L_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & -L_R \\ 0 & 0 & 0 & \cdots & L_R \end{bmatrix},\ \tilde{K} = \begin{bmatrix} K & 0 & \cdots & 0 \end{bmatrix}. \tag{30}$$

$\tilde{A}$ and $\tilde{B}$ appear in (27). The stability of (29) is equivalent to closed loop system (1) and (26) and $\bar{x}_1$ approaches $x(t+d)$ if $\tilde{x}_i$, $i = 2, \cdots, R$ converge to zero asymptotically, because $e_t(t) = \tilde{x}_2 + \cdots + \tilde{x}_R$. Based the same form of (29) and (14), proof can be followed same as proof of Theorem 1. □

To solve (11), using the modified CCL, from Schur complement, it is followed that (11) holds if

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & -\tilde{d}Y & \tilde{d}\bar{P}\tilde{A}^T & \bar{P}\tilde{E}^T + F^T E_b{}^T \\ * & \Omega_{22} & -\tilde{d}U & \tilde{d}M^T & 0 \\ * & * & -\tilde{d}T & 0 & 0 \\ * & * & * & -\tilde{d}\bar{S} + \varepsilon\tilde{D}\tilde{D}^T & 0 \\ * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{31}$$

where

$$-\bar{P}\bar{S}^{-1}\bar{P} < -\bar{T} \tag{32}$$

Taking $T = \bar{T}^{-1}$, $S = \bar{S}^{-1}$, and $P = \bar{P}^{-1}$, from Schur complement (32) becomes

$$\begin{bmatrix} T & P \\ * & S \end{bmatrix} > 0 \tag{33}$$

The existence of a solution for (31) and (33) is a sufficiently condition for the feasibility of (11), imposing some degree of conservativeness. However, using this technique, the original of non-convex problem has been casted into the following LMI based non-linear minimization problem:

Minimize $Tr(T\bar{T} + P\bar{P} + S\bar{S})$, subject to (33) and

$$\begin{bmatrix} T & P \\ * & S \end{bmatrix} > 0,\ \begin{bmatrix} \bar{T} & I \\ * & T \end{bmatrix} > 0,\ \begin{bmatrix} \bar{S} & I \\ * & S \end{bmatrix} > 0,\ \begin{bmatrix} \bar{P} & I \\ * & P \end{bmatrix} > 0 \tag{34}$$

Then, the modified CCL algorithm is used to solve it and find the maximum possible d as following procedure.

Step 1: Solve (31) and (34) for sufficiently small initial value of $\tilde{d}_0$ and find a feasible set $\{P_0, \bar{P}_0, S_0, \bar{S}_0, T_0, \bar{T}_0, \bar{Y}_0, \bar{U}_0, \bar{Q}_0, \cdots\}$ satisfying them. Set $j = 0,\ i = 0$.

Step 2: Solve the LMI (31) and (34) for all variables:

Minimize $Tr(T\bar{T}_i + \bar{T}T_i + P\bar{P}_i + \bar{P}P_i + S\bar{S}_i + \bar{S}S_i)$ subject to (31) and (34). Set $T_{i+1} = T$, $\bar{T}_{i+1} = \bar{T}$, $P_{i+1} = P$, $\bar{P}_{i+1} = \bar{P}$, $S_{i+1} = S$ and $\bar{S}_{i+1} = \bar{S}$.

Step 3: If (11) is satisfied and $d = \tilde{d}_i$ ($d = R\tilde{d}_i$ for Theorem 2), end. If (11) is not satisfied within a specified number of iterations (j), then exit with no solution. If (11) is satisfied but $d > \tilde{d}_i$ ($d > R\tilde{d}_i$ for Theorem 2), set $i = i + 1$, $j = 0$, increment $\tilde{d}_i$ and go to Step 2. Otherwise, set $i = i+1$, $j = j+1$ and go to Step 2.

Note that the number of sub-predictors, R, can be set sufficiently big to (11) becomes feasible. In the other words, if (11) is not satisfied for d in a usual number of CCL iterations, then R should be increased while it becomes feasible. Therefore this method can stabilize all unstable systems with long time delay and significant uncertainty. Next section presents a few examples to illustrate the capability of this method to stabilize unstable dead time systems with time varying uncertainty.

V. SIMULATION RESULTS

In this section a few examples are presented to shows the ability SSP to closed loop stability of dead time systems due to unstability of system, time delay value and weight of uncertainty.

Example 1: Consider system $\dot{x} = ax + bu(t-d)$, where a, b is scalar. SSP can forecast the states of systems for maximum delay $d = Rd_m$ that is shown in Table 1 (for iteration number 40 and increment and increment delay step 0.01).

Table 1: Maximum possible delay to prediction of state

$d = Rd_m$	$R=1$	$R=2$	$R=3$	$R=4$	$R=5$
$a=-1$	∞	∞	∞	∞	∞
$a=0.2$	4.93	9.77	14.55	19.33	23.05
$a=0.5$	1.96	3.85	5.72	7.56	9.13
$a=1$	0.97	1.90	2.8	3.69	4.51
$a=2$	0.48	0.93	1.36	1.78	2.15

Table 1 shows that maximum possible delay to stabilize of closed loop system is directly proportional to R and inversely proportional to a. Although the increasing the a, limits the bound of delay, but it is possible to compensate it by increasing the number of sub-predictor.

Example 2: Consider an unstable uncertain system with integrator term and non-minimum phase zero, defined as

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0+.02\sin(t) & 0 \\ 0 & 0.5+.02\sin(t) \end{bmatrix} x(t) + \begin{bmatrix} 2 \\ -2 \end{bmatrix} u(t-3) \\ y(t) = \begin{bmatrix} -1 & 1 \end{bmatrix} x(t), \qquad x(0) = \begin{bmatrix} 1 & 0 \end{bmatrix}^T \end{cases}$$

Using Theorem 2, the controller gain and SSP matrices are calculated as:

$$K = \begin{bmatrix} 1.15 & 4.59 \end{bmatrix},\ L_1 = \begin{bmatrix} -0.04 & 0.12 \\ -0.05 & -0.86 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} -0.08 & 0.23 \\ -0.01 & -0.82 \end{bmatrix},\ L_3 = \begin{bmatrix} -0.2 & 0.34 \\ -0.01 & -0.70 \end{bmatrix},$$

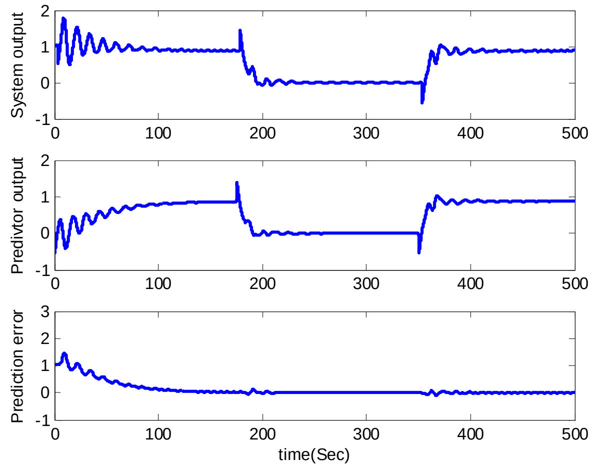

Figure.1 Pulse response of closed loop system

Note that the initial conditions of system are unknown in prediction and the initial conditions of SSP are set to zero. Figure 1 illustrates the pulse response of the closed loop system. This example shows the capability of this method to stabilize uncertain and unstable dead time systems with a long time delay.

VI. CONCLUSION

This paper suggests a method to stabilize uncertain dead time systems based on a new robust predictor. Moreover, this method is applied for unstable dead time systems with a long time delay by sequential sub-predictors. The main idea in this predictor is composed of a series of sub-predictor; each of them is for a partition of long time delay. This method can improve the flexibility of predictor to overcome any weight uncertainty. This method also can apply for nonlinear systems and output feedback that will be presented in future works.

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