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Smooth Second-Order Sliding Mode Control Design of PEM Fuel Cell System

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***Abstract*—A novel smooth second-order sliding mode (SSOSM) control is applied to tackle the problem of PEM fuel cell system output voltage stabilization along with controlling oxygen excess ratio. The oxygen excess ratio is controlled in the inner control loop using PID while the regulation of output voltage is being treated in the outer loop through smooth second-order sliding mode control. The control is applied on the validated nonlinear model of the PEM fuel cell system. The simulation results show that the scheme with SSOSM control provides smoothness (chattering free) with robust performance and the fuel consumption minimization.**

Keywords-component; Higher (Second) order Smooth Sliding Mode Control; PEM Fuel Cell System

I. INTRODUCTION

Petroleum is a fossil fuel and is obtained from the earth's resource of fossil which are finite. The current trend of petroleum consumption is highly increasing with respect to new explorations. If the oil discovery and consumption follow the current trends, estimation shows that the world oil reserves will hardly be used by 2038. Transportation sector is the main user of petroleum and is growing very fast specially in developing countries. This trend needs to optimal use of petroleum with improved efficiency and also require to switch to alternate energy resources.

Fuel cell is a promising alternate energy resource with zero emission environment friendly technology and recent researches in this area show that fuel cell is capable in wide range of application as power source. Different types of fuel cells are suitable for different applications. Polymer Electrolyte Membrane Fuel Cell (PEMFC) shows its potential in, stationary, portable and specially propulsion applications and now in the phase of commercialization.

Fuel Cell System is an electrochemical device that converts chemical energy into electricity directly. Hydrogen and oxygen are its inputs as fuel and oxidant respectively. The voltage is its output which doesn't inherit stabilized form. The voltage varies due to chemical reactions and fluid dynamics. It also degrades due to increase in the load. The variable and degraded voltage is not useful for all electric appliances because they need regulated voltage for their proper working and their long life. That is why; the stabilization of output voltage is a challenging problem for researchers.

The desired output voltage can be maintained through an effective control system. The controller can stabilize the voltage efficiently manipulating inputs. This manipulation can made the system economical avoiding conservative use of inputs. A typical propulsion system based on PEM fuel cell is shown in Fig. 1.

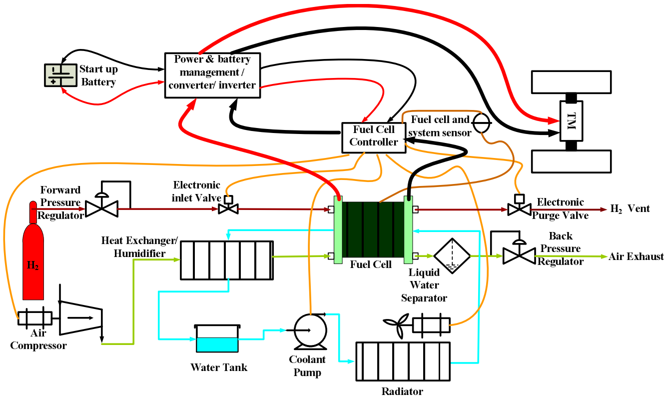

Figure 1. PEM Fuel Cell System for Propulsion

Most of the vast literature available on fuel cells deals with the steady-state conditions. Many researchers attempted to develop electrochemistry based models of PEM fuel cell [8, 9]. The work presented by Lu Ying Chiu and Tanrioven is aimed at developing dynamic models of PEM fuel cell [10, 11]. The dynamic model and design of combined fuel cell and ultra-capacitor system for stand-alone residential applications is developed in [12].

Some models focused on the description of the fuel cell stack and formed the basis for many 2D and 3D finite element models that allow predicting local reactant concentration, temperature and current density [13-15]. Kim et al, Lee et al, and Mann et al proposed generic models that can be adjusted to any fuel cell by adjusting a certain number of fitting parameters [16-18]. Rodatz developed a one dimensional, steady state fuel cell model. Based on stack temperature, current, anode and cathode humidity and pressures, it describes the cell voltage [19].

Although these models provide certain understanding of PEM fuel cell, they cannot be used to perform control algorithm development for PEM fuel cell system. Such models were intended for simulation or parameter estimation rather

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than control analysis. Other PEM fuel cell models available in the open literature cannot be used for simulations and control purposes because the information about the parameters was not listed completely. Some of them were incomplete and too complex as well as computationally intensive to be used for real-time applications. To develop refined control strategies for PEM fuel cell system, an accurate mathematical model of PEM fuel cell system is necessary.

In this wok, a dynamic non-linear mathematical model for 500Watt PEM fuel cell [6, 7], is used. This model describes the parameters that have a physical significance so that it can be adapted to a given system. It describes correctly the interaction between the different subsystems from a controls point of view. The results of the non-linear PEM fuel cell model simulation in the Simulink/Matlab environment ensure the perfectness of the Model.

The Avista SR-12 system is designed to produce power up to 500Watt (25V at 20A). The specifications of the SR-12 system, as provided by Avista [20], are shown in Appendix C [37].

In this work, PEM fuel cell model is studied in section II and a proposed scheme is developed in section III, controller synthesis and their stability proof along with simulation results for verification is provided in section IV, and then the final conclusion.

II. PEM FUEL CELL MODEL

The block diagram in Fig. 2 describes the relationship developed in mathematical model. The model has three inputs hydrogen pressure, oxygen pressure, disturbance load current and an exogenous input i.e. room temperature whereas it has only two outputs fuel cell voltage and stack temperature. The partial pressures of hydrogen, oxygen and water vapors are calculated by mass balance and material conservation equations. Then, the open circuit potential (V_{OFC}) of the fuel cell is determine by the Nernst equation. The voltage losses are calculated by ohmic voltage drop equation, activation voltage drop equations and concentration voltage drop equations. The terminal (output) voltage of the fuel cell is determined by voltage losses together with the double-layer charging effect equation. The thermodynamic equations are used to calculate stack temperature.

The states description of non-linear fuel cell system model [37] is as follows:

$x_1 = \left(m_{O_2\_net}\right)_{net}$ Net flow rate of Oxygen (mol/s)

$x_2 = \left(m_{H_2\_net}\right)_{net}$ Net flow rate of Hydrogen (mol/s)

$x_3 = \left(m_{H_2O\_net}\right)_{net}$ Net flow rate of Water (mol/s)

$x_4 = T$ Stack temperature (K)

$x_5 = P_{H_2}$ Partial pressure of hydrogen (atm)

$x_6 = P_{O_2}$ Partial pressure of oxygen (atm)

$x_7 = P_{H_2O}$ Partial pressure of water (atm)

$x_8 = Q_C$ Heat generated due to electrochemical reaction (J)

$x_9 = Q_E$ Heat generated due to electricity (J)

$x_{10} = Q_L$ Heat loss by air Convection (J)

$x_{11} = V_c$ Voltage across the Capacitor (V)

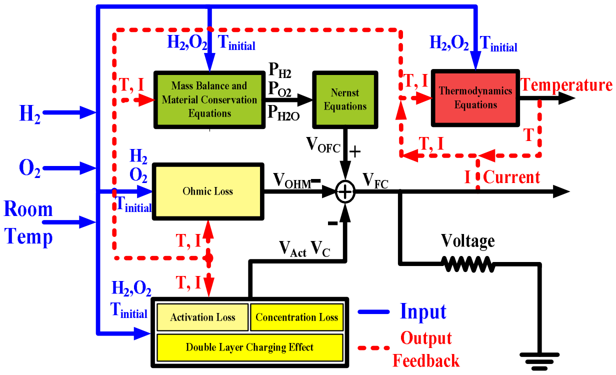

Figure 2. Block Diagram of PEM Fuel Cell Model

The V-I and P-I characteristic curves obtained in [37] of PEM fuel cell model is presented in Fig. 3.

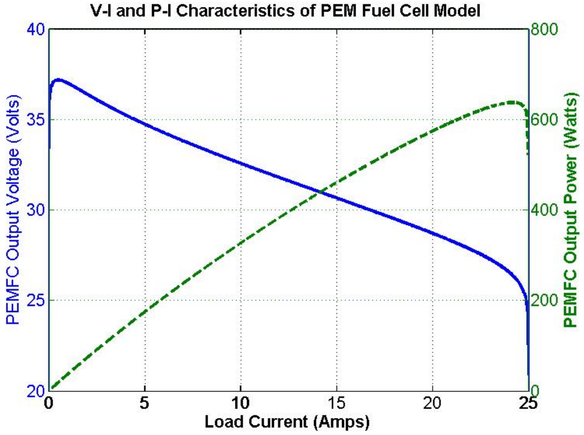

Figure 3. V-I & P-I Characteristics of PEM Fuel Cell Model

III. PROBLEM FORMULATION

The strategy contains two control loops. The first loop tries to maintain the stoichiometric oxygen excess ratio whereas the second loop stabilizes the output voltage of the system as shown in Fig. 4. The basic problem with the operation of fuel cell system is that it does not produce steady output voltage whereas most of the electrical appliances require stable one. There is a variation in the output voltage as the load varies and it drops with the passage of time even if the fuel supply is kept steady. It may conclude that to get rid of variable and degraded output voltage, there is an inevitable need of voltage stabilization.

The rapid and efficient control of air flow is required for avoiding oxygen starvation and the life of the stack [24]. The purpose of sustaining the excess ratio is to optimize the conversion of energy in the fuel cell and maximize the net power by the system operating under different load conditions, while minimizing the parasitic losses of feeding more oxygen into the cathode. If during the operation, the partial pressure of oxygen in the air stream of the cathode falls down to a certain critical level, a complicated phenomenon called oxygen starvation occurs [25].This phenomenon causes a sudden decrease in fuel cell output voltage, which can causes a hot spot or even burn the surface of a membrane in some severe situations [26].The sufficient mass flow of oxygen through the stack will satisfy the load demand. In this way not only the fuel consumption is minimized but also oxygen starvation will be avoided. The optimal mass flow of oxygen is achieved by maintaining the oxygen excess ratio to its optimal value at cathode. The stoichiometric ratio or the oxygen excess ratio is defined as [27].

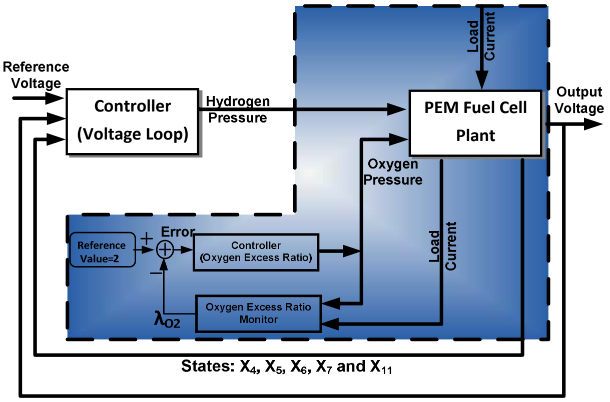

Figure 4. Control Scheme for PEM Fuel Cell System

$$\lambda_{O_2} = \frac{m_{O_2 in}}{m_{O_2 used}} \tag{1}$$

Where $m_{O_2 in}$ is the number of moles of oxygen going into the cathode, which depends on the pressure generated by the air blower and the vapors injected by the humidifier, while $m_{O_2 used}$ is the number of moles of oxygen consumed in the reaction. The total stack current depends upon this rate of oxygen consumption.

The optimum value of oxygen excess ratio is estimated by varying load to the stack on different values of stoichiometry operating as open-loop system [28]. It depends on the load condition and is normally taken as two which can be clearly seen from Fig. 5 that at higher load current, the peak power can be obtained at $\lambda_{O_2} = 2$ before and after that point, it decreases. Many fuel cell systems exhibits the same behavior with some ignorable deviations in the system operating range so λ_{O_2} can be kept constant. Otherwise, variable λ_{O_2} can easily be derived as a function of load current.

The advantage of the oxygen excess ratio control is to avoid oxygen starvation on the cathode side that can causes serious problems to the polymer membrane's life; the control will allow the oxygen to go below when it is needed [28, 29]. Another significance of maintaining oxygen excess ratio is that it maximizes or optimizes performance, efficiency and lifetime of the fuel cell.

On the other hand if the oxygen excess ratio crosses the optimal value then extra amount of energy is required to pump the oxygen into the cathode while the fuel cell system output power also degrades. Once the optimized stoichiometric ratio is obtained then keeping the oxygen excess ratio within optimal values by controlling the oxygen molar flow rate can be written as.

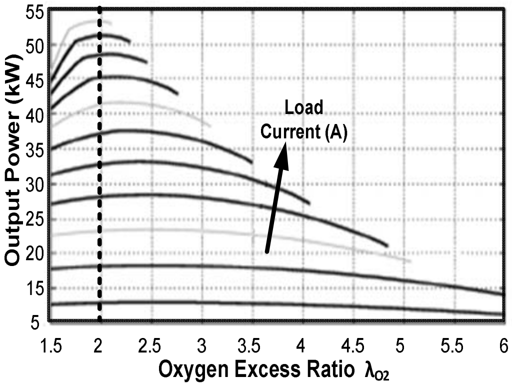

Figure 5. Analysis of oxygen excess ratio on net output power at different load conditions [28]

$$\lambda_{O_2} = \frac{4F 8.614x10^{-5} U_{pc}}{n_s\, I} \cong 2 \tag{2}$$

The output voltage stabilization is carried out via five different approaches i.e. Proportional, Integral and Derivative (PID) control, first order SMC, Higher Order Sliding Mode (HOSM) control using super twisting algorithm [37], Smooth Sliding Mode Control (SSMC) [38] and Smooth Second Order Sliding Mode Control. The simulation experiments are performed on a nonlinear state space mathematical model that is validated with experimental results available in public literature.

IV. CONTROLLER DESIGN

A. PID Control

To regulate the oxygen excess ratio a Proportional Differential and Integral (PID) controller is used. The reasons for selecting PID controller is that it is a simple controller and it can serve the particular task well without moving forward to advanced complicated robust controllers.

The control law for PID controller is given as:

$$U = K_p\, e + K_i \int e\, dt + K_d \frac{d}{dt} e \tag{3}$$

The PID controller is stabilizing the oxygen excess ratio at optimum value that is two in the presence of external varying

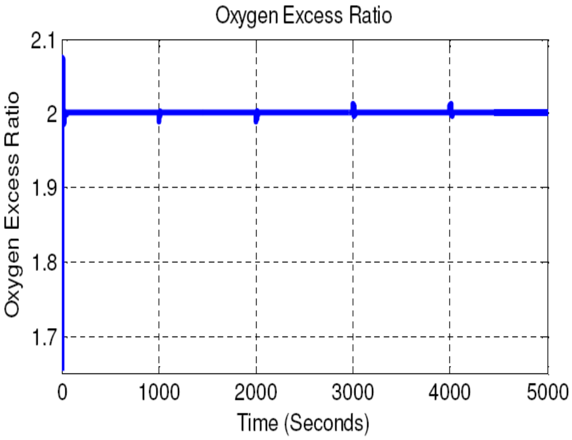

Figure 6. Controlled oxygen excess ratio

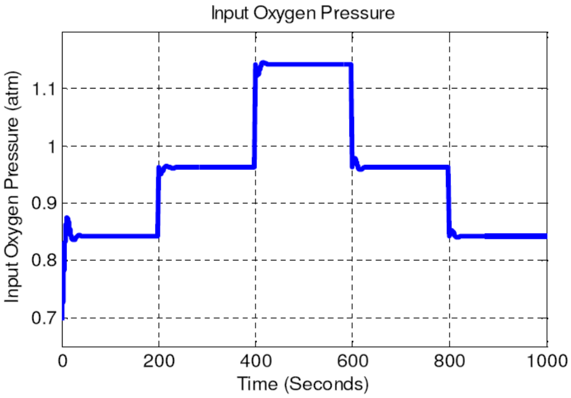

Figure 7. Input Oxygen Pressure (PID)

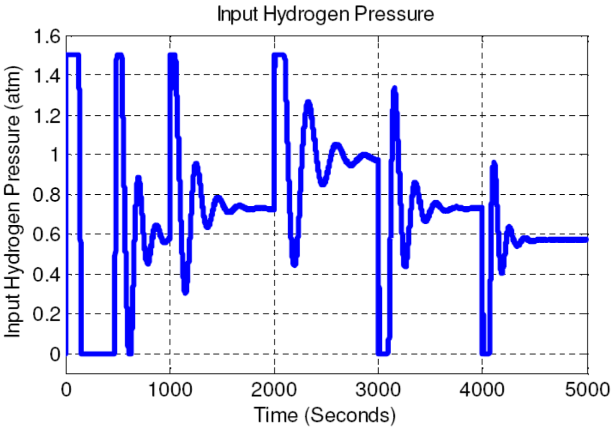

Figure 8. Input Hydrogen Pressure (PID)

disturbance. There are little over shots and under shots after each 1000 seconds interval. This is due to the step variation in the load current as in Fig. 6. This band of variation is bearable for the plant.

Controlled input oxygen pressure formed by the PID controller on oxygen excess ratio loop in order to maintain the oxygen excess ratio at 2. Oxygen excess ratio is maintained at an optimal value by an oxygen excess ratio controller as shown in Fig. 7. Oxygen excess ratio is regulating pretty well at optimal value via PID controller. A PID controller is applied on the voltage loop of the fuel cell to track the reference voltage in the presence of the oxygen excess ratio as shown in Fig. 9 and Fig. 8 represents the control effort generated by the PID controller to stabilize the fuel cell output voltages by manipulating input hydrogen pressure. The oscillations is because of the variation in load current. The settling time for the PID is high. The Fig. 10 shows error in the output voltage of the fuel cell; in ideal case error should be zero. The variations in the error are due to the load current variations on the output of fuel cell.

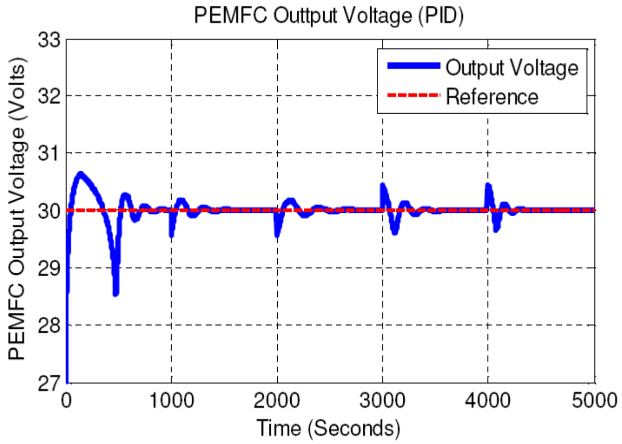

Figure 9. PEM Fuel Cell Output Voltage (PID)

Figure 10. PID Output Voltage Error

B. Smooth Second Oder Sliding Mode Control

A Higher-order Smooth Sliding Mode Control (HSSMC)/ Smooth Second-Order Sliding Mode Control (SSOSMC) strategy is investigated for Proton Exchange Membrane Fuel Cell (PEMFC) system to achieve the smoothness in control law. In this strategy first loop tries to maintain the stoichiometric ratio whereas the second loop stabilizes the output voltage of the system.

Smooth Second Order Sliding Mode Control is synthesized as follows. Consider SISO σ –dynamics

$$\dot{\sigma} = g(t) + u \tag{4}$$

Where σ is the sliding surface, the uncertainty *g(t)* is a smooth function and $u \in \Re$ is a control input. The sliding variable σ is the system motion along the system trajectories and $\sigma = 0$ is the system motion on the sliding surface. The smoothness in the control input u is the main objective of this synthesis. This synthesized control law u pushes the sliding variable and its derivative to zero, i.e. $\sigma, \dot{\sigma} \rightarrow 0$ in finite time. In this technique, by using Levant exact differentiator based observer, the drift term *g(t)* is estimated and cancelled. Here it is assumed that *g(t)* is available in real-time. The sliding surface σ in (4) can be selected as

$$\dot{x}_1 = \alpha_1 |x_1|^{(P-1)/P}\, sign(x_1) + x_2$$
$$\dot{x}_2 = \alpha_2 |x_1|^{(P-2)/P}\, sign(x_1);\ \sigma = x_1 \qquad (5)$$

Lemma Let $p \geq 2$, $\alpha_1, \alpha_2 > 0$ then the system (5) is finite time stable. The settling time is a continuous function and depends on the initial condition. Also it vanishes at the origin.

The Lemma can be proved by selecting the Lyapunov candidate function as

$$V = \frac{x_2{}^2}{2} + \int_0^{x_1} \alpha_2 |z|^{(P-2)/P}\, sign(z)\, dz \qquad (6)$$

and then applying LaSalle theorem [39]. It is also homogeneous with the dilation $dk{:}(x_1,\ x_2) \mapsto (k^p x_1,\ k^{p-1} x_2)$ and the degree of homogeneity is -1.

The term *g(t)* is unknown bounded disturbance and the sliding variable σ is very sensitive to this disturbance. It is assumed that the terms *g(t)* and *u(t)* are available, $g^{(m-1)}(t)$ *has* a known Lipshitz constant $L>0$ *by* $m-1$ times differentiability of *g(t)*. The *u(t)* is Lebesgue-measurable input control function and $\sigma(t)$ is continuous function defined $\forall t > 0$ i.e. it is understood in the Filippov sense. Then

$$\dot{\sigma} = \dot{V}_{fc} - \dot{V}_r \qquad (7)$$

Putting values $\dot{V}_{fc}$ (see appendix B) [37].

$$\dot{\sigma} = f_1\, \dot{x}_4 + f_2 \dot{x}_5 + f_3 \dot{x}_6 - f_4 \dot{x}_7 - n_s \dot{x}_{11} + f_5 - \dot{V}_r \qquad (8)$$

Now applying $\dot{x}_5$ values and then by simplifying we get

$$\dot{\sigma} = g(t) + b\, U_{P_A} \qquad (9)$$

Where

$$g(t) = f_1\, \dot{x}_4 + f_2\, [-2\xi_1(x_4) x_5 - \xi_2(x_4) I] + f_3 \dot{x}_6 - f_4 \dot{x}_7 - n_s \dot{x}_{11} + f_5 - \dot{V}_r \qquad (10)$$

$$b = 2 f_2\, \xi_1(x_4) \qquad (11)$$

The control law U_{P_A} that would maintain $\dot{\sigma} = 0$, i.e.

$$U_{P_A} = \frac{1}{b}[-g(t) + U_d] \qquad (12)$$

$$U_d = \alpha_1 |\sigma|^{2/3}\, sign(\sigma_1) + \alpha_2 \int |\sigma|^{1/3}\, sign(\sigma)\, d\tau \qquad (13)$$

Since the function $g(x)$ is differentiable, so it has a Lipshitz constant. As it is very difficult to estimate the exact value of the Lipshitz constant L for $\dot{g}(x) \leq L$, it is the only parameter for proposed observer, so can be estimated through some hit and trial method. $g(x)$ is estimated using observer with the paramters m=2, $\lambda_0 = 2$, $\lambda_1 = 1.5$, $\lambda_2 = 1.1$, [40].

$$\dot{z}_0 = \nu_0 + b\, U_{P_A}$$
$$\nu_0 = -\lambda_0 L^{1/3} |z_0 - \sigma|^{2/3}\, sign(z_0 - \sigma) + z_1\ ; \qquad \lambda_0 = 2$$
$$\dot{z}_1 = \nu_1$$
$$\nu_1 = -\lambda_1 L^{1/2} |z_1 - \nu_0|^{1/2}\, sign(z_1 - \nu_0) + z_2; \quad \lambda_1 = 1.5$$
$$\dot{z}_2 = -\lambda_2 L\, sign(z_2 - \nu_1); \qquad \lambda_2 = 1.1$$
$$\hat{g}(x) = z_1 \qquad (14)$$

If there is no noise at the input, after finite time as the transients die out, we obtain $\hat{g}(x) = g(x)$ in (14). The SSOSM Control Law in terms of hydrogen input at anode U_{P_A} with parameters $p = 3, m = 2$ will become

$$U_{P_A} = \frac{1}{b}\left(\alpha_1 |\sigma|^{2/3}\, sign(\sigma) + \alpha_2 \int |\sigma|^{1/3}\, sign(\sigma)\, d\tau + \hat{g}(x)\right) \qquad (15)$$

$$U_{P_A} = \frac{1}{b}\left(U_d + \hat{g}(x)\right) \qquad (16)$$

Which is the SSOSM Control with robust finite time convergent term, U_d.

C. Simulation Results:

The smooth control signal generated by HSSM controller to control input hydrogen pressure is represented in Fig. 11; there is no chattering at all. This result is the main objective of HSSM controller and is better than presented in [37,38].

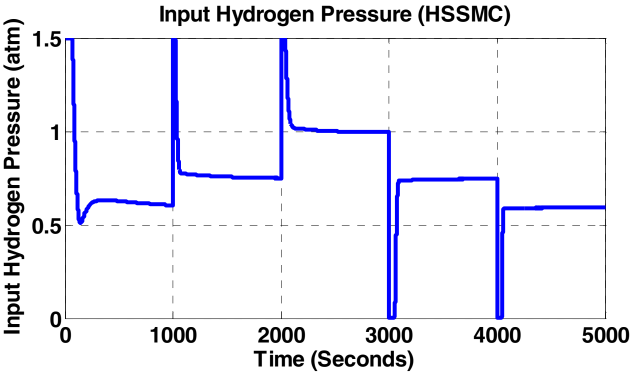

Figure 11. PEMFC Input Hydrogen Pressure HSSMC

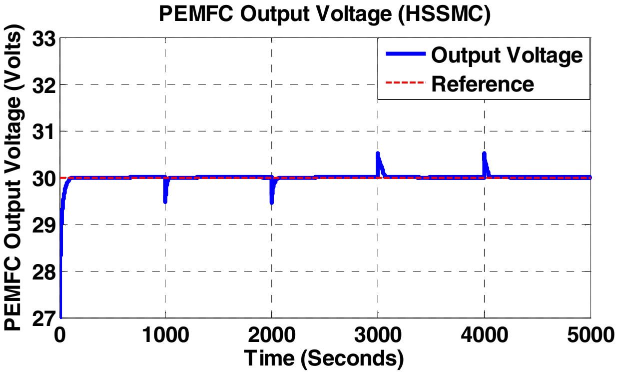

Figure 12. PEMFC Output Voltage HSSMC

The PEM fuel cell output voltage stabilized at 30 volts by HSSM controller is shown in Fig. 12. It can be seen that for two undershoots at 1000 and 2000 seconds the output voltage comes back to the reference value quickly where as in case of two overshoots at 3000 and 4000 seconds the output takes little more time to returns the reference value. The load current increases at 1000 and 2000 whereas it decreases at 3000 and 4000 seconds. When load current increases the fuel cell needs more fuel to maintain the voltage level that surplus amount can be rapidly provided by the controller where as in the case when the load current decreases the high pressure fuel stored in the gas chambers need some time to consume it up.

The sliding surface in case of HSSM controller ideally it remains at zero. The peaks are due to the load current profile given to the system. The Fig. 13 shows the robustness of the controller. The sliding surface leaves the manifold to cater the load disturbance for a few seconds and remains there forever.

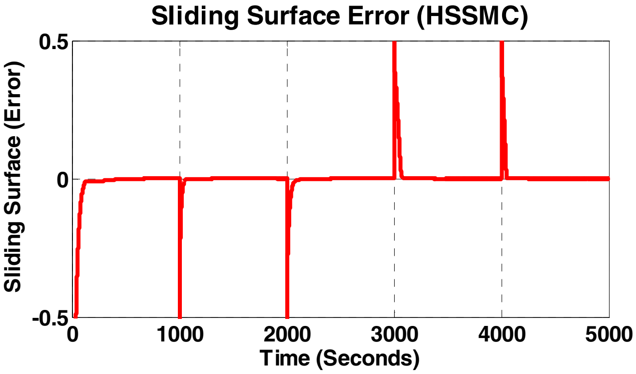

Figure 13. Sliding Surface HSSMC

Conclusion

A novel chattering free smooth second order sliding mode control (SSOSMC)/HSSMC is studied to tackle the problem of PEM fuel cell system output voltage stabilization problems along with controlling oxygen excess ratio. The oxygen excess ratio is controlled in the inner control loop using PID while the regulation of output voltage is being treated in the outer loop through HSSM control. The control law is designed for the formulated problem and stability analysis is provided. The simulation is carried out on the nonlinear, validated model. Finally, it is shown from the simulation that HSSM controller provides the smooth/ chattering free control which is robust against uncertain disturbances and minimize fuel consumption by showing minimal control efforts. The proposed controller is far better in avoiding chattering and easy for implementation than the previously applied different techniques for the formulated scheme.

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