UKACC International Conference on Control 2012
Cardiff, UK, 3-5 September 2012

Rapid Alignment Method of INS with Large Initial Azimuth Uncertainty under Complex Dynamic Disturbances

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Abstract—**For the rapid alignment of the ship-borne weapon INS with large initial azimuth attitude error under complex environment disturbances, a nonlinear error propagation model augmented by sensor errors and disturbance sources was proposed. Velocity plus angular rate matching method was applied in the implementation of the alignment. Simulation results show that comparing with the conventional solutions, this method can accomplish the transfer alignment of a mooring ship's weapon INS with large heading error rapidly and accurately. Meanwhile, it has strong adaptability to the sea condition and can improve the precision of the alignment under complex environment disturbances.**

I. Introduction

Aligning SINS (slave inertial navigation system) of lower accuracy by MINS (master inertial navigation system) is called transfer alignment (TA). TA method can align the SINS with short time and high accuracy. Conventional TA error propagation model based on the hypothesis of small misalignment angle between SINS and MINS is linear; however, there might be large heading error between the frames of the two INSs in ship-borne TA process[1],[2]. Thus, it is necessary to study the methodology of TA with large azimuth misalignment.

In the works [3]and [4], velocity matching was used to execute the TA with large heading misalignment angle of SINS, but the long alignment time cannot meet the operational requirements. In the research [5] , velocity plus angular rate matching TA method estimated attitude errors accurately and rapidly only by virtue of swaying movement caused by sea waves, when the misalignment angles are small. Meanwhile, it can give good estimations of gyro-drifts. However, there are few researches on this matching method under the condition of large azimuth attitude error between SINS and MINS.

There are some special problems for the transfer alignment between the ship-borne weapon INS and the Master INS. On the one hand, it is inappropriate for large warship to execute maneuver in order to accomplish the alignment. On the other hand, ship-borne TA is badly affected by environment disturbance. Based on the above considerations, a new approach of the TA for ship-borne weapon with large azimuth error was studied in this paper. This alignment method used the differences of velocity and angular rates between SINS and MINS as the measurements. Meanwhile, dynamic and static lever arm errors and flexure deformation are augmented to system model of the TA. The alignment method was executed by UKF. Performance of this approach was tested under two typical sea conditions by the simulation.

II. Error propagation Model of the Transfer Alignment

A. State-Equations of Transfer Alignment

According to [6], the error propagation equation with the large azimuth misalignment is

$$\dot{\phi} = (I - C_n^{n'})\omega_{in}^n - C_b^{n'}\varepsilon + \eta_\phi \tag{1}$$

In the research [8], lever arm effects were compensated in the velocity error propagation equation, it needs angular acceleration information to calculate compensation item, and however, the angular acceleration cannot be obtained directly. Difference calculation is needed in order to get angular acceleration, but it will bring extra alignment error. Therefore, in this paper, instead of compensating lever-arm effects in velocity error propagation equation, those errors were compensated in the velocity measurement. Thus, the velocity error propagation equation is

$$\delta\dot{v}_s^n = [I - (C_n^{n'})^T]C_b^{n'}f^b - (2\omega_{ie}^n + \omega_{en}^n) \times \delta v_s^n + C_b^{n'}\bigtriangledown^b + \eta_v \tag{2}$$

In (1)and (2), ϕ is the attitude misalign vector between SINS′s and MINS′s navigation frames, δv_s^n is the velocity error vector of SINS, $\eta_\phi$and η_v are the noise vectors of attitude and velocity, $C_n^{n'}$ is the transform matrix from the true navigation frame(n) to the SINS′s navigation frame(n'), $C_b^{n'}$ is the body-axes-to-navigation direction cosine matrix of SINS. When the azimuth attitude error is large, $C_{n'}^n$ is given by the following form

$$C_{n'}^n = \begin{bmatrix} cos\phi_z & -sin\phi_z & \phi_y cos\phi_z + \phi_x sin\phi_z \\ sin\phi_z & cos\phi_z & \phi_y sin\phi_z - \phi_x cos\phi_z \\ -\phi_y & \phi_x & 1 \end{bmatrix} \tag{3}$$

The flexure deformation, which is generated by sea waves, winds and engine vibration, can be approximated by a second-order Markov process [6] [7]

$$\ddot{\theta}_{fi} + 2\beta_i \dot{\theta}_{fi} + \beta_i^2 \theta = \eta_i \quad (i = x, y, z) \tag{4}$$

Where, $\theta = [\theta_x, \theta_y, \theta_z]^T$ is the flexure deformation vector, and its variance is $\sigma = [\sigma_x, \sigma_y, \sigma_z]^T$, $\eta = [\eta_x, \eta_y, \eta_z]^T$ is a white noise process with strength given by the diagonal matrix Q_n. $\beta = [\beta_x, \beta_y, \beta_z]^T$ is a constant vector. The relationship between Q_{ni}, $\sigma_i$and β_i is $Q_{ni} = 4\beta_i^3 \sigma_i^2 \quad (i = x, y, z)$,and the correlation time τ_i of the stochastic process is related with β_i by $\beta_i = 2.146/\tau_i \quad (i = x, y, z)$.

For the time of the TA is short, the random errors can be treated as white noises besides the zero-biases of the gyros and accelerometers. Thus, the models of gyro and accelerometer can be approximated as

$$\varepsilon = \varepsilon_b + \varepsilon_w \tag{5}$$

$$\bigtriangledown = \bigtriangledown_b + \bigtriangledown_w \tag{6}$$

where, ε_b is constant zero-drift, ε_w is Gaussian white noise. $\bigtriangledown_b$ is constant zero-bias, $\bigtriangledown_w$ is Gaussian white noise.

The measurement error vector of static lever arm is constant, thus $\delta \dot{r}_{meas}^T = 0$. According to the investigation [8], since the deformation angle is small, the dynamic lever arm length (δr_f), which is caused by the deformation, can be described as $\delta r_f = R\theta_f$, thus

$$\delta \dot{r}_f = R\dot{\theta}_f = R\omega_f \tag{7}$$

where,

$$H = \begin{bmatrix} 0 & z_0 & 0 \\ 0 & 0 & x_0 \\ y_0 & 0 & 0 \end{bmatrix} \tag{8}$$

Comparing with the conventional TA model of large azimuth misalign, in order to estimate those environment disturbances in the alignment process, this augmented model is established.

B. Measurement Equations

The measurements of this TA method are velocity and angular rate differences of of SINS and MINS.

As discussed above, the lever-arm effects should be compensated in the velocity matching measurement, thus

$$\Delta v = v_s^n - v_m^n = \delta v_s^n + v_r^n \tag{9}$$

The compensation for velocity measurement is governed by (10), if the measurement of the lever-arm is accurate

$$v_r^n = C_m^n(\omega_{im}^m \times r_0^m) + C_m^n \dot{r}_0^m \tag{10}$$

where, r_0^m is the deterministic lever-arm in the body-frame of MINS, which can be measured beforehand, and $\dot{r}_0^m = 0$ in this situation,$\times$ stands for the skew matrix of the vector.

However, there are some uncertain errors, which cannot be measured accurately. These errors are the static lever-arm measurement error (δr_{meas}) and the dynamic lever-arm error (δr_f) caused by deformation. Thus, the real lever-arm is as following

$$r = r_0 + \delta r_{meas} + \delta r_f \tag{11}$$

Accordingly, the compensation item of the velocity measurement is

$$v_r^n = C_m^n[\omega_{im}^m \times (r_0^m + \delta r_{meas}^m + \delta r_f^m)] + C_m^n \delta \dot{r}_f \tag{12}$$

where, $\omega_{im}^m \times$ is the skew matrix of vector ω_{im}^m.

After compensating the deterministic lever-arm effect by (10), the velocity matching measurement is still affected by δr_{meas} and δr_f, and it is governed by (13)

$$\begin{aligned} \Delta v &= \delta v_s^n + C_m^n[\omega_{im}^m \times (\delta r_{meas}^m + \delta r_f^m)] + C_m^n \delta \dot{r}_f \\ &= \delta v_s^n + C_m^n[\omega_{im}^m \times (\delta r_{meas}^m + \delta r_f^m)] + C_m^n R\omega_f \end{aligned} \tag{13}$$

The angular rate measurement is the difference of angular rates of SINS and MINS in each navigation frame, therefore

$$\Delta\omega = \omega_{ibs}^{n'} - \omega_{ibm}^n = C_{bs}^{n'}\omega_{ibs}^{bs} - C_{bm}^n \omega_{ibm}^{bm} \tag{14}$$

where, bm and bs are the body frames of MINS and SINS respectively. ω_{ibs}^{bs} and $\omega_{ibm}^{bm}$are the outputs of angular rates of SINS and MINS respectively. The relationship of ω_{ibs}^{bs} and ω_{ibm}^{bm} is

$$\omega_{ibs}^{bs} = C_{bm}^{bs}(\omega_{ibm}^{bm} + \omega_f^{bm}) + \varepsilon \tag{15}$$

Substituting (15) into (14), then the

$$\begin{aligned} \Delta\omega &= C_{bm}^{n'}\omega_{ibm}^{bm} + C_{bs}^{n'}\omega_f^{bs} + C_{bs}^{n'}\varepsilon - C_{bm}^n \omega_{ibm}^{bm} \\ &= C_n^{n'}\omega_{ibm}^n - \omega_{ibm}^n + C_{bs}^{n'}\omega_f^{bs} + C_{bs}^{n'}\varepsilon \\ &= (C_n^{n'} - I)\omega_{ibm}^n + C_{bs}^{n'}\omega_f^{bs} + C_{bs}^{n'}\varepsilon \end{aligned} \tag{16}$$

Comparing with the traditional TA method, the circumstance disturbances are augmented to this TA model, and these errors can also be reflected in the measurement equations.

III. Simulation verification of the TA method

Augmenting the deformation and lever-arm errors to the TA model, the states of the TA filter are as follows

$$X = \begin{bmatrix} \phi^T & \delta v^T & \varepsilon^T & \bigtriangledown^T & \theta_f^T & \omega_f^T & \delta r_{meas}^T & \delta r_f^T \end{bmatrix}^T \tag{17}$$

Summarizing the TA filter model, the state vector is defined as (17), and the corresponding state equation consists of the attitude error (1), velocity error (2), the disturbances (4) and (7) ,and constant zero-bias of IMU (5)(6).

Adding velocity measurement noise and angular rate measurement noise to (13) and (16), the measurement equations of this TA filter are

$$\begin{cases} Z_{\Delta v} = \Delta v + \lambda_{\Delta v} \\ Z_{\Delta\omega} = \Delta\omega + \lambda_{\Delta\omega} \end{cases} \tag{18}$$

where, $\lambda_{\Delta v}$ and $\lambda_{\Delta\omega}$ are the measurement noises of velocity matching and angular rate matching respectively.

The implementation of this TA method is by UKF [4], [10].

The ship is mooring in the TA process. Under the waves excitation, the ship sways around the pitch axis, the roll axis

TABLE I
PARAMETER SETTINGS OF FLEXURE DEFORMATION

Sea condition	Parameter			
	Variable	Roll	Pitch	Yaw
Medium sea	Amplitude	$1'$	$15'$	$3'$
	Correlation time	60s	60s	60s
Peaceful sea	Amplitude	$1'$	$6'$	$2'$
	Correlation time	60s	60s	60s

TABLE II
ERRORS OF THE SINSs IMUs

Sensor error	Accelerometer		Gyro	
	Constant zero-bias	Random walk coefficient	Constant zero-drift	Random walk coefficient
Degree1	100 μg	10μg/$\sqrt{h}$	0.01°/h	0.001°/$\sqrt{h}$
Degree2	400μg	40 μg/$\sqrt{h}$	1°/h	0.05°/$\sqrt{h}$

and the heading axis separately for sinusoidal motion. The model is given as follows

$$\begin{cases} \psi = \psi_m \sin(\omega_p t + \varphi_p) \\ \gamma = \gamma_m \sin(\omega_r t + \varphi_r) \\ H = H_m \sin(\omega_h t + \varphi_h) \end{cases} \tag{19}$$

where, ψ_m ,γ_m and H_m are the amplitudes of the sway. $\omega_i = 2\pi/T_i (i = p, r, h)$,and T_i is the period of sways. $\varphi_i (i = p, r, h)$ is the initial phase, and $\varphi_i(t)$ is a random value in $(-2\pi, 2\pi)$. Parameters of the above model are determined by the sea condition, two sea conditions are studied in this paper, one is medium sea condition, and another is peaceful sea condition, the parameters of these two conditions are

For medium sea condition,
$\psi_m = 2.5°$,$\gamma_m = 12°$,$H_m = 4°$;
$T_p = 8s$,$T_r = 13s$,$T_h = 40s$.
For peaceful sea condition,
$\psi_m = 0.5°$,$\gamma_m = 2°$,$H_m = 2°$;
$T_p = 5s$,$T_r = 10s$,$T_h = 60s$

The parameter settings of the flexure deformation are shown in table I. The TA method is also tested by two different accuracies (see table II) of the SINSs IMUs

This TA filter can estimate flexure deformation angular rate accurately, the results are shown in Figure 1.

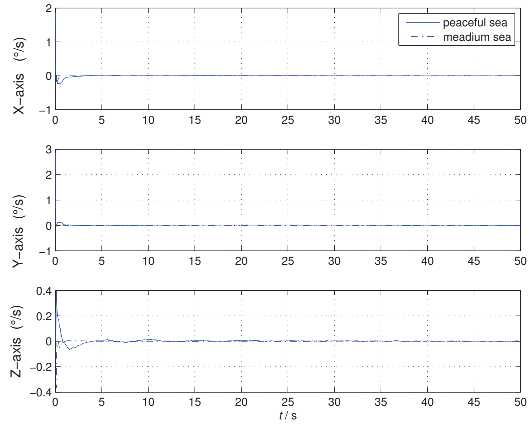

Fig. 1. Estimation bias of flexure deformation angular rate.

The disturbance of the lever-arm effect becomes bigger as the strength of the ships sway increases. Thus, in the following simulation, the influence of the lever-arm error is analyzed in the medium sea condition. where, let $\delta r^m_{meas} = [0.5\text{m}, 0.5\text{m}, 0.5\text{m}]^T$. The velocity errors caused by δr_{meas} and δr_f are shown in Figure2 and Figure3.

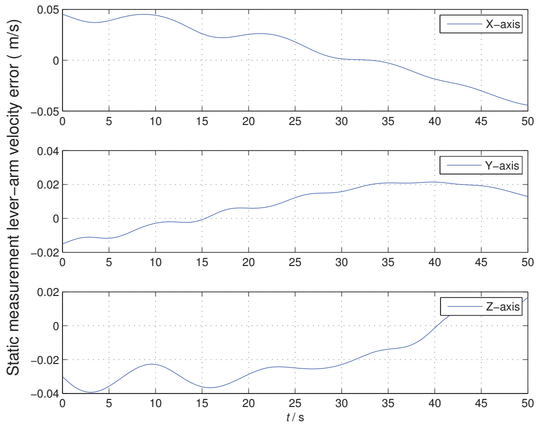

Fig. 2. Velocity error caused by the static lever-arm measurement error.

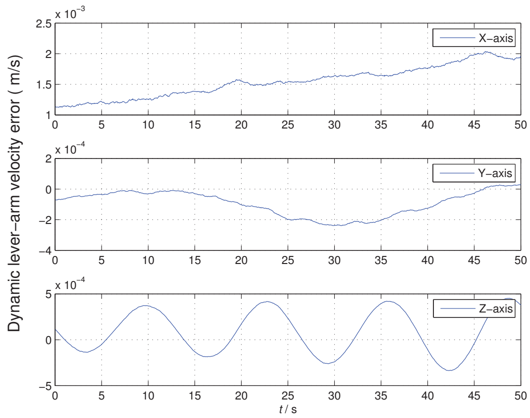

Fig. 3. Velocity error caused by the dynamic lever-arm error.

From Figure2 to Figure5, we can see that the velocity error caused by δr_{meas} and δr_f influenced the velocity measurement in certain degree, and this disturbance is even more harmful to SINS of high sensor accuracy (such as a shipboard aircraft). Thus, in order to get useful velocity measurement for the TA filter, these errors must be compensated. This TA method can estimate δr_{meas} and δr_f accurately, the estimation errors of them are shown in Figure6 and Figure7.

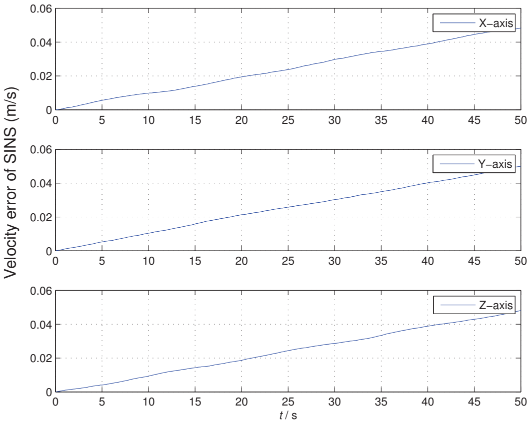

Fig. 4. Velocity error of SINS with IMU error degree 1.

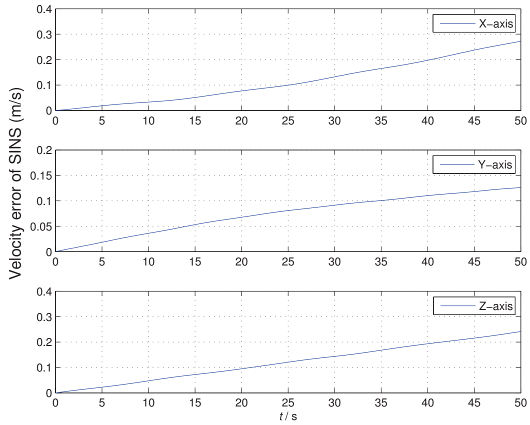

Fig. 5. Velocity error of SINS with IMU error degree 2.

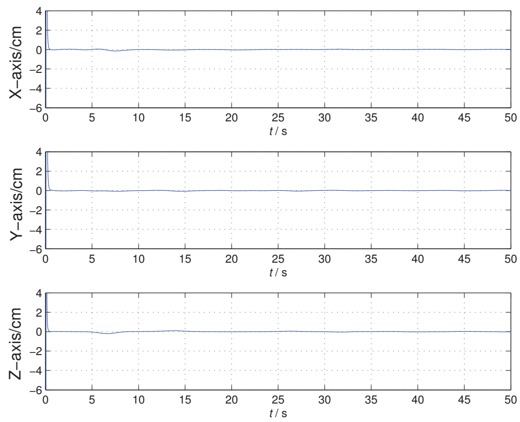

Fig. 6. Estimation bias of the static lever-arm measurement error.

The static lever-arm measurement error of a shipboard aircraft can be more indeterminate, thus, let $\delta r^m_{meas} = [1.5\text{m}, 1.5\text{m}, 1.5\text{m}]^T$ in the following simulation. Table III shows the TA results of partial or all disturbance compensations. In situation 1, there are no compensations of both lever-arm error and deformation. In situation 2, there is only lever-arm error compensation. In situation 3, there is only deformation compensation. In situation 4, all the disturbances are compensated by the method of this paper.

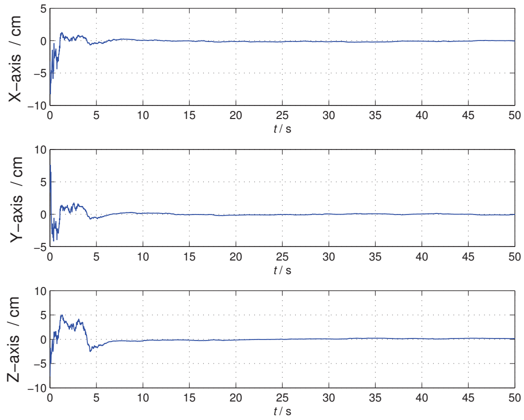

Fig. 7. Estimation bias of the dynamic lever-arm error.

TABLE III
TA RESULTS OF DIFFERENT COMPENSATION SITUATIONS

Estimation error	Different compensation situations			
	Situation1	Situation2	Situation3	Situation4
ϕ_x	$9.9701'$	$9.4511'$	$5.1821'$	$4.0645'$
ϕ_y	$16.5467'$	$15.9928'$	$4.3780'$	$3.9019'$
ϕ_z	$13.7641'$	$12.7643'$	$6.4335'$	$5.0075'$

The simulation results shows that both flexure deformation and lever-arm error affect the accuracy of the TA in some degree, see Table III, whereas the TA method in this paper can compensate all these disturbances very well, and the alignment accuracy can be improved a lot. From the alignment results in the four situations of different sea condition and sensor error , as shown in Figure8 through Figure11, we can get that this TA method has strong adaptability to the sea condition, and the degree of sensor error does not affect the TA results too much (see table IV).

The above results are acquired in the situation that the measurement noises of actual value and the TA filter are same.

TABLE IV
ESTIMATION BIASES OF MISALIGNMENT ANGLES OF THIS TA METHOD

Sea condition	Sensor error degree	ϕ_x	ϕ_y	ϕ_z
Medium sea	Degree 1	$4.0645'$	$3.9019'$	$5.0075'$
	Degree 2	$4.5735'$	$4.3017'$	$5.5971'$
Peaceful sea	Degree 1	$4.8917'$	$4.0133'$	$5.4335'$
	Degree 2	$5.0103'$	$4.3855'$	$5.8011'$

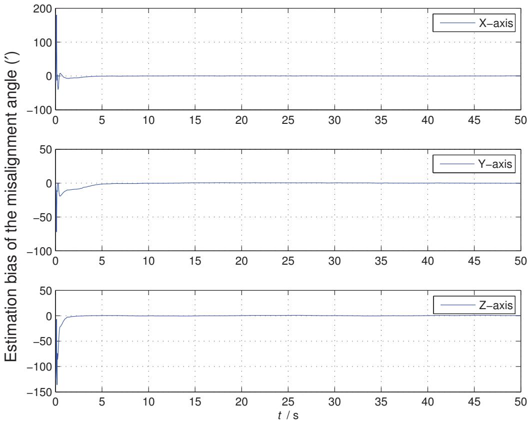

Fig. 8. Estimation bias of the misalignment angle, in the condition of medium sea and sensor error degree 1.

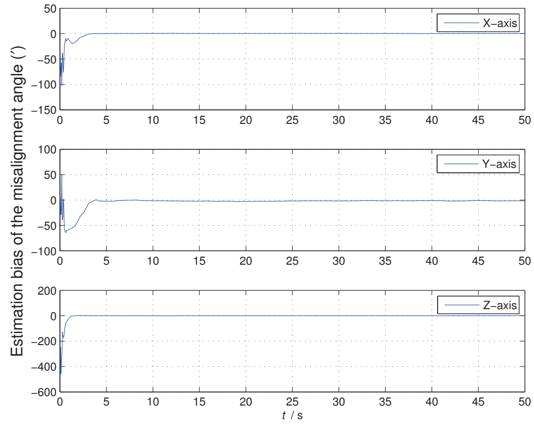

Fig. 10. Estimation bias of the misalignment angle, in the condition of peaceful sea and sensor error degree 1.

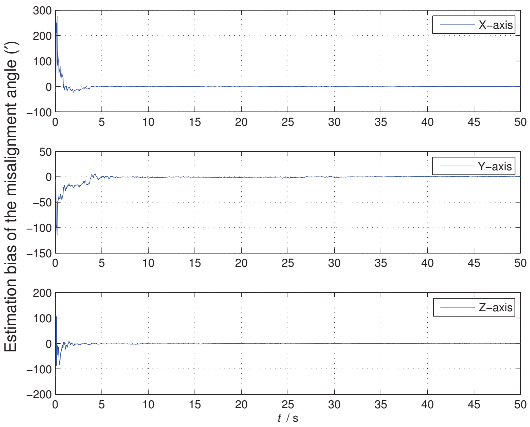

Fig. 9. Estimation bias of the misalignment angle, in the condition of medium sea and sensor error degree2 .

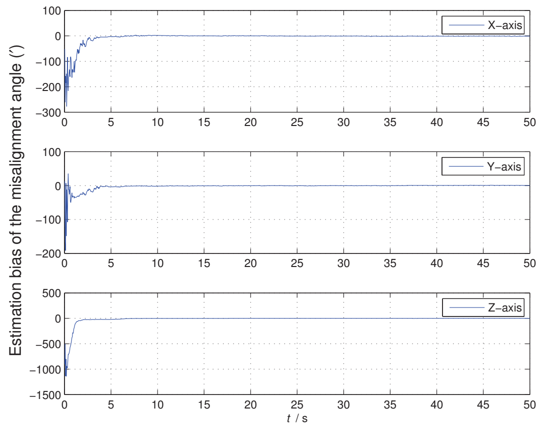

Fig. 11. Estimation bias of the misalignment angle, in the condition of peaceful sea and sensor error degree 2.

From(16), we can see that deformation angular rate is a main disturbance of angular rate matching measurement. In the TA method of this paper, the flexure deformation disturbances are augmented to the states of the TA filter, the measurement disturbance could be compensated to some extent in the process of alignment filtering. Thus, the actual measurement noise will just change in a small range. Assuming that the actual measurement noise is two times larger than the TA filter's. The comparison simulations are carried out in the condition of peaceful sea and sensor error degree 1, and the results are shown in Figure 12. The estimation bias of the misalignment angle is $\delta\phi = [5.5011', 4.7566', 5.6179']^T$.

The simulation results show that the bigger actual measurement noise can cause lower accuracy of the misalignment angle estimation. Meanwhile, the research [11] has shown that the adaptive Kalman filter could be one solution of this problem.

Besides, time-delay is another influencing factor of TA. According to the research [12] , time delay may cause some misalignment between the MINS and SINS, which can reduce the accuracy of the TA. Recent research [12] has shown that the fire control computer of the MINS can give the time-delay information to the SINS, then the data can be processed by Kalman filtering and the time origin is unified to the moment corresponding to the MINS data. Therefore, the time-delay barely affects the TA process in this situation.

IV. Conclusion

This transfer alignment method is for the purpose of reducing alignment errors induced by the disturbances and aligning the weapon INS with large azimuth attitude error on a mooring ship. The method has been established through the disturbances state augmentation and velocity plus angular rate matching, and via qualitative explanations and interpreting computer simulation results, it turns out to be that comparing with the conventional TA method the present one is more

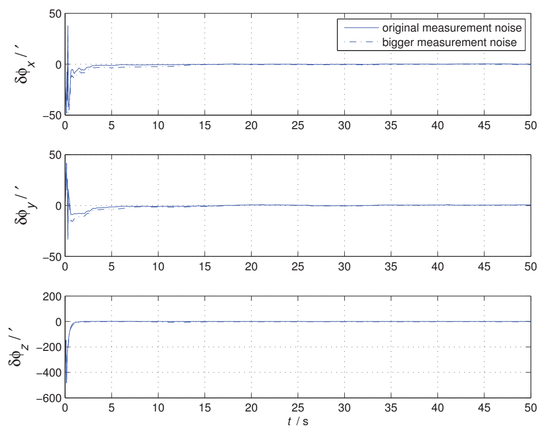

Fig. 12. Comparison of estimation bias of the misalignment angle under two different measurement noise situations

suitable for weapon INS with large azimuth attitude error of a mooring ship under complex environment disturbances.

A related further research of TA with large azimuth attitude error should develop an adaptive state estimation method in face of uncertain measurement noise and an error compensation scheme based on uncertain time-delay.

ACKNOWLEDGMENT

This work is supported by the grant from National Program on Key Basic Research Project (973 Program) (No.613121) and National Natural Science Foundation of China (No. 61104189).

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