UKACC International Conference on Control 2012
Cardiff, UK, 3-5 September 2012

Backstepping based adaptive sliding mode control for spacecraft attitude maneuvers

Binglong Cong, Xiangdong Liu and Zhen Chen
Key laboratory for Intelligent Control & Decision of Complex Systems, School of Automation
Beijing Institute of Technology, Beijing, China 100081
Email: {cbl, xdliu, chenzhen76}@bit.edu.cn

***Abstract*—This paper aims to address the robust control problem of rigid spacecraft attitude maneuvers in the presence of inertia matrix uncertainty and external disturbance. A backstepping based adaptive sliding mode control (B-ASMC) design is proposed as a solution, where the upper bounds of the parametric uncertainty and disturbance are not required in advance. Compared to current adaptive sliding mode control (ASMC) design, the B-ASMC design has two advantages. Theoretically, the asymptotical stability of the attitude states rather than the sliding function is guaranteed. Practically, the over-adaptation problem in current ASMC design is alleviated and the system performance is improved. Detailed design principle and rigorous closed-loop system stability analysis are provided. A large angle attitude maneuver is employed in the numerical simulation to verify the effectiveness of the proposed algorithm.**

***Index Terms*—attitude maneuver, adaptive sliding mode control, backstepping, over-adaptation.**

I. Introduction

With the development of aerospace technologies, more and more space missions require that the involved spacecraft implements attitude maneuvers with large angles. Design of an attitude control system for such case poses a challenging problem, including the nonlinear characteristics in the attitude dynamics & kinematics, modeling uncertainty and unexpected external disturbances. Thus, in order to guarantee the control performance, it is necessary to employ nonlinear robust control methods. Sliding mode control (SMC) is a powerful nonlinear control method that is well known for its strong robustness. SMC can provide many good properties, such as insensitivity to model uncertainty, disturbance rejection, and fast dynamic response, which make it a welcome approach for spacecraft attitude control [1]–[4].

According to the equivalent control concept, current SMC algorithms generally consist of two parts, the continuous equivalent control component and the discontinuous switching control component. In order to satisfy the reaching condition, the switching gain should be larger than the upper bounds of the model uncertainty and disturbance. However, those bounds are hard to find in many practical situations. Therefore, conservative design is generally adopted, where the switching gain is selected sufficiently large, such as those in [1]–[4]. Nonetheless, a large switching control component may aggravate the chattering problem which could excite the unmodelled dynamics and may lead to instability.

To eliminate the need of uncertainty and disturbance bounds, adaptive scheme is integrated into SMC design, which is known as the ASMC technique. At the initial stage, it is assumed that the lumped uncertainty was bounded by a linear function of the state-norm. Correspondingly, adaptive laws were designed for the linear function coefficients, as suggested in [5]–[7]. In particular, in [7], an ASMC algorithm was proposed for the attitude stabilization of a rigid spacecraft, where the lumped uncertainty is assumed to be bounded by a linear function of the norms of angular velocity and quaternion. Subsequently, the lumped uncertainty was assumed to be bounded by an unknown constant and consequently a simple adaptive law was proposed for the switching gain calculation in [8]. Subsequent results can be found in many applications such as internal combustion engines [9], induction servomotor [10], planetary gear-type inverted-pendulum [11], etc. However, on the basis of Barbalat lemma, all the ASMC algorithms mentioned above can only guarantee that the sliding function is asymptotically stable but not the system states. And the system performance has not been taken into account.

On the other hand, the backstepping design technique has been widely used to control nonlinear systems with matched or unmatched uncertainties in recent years (see [12] and references therein). The key feature of backstepping design is that it stabilizes the system states through a step-by-step recursive process. Once the final step is completed, the stability of the entire system is guaranteed naturally. However, conventional backstepping design mainly assume that the lumped uncertainty is constant or slowly changing. When the derivative of the lumped uncertainty cannot be regarded as zero, backstepping design with integral adaptive laws are no longer applicable. Recently, there has been continuous efforts to combine the backstepping technique with the SMC method, such as [13]–[15]. Unfortunately, a prior knowledge of the lumped uncertainty bound is required.

Considering the characteristics of both the ASMC method and the backstepping technique, it is natural to combine those two design methodologies to preserve their advantages and at the same time overcome their drawbacks mentioned above, which leads to the proposed B-ASMC design. In this paper, we focus on the robust attitude control for a rigid spacecraft, where the inertia matrix uncertainty and external disturbance are considered. Noticing the cascade structure of the attitude control system, the attitude controller is designed in the backstepping framework, where the ASMC algorithm is designed in the final step to deal with the lumped uncertainty. By virtue of the backstepping design procedure, the proposed B-ASMC algorithm can guarantee the asymptotical stability of the closed-loop system not just the sliding function. Moreover, the system performance can be improved by the proposed algorithm.

II. Preliminaries

A. Mathematical Model

Consider a thruster control rigid spacecraft, whose attitude dynamics is governed by the following equation:

$$J\dot{\omega}_b + S(\omega_b)J\omega_b = T_b + T_d \tag{1}$$

where $J \in \mathbb{R}^{3\times3}$ is the spacecraft inertia matrix, $\omega_b \in \mathbb{R}^3$ denotes the angular velocity vector of $\mathcal{F}_B$, the body-fixed frame, with respect to $\mathcal{F}_I$, the inertia frame. $S(\cdot)$ is the skew-symmetric matrix operator, which is operated as follows:

$$S(\alpha)\beta = \alpha \times \beta$$

where α and β are the vectors in $\mathbb{R}^3$. $T_b \in \mathbb{R}^3$ is the vector of control torque provided by the thrusters, $T_d \in \mathbb{R}^3$ is the time-varying external disturbance vector, including environmental and non-environmental disturbance torques. Furthermore, the inertia matrix uncertainty is considered. Let $J = \hat{J} + \Delta J$ with ΔJ the uncertainty caused by the change in mass properties and $\hat{J} = \mathrm{diag}(J_1, J_2, J_3)$ the nominal inertia matrix. Then (1) is described as:

$$\hat{J}\dot{\omega}_b + S(\omega_b)\hat{J}\omega_b = T_b + T_d - \Delta J\dot{\omega}_b - S(\omega_b)\Delta J\omega_b \tag{2}$$

According to the structural feature in (2), one can merge all the elements caused by inertia matrix uncertainty and external disturbance as the lumped uncertainty, i.e., let $d = T_d - \Delta J\dot{\omega}_b - S(\omega_b)\Delta J\omega_b$. Correspondingly, the attitude dynamics is rewritten as:

$$\hat{J}\dot{\omega}_b + S(\omega_b)\hat{J}\omega_b = T_b + d \tag{3}$$

From (3), it is clear that the lumped uncertainty is matched to the system. Without loss of generality, it is assumed that d is smooth and satisfies $\|d\|_\infty < d_{\max}$ with $d_{\max}$ the unknown upper bound and $\|\cdot\|_\infty$ the vector infinity-norm.

As for the attitude representation, quaternion and modified Rodrigues parameters (MRPs) are the two most popular parameters. Quaternion is characterized by its global non-singularity. However, the use of quaternion requires an extra parameter, which leads to a non-minimal parameterization [4]. For the attitude maneuver whose principal angle is within $(-2\pi, 2\pi)$, MRPs can provide a nonsingular minimal attitude description. Moreover, by introducing the shadow MRPs and a switching mechanism, MRPs turn out to be a nonsingular, bounded, minimal attitude representation. Therefore, MRPs are utilized in this paper, whose kinematics is:

$$\dot{\sigma}_b = M(\sigma_b)\omega_b \tag{4}$$

where $\sigma_b \in \mathbb{R}^3$ denotes the inertial MRPs vector of $\mathcal{F}_B$ with respect to $\mathcal{F}_I$. $M(\sigma_b)$ is the Jacobian matrix in the form of $M(\sigma_b) = \dfrac{(1-\|\sigma_b\|^2)I_3 + 2S(\sigma_b) + 2\sigma_b\sigma_b^T}{4}$ with $\|\cdot\|$ the vector 2-norm and I_3 the 3×3 identity matrix. Moreover, $M^T(\sigma_b)M(\sigma_b) = m(\sigma_b)I_3$ with $m(\sigma_b) = (1+\|\sigma_b\|^2)^2/16$. The transition matrix from $\mathcal{F}_I$ to $\mathcal{F}_B$ in terms of σ_b is:

$$R(\sigma_b) = I_3 + \frac{8S(\sigma_b)S(\sigma_b) - 4(1-\|\sigma_b\|^2)S(\sigma_b)}{(1+\|\sigma_b\|^2)^2} \tag{5}$$

In this paper, the attitude reorientation control problem is considered. Our goal is reorienting the spacecraft from an arbitrary stationary attitude to a desired attitude with zero angular velocity. Denoting the attitude variables of $\mathcal{F}_D$, the desired frame, as $\sigma_d \in \mathbb{R}^3$ and $\omega_d \in \mathbb{R}^3$, the error attitude variables are defined as follows:

$$\sigma_e = \sigma_b \oplus \sigma_d^* \tag{6}$$

$$\omega_e = \omega_b - R(\sigma_e)\omega_d \tag{7}$$

where $\sigma_e \in \mathbb{R}^3$ is the error MRPs, $\oplus$ is the MRPs production operator, characterizing the successive rotations. For two MRPs expressed in their corresponding frames, e.g., $\sigma_1 \in \mathbb{R}^3$ and $\sigma_2 \in \mathbb{R}^3$, it is operated as follows:

$$\sigma_1 \oplus \sigma_2 = \frac{(1-\|\sigma_2\|^2)\sigma_1 + (1-\|\sigma_1\|^2)\sigma_2 - 2S(\sigma_1)\sigma_2}{1 + \|\sigma_2\|^2\|\sigma_1\|^2 - 2\sigma_2^T\sigma_1}$$

σ_d^* is the inverse of σ_d, which is extracted from $R^{-1}(\sigma_d)$ and $\sigma_d^* = -\sigma_d$, $R(\sigma_e)$ and $R(\sigma_d)$ are the transition matrices from $\mathcal{F}_D$ to $\mathcal{F}_B$ and from $\mathcal{F}_I$ to $\mathcal{F}_D$, and their expressions in terms of σ_e and σ_d can be obtained by replacing σ_b by σ_e and σ_d in (5). As $\omega_d = 0$, one has $\omega_e = \omega_b$. Therefore, the error attitude dynamics is expressed same as (3). As mentioned in [17], if the attitude variables pairs (σ_b, ω_b) and (σ_d, ω_d) satisfy the MRPs kinematics formulation described in (4), then the error attitude variables pair (σ_e, ω_e) also satisfies the MRPs kinematics formulation. Then, the attitude control system is governed by the following equations:

$$\begin{cases} \hat{J}\dot{\omega}_b + S(\omega_b)\hat{J}\omega_b = T_b + d \\ \dot{\sigma}_e = M(\sigma_e)\omega_b \end{cases} \tag{8}$$

B. Problem Statement

The control objective can be summarized as follows: design a robust control algorithm to steer the attitude variables pair (σ_b, ω_b) from $(\sigma_b(0), 0)$ to $(\sigma_d, 0)$ (or equivalently render $\lim_{t\to\infty} \sigma_e = \lim_{t\to\infty} \omega_b = 0$) when the lumped uncertainty upper bound $d_{\max}$ is unknown in advance.

III. Main Results

A. Conventional ASMC Algorithm Design

In this section, the ASMC algorithm is applied to the attitude control problem under consideration and its major drawback will be revealed. First, define the following nonlinear sliding function $s \in \mathbb{R}^3$:

$$s = \omega_b + \lambda\frac{M^T(\sigma_e)}{m(\sigma_e)}\sigma_e \tag{9}$$

where $\lambda > 0$ is the sliding function gain and $m(\sigma_e)$ can be obtained by replacing σ_b in (4) by σ_e.

According to the design principle presented in [8], following ASMC algorithm can be obtained:

$$T_b = S(\omega_b)\hat{J}\omega_b - \lambda\hat{J}\frac{4M(\sigma_e) - 2\sigma_e\sigma_e^T}{1 + \|\sigma_e\|^2}\omega_b - \hat{d}\mathrm{sgn}(s) \quad (10a)$$

with $\hat{d}$ the estimation of $d_{\max}$ which is given by

$$\hat{d} = c\int_0^t \|s\|_1 d\tau \quad (10b)$$

where $c > 0$ is the adaptive gain, $\mathrm{sgn}(\cdot)$ is the sign function and $\|s\|_1 = s^T\mathrm{sgn}(s)$ denotes the vector 1-norm of s.

By selecting the Lyapunov candidate function in the form of

$$V = \frac{1}{2}s^T\hat{J}s + \frac{1}{2c}\tilde{d}^2$$

where $\tilde{d} = \hat{d} - d_{\max}$ denotes the estimation error, it is easy to obtain that the time derivative of the above Lyapunov function is

$$\begin{aligned}\dot{V} &= s^T d - d_{\max}s^T\mathrm{sgn}(s)\\ &\leq -\eta\|s\|_1\end{aligned} \quad (11)$$

where $\eta = d_{\max} - \|d\|_\infty$. On the basis of the Barbalat lemma, one can conclude that $\lim\limits_{t\to\infty} s = 0$.

There are two major problems of the above ASMC algorithm. Theoretically speaking, $\lim\limits_{t\to\infty} s = 0$ cannot rigorously guarantee $\lim\limits_{t\to\infty} \omega_b = \lim\limits_{t\to\infty} \sigma_e = 0$, i.e., the asymptotic stability of the closed-loop system is not actually achieved. Practically speaking, the ASMC algorithm does not consider the dynamics of the reaching phase, which may arise an over-adaptation of the switching gain with respect to the lumped uncertainty bound and lead to an undesirable system performance. In the following, we try to address those problems by combining the ASMC technique with the backstepping design and present the B-ASMC algorithm.

B. B-ASMC Algorithm Design

As demonstrated in [18], an important property of the system in (8) is that it describes the attitude control system in a *cascade interconnection*, which accords with the strict feedback form in backstepping design. With this in mind, it is possible to design the ASMC algorithm in the backstepping framework and use its key feature to guarantee the stability of the closed-loop system.

First, in the attitude kinematics subsystem, treat the angular velocity as an independent input, then there exists a state feedback stabilizing control law $\omega_b^*(\sigma_e)$ in the form of

$$\omega_b^*(\sigma_e) = -k_\sigma\frac{M^T(\sigma_e)}{m(\sigma_e)}\sigma_e = -k_\sigma\frac{4}{1 + \|\sigma_e\|^2}\sigma_e \quad (12)$$

with $k_\sigma > 0$. Now, consider a Lyapunov candidate function for the attitude kinematics subsystem with the form of $V_\sigma = \|\sigma_e\|^2/2 = \sigma_e^T\sigma_e/2$. From (8) and (12), the derivative of the above Lyapunov candidate is

$$\dot{V}_\sigma = -k_\sigma\sigma_e^T\sigma_e \leq 0 \quad (13)$$

If the angular velocity ω_b is identical to $\omega_b^*(\sigma_e)$, the attitude kinematics subsystem response is characterized by

$$\sigma_e = \exp(-k_\sigma(t - t_i))\sigma_e(t_i) \quad (14)$$

with t_i the time when $\omega_b = \omega_b^*(\sigma_e)$, which implies a good error MRPs response would be achieved.

Then, in order to guarantee ω_b can track $\omega_b^*(\sigma_e)$, a coordinated transformation is utilized. Let $z = \omega_b - \omega_b^*(\sigma_e) \in \mathbb{R}^3$, the attitude control system described by σ_e and z is represented as:

$$\begin{cases}\hat{J}\dot{z} = T_b - S\left(\omega_b^*(\sigma_e) + z\right)\hat{J}\left(\omega_b^*(\sigma_e) + z\right) + d - \hat{J}\dot{\omega}_b^*(\sigma_e)\\ \dot{\sigma}_e = M(\sigma_e)z + M(\sigma_e)\omega_b^*(\sigma_e)\end{cases} \quad (15)$$

where $\dot{\omega}_b^*(\sigma_e)$ can be analytically expressed as

$$\dot{\omega}_b^*(\sigma_e) = -k_\sigma\frac{4M(\sigma_e) - 2\sigma_e\sigma_e^T}{1 + \|\sigma_e\|^2}\left(\omega_b^*(\sigma_e) + z\right)$$

Here, using the ASMC methodology, the attitude control law for the attitude dynamics subsystem is designed as:

$$\begin{aligned}T_b =& S\left(\omega_b^*(\sigma_e) + z\right)\hat{J}\left(\omega_b^*(\sigma_e) + z\right) + \hat{J}\dot{\omega}_b^*(\sigma_e)\\ &- k_\omega\hat{J}z - \hat{d}\mathrm{sgn}(z)\end{aligned} \quad (16a)$$

with

$$\hat{d} = c\int_0^t \|z\|_1 d\tau \quad (16b)$$

Consider a Lyapunov candidate function for the attitude dynamics subsystem in the form of

$$V_\omega = \frac{1}{2}z^T\hat{J}z + \frac{1}{2c}\tilde{d}^2 \quad (17)$$

According to (16a) and (16b), the derivative of the above Lyapunov function is

$$\begin{aligned}\dot{V}_\omega &= -k_\omega z^T\hat{J}z + z^T\left[d - \hat{d}\mathrm{sgn}(z)\right] + (\hat{d} - d_{\max})z^T\mathrm{sgn}(z)\\ &= -k_\omega z^T\hat{J}z + z^T d - d_{\max}z^T\mathrm{sgn}(z)\\ &\leq -k_\omega z^T\hat{J}z - \eta\|z\|_1 \leq -k_\omega z^T\hat{J}z\end{aligned}$$

In order to obtain the control law for the entire system, we should explore the interconnection between the virtual control law in (12) and the attitude control law in (16a) and (16b). Thus, the B-ASMC algorithm is presented as:

$$\begin{aligned}T_b =& S\left(\omega_b^*(\sigma_e) + z\right)\hat{J}\left(\omega_b^*(\sigma_e) + z\right) + \hat{J}\dot{\omega}_b^*(\sigma_e)\\ &- M^T(\sigma_e)\sigma_e - k_\omega\hat{J}z - \hat{d}\mathrm{sgn}(z)\end{aligned} \quad (18a)$$

with

$$\hat{d} = c\int_0^t \|z\|_1 d\tau \quad (18b)$$

Now, we are ready to state the following theorem:

Theorem 1: For the attitude control system described in (8), the B-ASMC algorithm in (18a) and (18b) can globally asymptotically stabilize the closed-loop system in the presence that the lumped uncertainty upper bound $d_{\max}$ is unknown in advance.

Proof 1: Chose the Lyapunov function for the entire system as

$$V = V_\sigma + V_\omega \tag{19}$$

By taking the time derivative along the system trajectory, one has:

$$\begin{aligned}\dot{V} =&\sigma_e^T M(\sigma_e) z + \sigma_e^T M(\sigma_e)\omega_b^*(\sigma_e) \\ &- k_\omega z^T \hat{J} z - z^T M^T(\sigma_e)\sigma_e + z^T \left[d - d_{\max}\mathrm{sgn}(z)\right] \\ =& - k_\sigma \sigma_e^T \sigma_e - k_\omega z^T \hat{J} z + z^T d - d_{\max} z^T \mathrm{sgn}(z) \\ \leq& - k_\sigma \sigma_e^T \sigma_e - k_\omega z^T \hat{J} z - \eta \|z\|_1 \\ \leq& - k_\sigma \sigma_e^T \sigma_e - k_\omega z^T \hat{J} z\end{aligned}$$

where we have used the fact that $\sigma_e^T M(\sigma_e) z = z^T M^T(\sigma_e)\sigma_e$ and $\|d\|_\infty < d_{\max}$.

Let $\chi = k_\sigma \sigma_e^T \sigma_e + k_\omega z^T \hat{J} z$. It is obvious that χ is uniformly continuous. By integrating the above equation from zero to t, one has:

$$\int_0^t \dot{V} d\tau \leq - \int_0^t \chi d\tau \Rightarrow V(0) \geq \int_0^t \chi d\tau \tag{20}$$

Taking the limits as $t \to \infty$ on both sides of (20) gives

$$\infty > V(0) \geq \lim_{t\to\infty} \int_0^t \chi d\tau \tag{21}$$

On the basis of Barbalat lemma, we can obtain $\lim_{t\to\infty} \chi = 0$, which implies that $\lim_{t\to\infty} \sigma_e = \lim_{t\to\infty} z = 0$. As $\lim_{t\to\infty} \sigma_e = 0$, one has $\lim_{t\to\infty} \omega_b^*(\sigma_e) = 0$. According to the definition of z, it is easy to obtain that $\lim_{t\to\infty} \omega_b = 0$. As V is radially unbounded, then we can obtain the conclusion.

C. Discussions

Here are some remarks:

Remark 1: By comparing the ASMC algorithm with the B-ASMC algorithm, one can find that the transformed variable z is actually the sliding function s. Therefore, if we rewrite the B-ASMC algorithm in terms of σ_e and s, there are two additional terms of the B-ASMC algorithm as compared to the ASMC algorithm, $-k_\omega \hat{J} s$ and $-M^T(\sigma_e)\sigma_e$.

The first term is used to improve the system performance by specifying dynamics in the reaching phase. Such a strategy belongs to the so called the *reaching law* method, which was presented in [19]. The *reaching law* is a differential equation which specifies the dynamics of the sliding function. When it is used in the ASMC design, additional benefit can be shown, which has not been fully explored in the literature. For this case, the sliding function dynamics is governed by $\hat{J}\dot{s} = -k_\omega \hat{J} s - \hat{d}\mathrm{sgn}(s)$. As the initial value of $\hat{d}$ is zero, before the adaptation scheme can produce a large enough $\hat{d}$ to satisfy the reaching condition, the term $-k_\omega \hat{J} s$ provides a necessary damping to speed up the reaching phase. On the other hand, it is well known that the basic idea of the ASMC method lies in that the switching gain can be adjusted by the departure from the sliding surface. However, from the adaptive law in (10b), one can see that the integral action starts from the very beginning and any departure from the sliding surface will results in an increase of the switching gain $\hat{d}$. Therefore, if the initial system error is large, or equivalently the initial system trajectory is located far from the sliding surface, the resulting $\hat{d}$ generated by the ASMC algorithm is much larger than the necessary value. Due to the fact that the chattering level is directly determined by the switching gain, the chattering phenomenon is serious in current ASMC design. However, by virtue of $-k_\omega \hat{J} s$, such an over adaptation problem would be weakened due to the fact that part of role of impelling the system trajectory to the sliding surface has been transferred from the $-\hat{d}\mathrm{sgn}(s)$ term to the $-k_\omega \hat{J} s$ term. The parameter k_ω serves as a tuning parameter dealing with the trade-off between the chattering level and control torque amplitude.

The second term, $-M^T(\sigma_e)\sigma_e$, is used to guarantee the asymptotical stability of the closed-loop system, which has already been verified in the above proof.

Remark 2: In [20], the sliding motion with an infinite frequency of the control switching is defined as the *ideal sliding*. In *ideal sliding*, the system trajectory is strictly constrained on the sliding surface. Whereas, due to the switching imperfection, i.e., the switching frequency is finite, sliding motion only takes place in a small neighborhood of the sliding surface, which is defined as the *real sliding*. Recalling the adaptive law in (18b), the switching gain will converge to a bounded value only in *ideal sliding*. However, in *real sliding*, as the sliding function is not identically equal to zero, $\hat{d}$ will become unbounded. For implementation in practice, the adaptive law has to be modified to get a bounded switching gain, such as the so-called σ-modification in [6]. In this paper, the approach proposed in [20] will be used, where the adaptive law in (18b) is modified as:

$$\hat{d} = \begin{cases} c\int_0^t \|s\|_1 \mathrm{sgn}(\|s\|_1 - \epsilon) d\tau & \text{if} \quad \hat{d} > \mu \\ \int_0^t \mu d\tau & \text{if} \quad \hat{d} \leq \mu \end{cases} \tag{22}$$

where $\mu > 0$ is a very small scalar to ensure $\hat{d}$ is positive and $\epsilon > 0$ is carefully chosen to deal with the trade-off in control accuracy and bounded switching gain. Further details on ϵ tuning can refer to [20].

IV. Numerical Simulation

In this section, a large angle attitude maneuver is employed to verify the effectiveness of the proposed B-ASMC algorithm by comparing it with the ASMC algorithm.

The spacecraft inertia matrix for the controller design is $\hat{J} = \mathrm{diag}(48, 25, 61.8)$ (kg.m) and the uncertainty is 10% of the nominal value. $T_d = [\sin(0.2t), 2\cos(0.3t), 3\sin(0.4t)]^T \times 10^{-1}$ (N.m) is the external disturbance. The initial attitude variables of the spacecraft are $\sigma_b(0) = [-0.2, 0.3, 0.1]^T$ and $\omega_b(0) = [0, 0, 0]^T$ (rad/s). The desired attitude is $\sigma_d = [0.1, 0.2, -0.3]^T$ with the desired angular velocity $\omega_d = [0, 0, 0]^T$ (rad/s). The B-ASMC parameters are $k_\sigma = 0.2$, $k_\omega = 0.6$ and the adaptive gain is selected as $c = 1$. For comparison, the ASMC parameter is $\lambda = 0.2 = k_\sigma$ and the adaptive gain is also selected as $c = 1$. The simulation results are shown in Fig.1–Fig.5, where the superscripts x, y, z denote the triaxial components of related vectors.

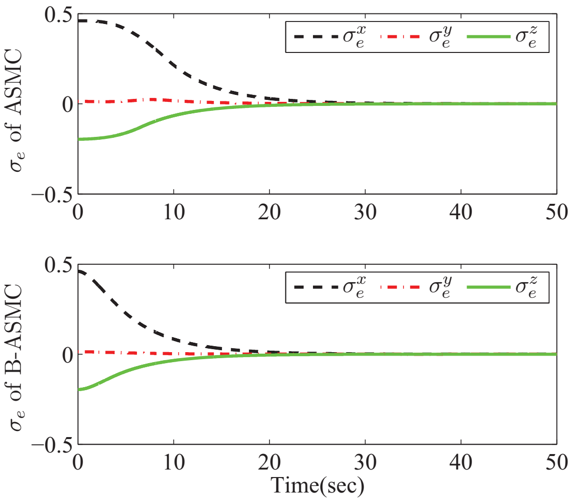

Fig. 1. Error MRPs response comparison

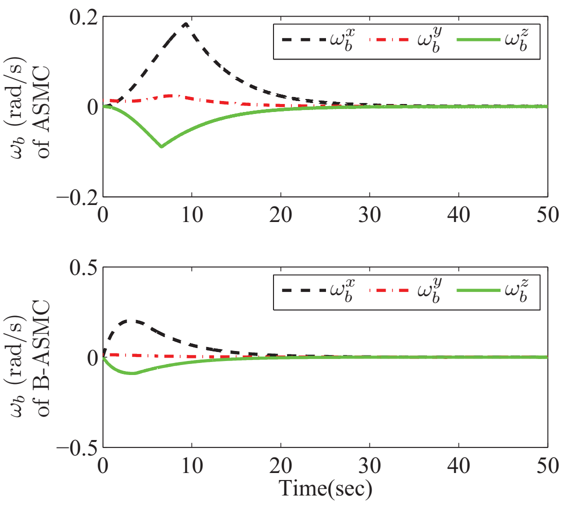

Fig. 2. Angular velocity response comparison

Fig.1 and Fig.2 illustrate the evolutions of the reorientation maneuver controlled by the ASMC algorithm and the B-ASMC algorithm in terms of error MRPs and angular velocity, with the corresponding control torque compared in Fig.3. From Fig.1, we can see that the convergence of the error MRPs controlled by the B-ASMC algorithm is faster than the ASMC algorithm. The fact is that the control torque computed by the ASMC algorithm is zero at the initial time according to (10a) and (10b). Therefore, the convergence is very slow at the beginning.

Fig.3 illustrates the control torque comparison, where the chattering problem in the ASMC algorithm is more serious

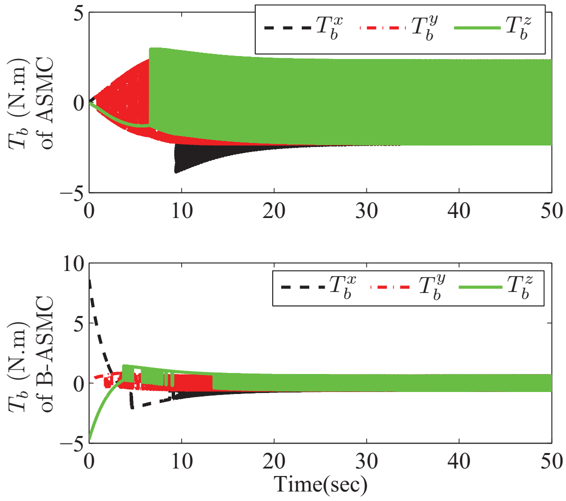

Fig. 3. Control torque response comparison

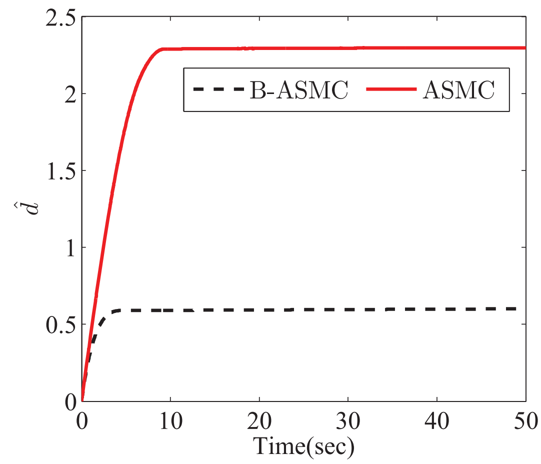

Fig. 4. $\hat{d}$ comparison between two algorithms

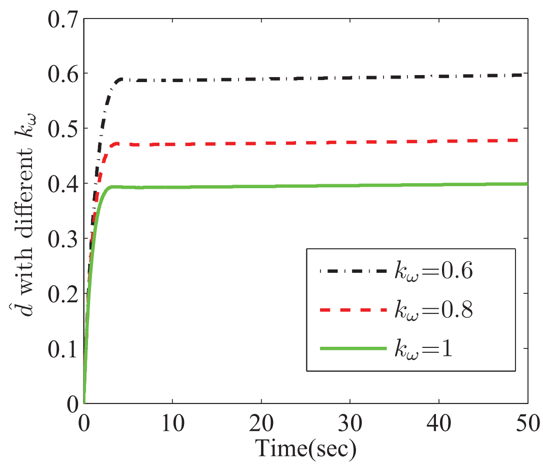

Fig. 5. $\hat{d}$ with different k_ω in the B-ASMC algorithm

than the B-ASMC algorithm. By examining the switching gains generated by the two control algorithms, as shown in Fig.4, it is clear that the resulting $\hat{d}$ updated by the B-ASMC algorithm is much smaller than the ASMC algorithm, which verifies that the B-ASMC algorithm can weaken the over adaptation problem in the ASMC algorithm and the chattering phenomena is correspondingly reduced. Moreover, $\hat{d}$ generated by the B-ASMC algorithm with different k_ω is illustrated in Fig.5, from which we can see that the larger the k_ω is, the smaller the $\hat{d}$ will be and consequently the lower chattering phenomena. However, it should be pointed out that large k_ω will result in a large initial control torque, as shown in Fig.3.

V. Conclusion

This paper presents a B-ASMC design for the spacecraft attitude control problem. The proposed algorithm solves the theoretical inadequacy in current ASMC design, where the asymptotical stabilities of the sliding function and the entire closed-loop system are achieved. The system performance is improved by virtue of two additional terms in the control law as compared to current ASMC algorithms. Moreover, the over adaptation problem in ASMC design is also considered and a lower-chattering control signal is achieved. The issue of large initial control torque requirement in B-ASMC will be a topic of in the future work.

References

[1] S.R. Vadali, "Variable structure control of spacecraft large-angle maneuvers", *J. Guidance*, vol. 9, no. 2, pp. 235-239, 1986.

[2] S.C. Lo, Y.P. Chen, "Smooth sliding-mode control for spacecraft attitude tracking maneuvers", *Journal of Guidance, Control, and Dynamics*, vol 18, no. 6, pp. 1345-1349, 1995.

[3] S.A. Kowalchuk, C.D. Hall, "Spacecraft attitude sliding mode controller using reaction wheels", in *AIAA/AAS Astrodynamics Specialist Conference and Exhibit*, Honolulu, Hawaii, AIAA 2008-6260, 2008.

[4] J.L. Crassidis, "Sliding mode control using modified Rodrgiues parameters", *Journal of Guidance, Control, and Dynamics*, vol. 19, no. 6, pp. 1381-1383, 1996.

[5] D.S. Yoo, M.J. Chung, "A variable structure control with simple adaptation laws for upper bounds on the norm of the uncertainties", *IEEE Transactions on Automatic Control*, vol. 7, no. 6, pp. 860–865, 1992.

[6] G. Wheeler, C.Y. Su, Y. Stepanenko, "A sliding mode controller with improved adaption laws for the upper bounds on the norm of uncertainties", in *Proc. 1996 IEEE Workshop on Variable Structure Systems*, Tokyo, Japan, pp. 154–159, 1996.

[7] Z. Zhu, Y.Q. Xia, and M.Y. Fu, "Adaptive sliding mode control for attitude stabilization with actuator saturation", *IEEE Transaction on Industrial Electronics*, vol. 58, no. 10, pp. 4898–4907, 2011.

[8] F.J. Lin, S.L. Chiu, "Novel sliding mode controller for synchronous motor drive", *IEEE Transactions on Aerospace and Electronic Systems*, vol. 34, no. 2, pp. 532–542, 1998.

[9] J.S. Souder, J.K. Hedrick, "Adaptive sliding mode control of air-fuel ratio in internal combustion engines", *Int. J. Robust Nonlinear Control*, vol. 14, no. 6, pp. 525–541, 2004.

[10] R.J. Wai, "Adaptive sliding mode for induction servomotor drive", *IEE Proc.-Electr. Power Appl.*, vol. 147, no. 6, pp. 553–562, 2000.

[11] Y.J. Huang, T.C. Kuo and S.H. Chang, "Adaptive sliding-mode conrol for nonlinear systems with uncertain parameters", *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics*, vol. 38, no. 2, pp. 534–539, 2008.

[12] M. Krstić, Kanellakopoulos, and P. Kokotović, *Nonlinear and Adaptive Control Design*, New York: Wiley, 1995.

[13] G. Bartolini, A. Ferrara, L. Giacomini, E. Usai, "A combined backstepping/second order sliding mode approach to control a class of nonlinear systems", in *Proc. 1996 IEEE Workshop on Variable Structure Systems*, Tokyo, Japan, pp. 205–210, 1996.

[14] G. Bartolini, A. Ferrara, L. Giacomini, E. Usai, "Properties of a Combined Adaptive/Second-Order Sliding Mode Control Algorithm for some Classes of Uncertain Nonlinear Systems", *IEEE Transactions on Automatic Control*, vol.45, no. 7, pp. 1334–1341, 2000.

[15] C.C. Peng, W.T. Hsue Alber, C.L. Chen, "Variable structure based robust backstepping controller design for nonlinear systems", *Nonlinear Dynamics*, vol. 63, pp. 253-262, 2011.

[16] H. Schaub, J.L. Junkins, *Analytical Mechanics of Space Systems*, AIAA, Virginia, 2009.

[17] J.T.-Y. Wen, K. Kreutz-Delgado, "The attitude control problem", *IEEE Transactions on Automatic Control*, vol. 36, no. 10, pp. 1148–1162, 1991.

[18] P. Tsiotras, "Further passivity results for the attitude control problem", *IEEE Transactions on Automatic Control*, vol. 43, no. 11, pp. 1597–1600, 1998.

[19] W.B. Gao, J.C. Hung, "Variable Structure Control of Nonlinear Systems: A new Approach,", *IEEE Transactions on Industrial Electronics*, vol. 40, no. 1, pp. 45–55, 1993.

[20] F. Plestan, Y. Shtessel, et al, "New methodologies for adaptive sliding mode control", *International Journal of Control*, vol. 83, no. 9, pp. 1907–1919, 2010.

[21] A. Lavant, "Sliding order and sliding accuracy in sliding mode control", *International Journal of Control*, vol. 58, no.6, pp. 1247–1263, 1993.