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Using MATLAB GUIs to improve the learning of frequency response methods

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Abstract—**This paper describes two MATLAB GUIs that have been designed to improve student learning of frequency response methods. One GUI is focused on plotting using asymptotes and one on systems identification. The paper includes positive student feedback on how they felt these GUIs helped their understanding, and useful suggestions on how they can be improved.**

Keywords- Education, Frequency Response, Bode diagrams

I. INTRODUCTION

A recent survey of Control curricula[1] in the UK reaffirmed that Frequency Response methods were an integral part of the degree, typically taught as part of the second course in control. In the past, as shown by books such as Atkinson[2], much emphasis was given to the calculations for plotting and designing using Bode and Nyquist diagrams. Nowadays, programs such as MATLAB can do much of the work for students, but there is the danger that students do not fully understand what is happening. In [1] specific views are given:

"Today however the scenario is different. Students need to have an understanding of how to sketch Bode and Nyquist plots and root-loci, but need to have less concern for computing the numerical details; software can be used."

"I think there is great value in teaching the theory of Bode diagrams and Nyquist plots. This provides the basis of understanding the plant dynamics and the effects of closed loop control and the controller settings. However, although I think students should understand the mechanics of calculating the frequency response of a system, the use of MATLAB should enable the calculations to be done quickly so that time can be devoted to controller design. It would also allow higher order systems to be investigated."

"Computer assessment using MATLAB looks interesting … though still have a hankering after asymptotic Bode sketches as a first start - confirm with MATLAB once or twice then leave it to computer."

This year, after a teaching review, a System Identification and Control module was formed, for which the author gave ten lectures and two assignments on frequency response methods. These covered plotting Bode and Nyquist diagrams, and the design of controllers and identification of linear systems using frequency domain data.

This paper describes the assignments set, to students taking Cybernetics, Robotics and Electronic Engineering degrees at Reading, including the use of two MATLAB GUIs developed to assist students to learn about the plotting and identification, and the associated programming of controllers.

These GUIs involve the use of asymptotes, as they help the understanding of Bode diagrams, for which the author has developed extra asymptotes to help as regards phase plots. These extra asymptotes are also described.

The paper is organized as follows. In the next section, some comments are made as regards frequency response, asymptotes and identification using Bode plots. Section III describes the GUI used for plotting asymptotes and Section IV the GUI for identification. Section V contains details of the assignments, and in section VI student feedback on the GUIs is given. Section VII has the author's reflections on the feedback, and then concluding remarks are made.

II. ON FREQUENCY RESPONSE

The frequency response of a system is the variation of its gain and phase as the angular frequency ω of its sinusoidal input is varied. The results can be plotted on Bode diagrams, where log(gain) and phase are plotted separately against log(ω), or on a Nyquist diagram, where the gain and phase provide the polar coordinates of the locus. In this paper, Bode plots are considered, which are used to plot the 'open loop' transfer function (that round the feedback loop), to assess stability, design controllers and for identification.

Here, linear systems are modeled as a series of single or quadratic poles or zeros, operating around corner frequencies. Knowing these corner frequencies, it is possible to calculate values so as to sketch the Bode plots. Similarly, the corner frequencies can be estimated from the Bode plots of a system. In both processes one considers what happens at low frequencies and then at each corner frequency in turn, until the highest corner frequency has been processed. It is thus appropriate to develop computer based systems for both the plotting of Bode plots and system identification from Bode plots.

In Thorne[3], laboratory practicals are described in which students learn to identity systems comprising multiple quadratic poles only. Thorne also believes that identification helps students in the understanding of Bode plots.

Bode plots can be approximated by straight lines, the asymptotes, which can be derived from Bode's fundamental work[4]. For a stable minimum phase system with transfer function P(jω), its phase at ω_c is given by (1): $u = \ln(\omega/\omega_c)$

$$\angle P(j\omega_c) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{d|P(j\omega)|}{du} \ln\coth\frac{|u|}{2}\,du \qquad (1)$$

By assuming that the gain is constant around ω_c, the derivative tern can be brought outside the integral. The remaining integral evaluates as ½ π^2. Changing the derivative variable from u to ω, (1) can be rewritten as:

$$\angle P(j\omega_c) \approx \omega \frac{d|P(j\omega)|}{d\omega}\Big|_{\omega_c} * \frac{\pi}{2} \qquad (2)$$

Hence the Bode gain plot can be approximated as straight lines, between the corner frequencies of P(jω), whose gradients are integers, and the phase plot can be approximated as constant integer multiples of ½ π radians (or 90^O).

Sketches of gain plots start and end on the first and last asymptotes, and can be close to all the asymptotes, depending on the proximity of the system corner frequencies. This is less true of the phase, due to step changes of the asymptotes at the corner frequencies. As such, books like Dorf and Bishop[5] or Nise[6] suggest that these phase asymptotes should be joined by 'diagonal' asymptotes between one tenth of and ten times the associated corner frequency, that is over a range of 100 rad/s. The author has developed a better approach, where the range for such diagonal asymptotes is set so that their slope equals that of the phase plot at the corner frequency. For a single pole or zero, this range is $e^{\pi} \approx 23$rad/s. For a quadratic pole or zero with damping ratio ζ, the range is $e^{\zeta\pi} = 4.8$ rad/s if ζ = 0.5 or 2.2 rad/s if ζ = 0.25. These values and the approach are justified in the appendix, and used in the GUIs.

Consider the following system which illustrates these extra asymptotes, and the approach for plotting and identification. The system being analysed is

$$\frac{50(1 + s/2)}{(1 + s/0.07)(1 + s/0.4)(s^2/20^2 + 0.6s/20 + 1)} \qquad (3)$$

By inspection the corner frequencies are 0.07, 0.4, 2 and 20 rad/s. The gain asymptote has value 50 and slope 0 until 0.07 rad/s, the slope is -1 until 0.4 rad/s, -2 until 2 rad/s, -1 until 20 rad/s and -3 thereafter. In these ranges, the phase asymptotes have values 0, -90, -180, -90 and -270 degrees. The range of the extra phase asymptotes around each corner frequency is 23 rad/s except at 20 rad/s where it is 2.6 rad/s. These can be seen in Fig 1, where the actual Bode plot is also superimposed. The gain plot is easy to sketch from the asymptotes. The extra phase asymptotes make it easier to sketch the phase plot.

As regards identification, the shape of the phase plot here could be used. As the phase moves from 0^O past -90^O it is apparent that there are two low frequency poles. The phase then rises, suggesting a zero, and then the phase decreases more rapidly towards -270^O suggesting a quadratic pole: the steepness of the plot at this point, which is consistent with the diagonal asymptote, suggests that the pole is underdamped.

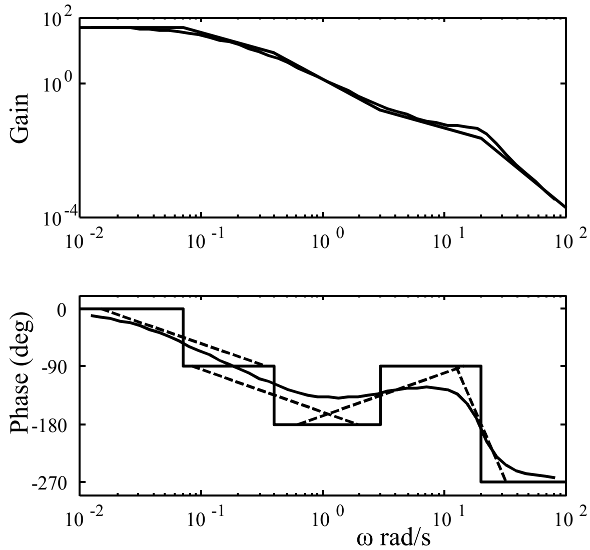

Figure 1 Bode plot of example system including asymptotes

In this example, the corner frequencies are well spaced, making sketching and identification easier as each pole or zero has a relatively small impact on the others. In the GUI described later, it is shown how closer poles and zeroes are addressed, for instance, by identifying two such poles by a quadratic pole, or a pole near a zero by a lead-lag element. First, however, the GUI for plotting is described.

III. GUI FOR PLOTTING

The first GUI is aimed at helping students to understand the frequency response, by drawing the asymptotes for the Bode gain and phase plots. In the lectures various transfer functions are discussed and the corner frequencies identified. It is stated that the slope of the gain asymptotes are integers and that the phase asymptotes constant values being that slope multiplied by 90^O. It is explained that, for example, when the corner frequency is that of a single pole, the slope decreases by 1, and hence there is thereafter an extra 90^O phase lag. In the past, the emphasis has been on sketching gain asymptotes approximately using these integer slopes. However, in the author's experience, even some of the better students did not appreciate what was happening: this is consistent with[1].

The GUI was therefore designed to require the students to inspect a transfer function and enter each corner frequency in turn and relevant information about the gain and phase.

Initially it was planned that the students would enter the slope of the gain plot and the asymptotic phase immediately after each corner frequency. However, as phase is the gain slope times 90^O it was perhaps redundant to require both. The author thought that students would learn more if they had to enter at the corner frequency the asymptotic gain there as well as the asymptotic phase. Thus if the gain was G at corner frequency ω_1, after which the slope was s, then at the next corner frequency ω_2 they would have to calculate the gain as

$$G * \left(\frac{\omega_2}{\omega_1}\right)^s \qquad (4)$$

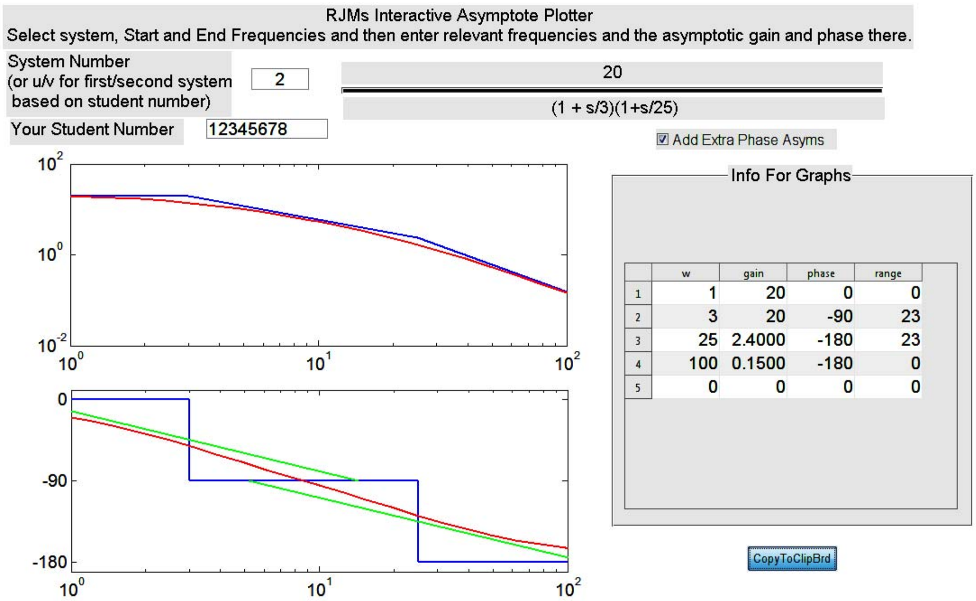

Figure 2 Plotter GUI after student has used system

Overall, when given a transfer function, the student first has to enter a suitable range of frequencies for the plot, being before the first and after the last corner frequencies. The student then enters for this first frequency, each corner frequency in turn, and the last frequency, the gain of the asymptote there, and the asymptotic phase after that frequency. If the extra diagonal asymptotes are used, the student has to enter the range of the phase asymptote around that frequency.

As each frequency is added, the asymptotes up to and including this frequency are shown. The student can move to the next corner frequency, or 'undo' the last entry.

At the end, the actual Bode plots are superimposed on top of the asymptotes, and the student can assess how well they have entered the data. As an example, suppose the system is

$$\frac{20}{(1 + s/3)(1 + s/25)} \tag{5}$$

The corner frequencies are 3 and 25 rad/s, and so a suitable range of frequencies is 1 to 100 rad/s. The student should then enter the following (the last column being needed for the extra phase asymptotes)

ω	Gain	Phase	Range
1	20	0	0
3	20	-90	23
25	2.4	-180	23
100	0.15	-180	0

Figure 2 shows the GUI after these data have been input.

The GUI has a variety of systems built into it which the student can be tested upon. In addition, as the GUI is used as part of marked coursework, potential plagiarism needs to be addressed. Thus the GUI can also be invoked with systems whose structure is fixed, but whose corner frequencies are calculated from the student's unique student number.

IV. GUI FOR IDENTIFICATION

For identification, the process is straightforward, with the result being built up as a series of elements starting at low frequencies, and continuing until after the last corner frequency is found. The user selects that the next element is a constant gain, an integrator, a single pole or zero, a quadratic pole or zero, or a lead-lag element.

For any system to be identified, three arrays are provided, having a series of angular frequencies and the gain and the phase at each of these frequencies. In the identification process, two more gain and phase arrays are calculated: one set has the gain and phase of the identified model, and one has the gain and phase of the model that is still to be identified. The former set is initialised with gains of 1 and phases of 0; the latter set is initialised with the original gain and phase arrays provided.

Once an element of the system model is estimated, the gain and phase of that element are calculated at each of the angular frequencies. The 'already identified' gain array is multiplied by the gain found, and its phase array has the new phase values added. The 'to-be-identified' gain array is divided by the gain found and its phase array has the new phase values subtracted.

Two Bode plots are provided. One shows the frequency response data provided, superimposed on to which is the estimate of the system so far. The other has the response yet to be identified. The user interacting with the system should see the identified system following the original plot at lower frequencies. Deviations of the to-be-identified plot from its low frequency response are used to identify the next element.

As used this year, in the GUI the user specifies the type of element, and the system automatically estimates the parameter(s) associated with that element, such as gains or time constants, which the user can accept, modify or reject. In the final section of the paper consideration is given as to how the user could be required to calculate these parameters.

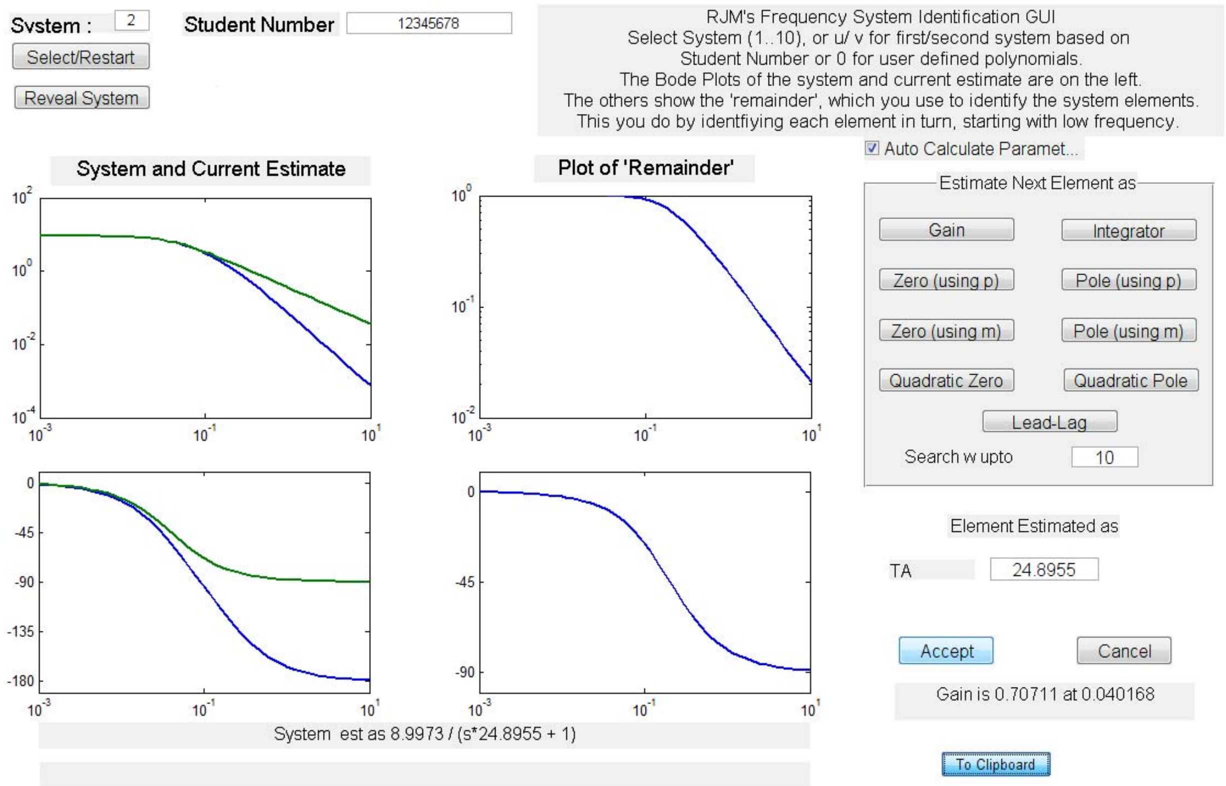

Figure 3 Part way through identification of two pole system

When the user selects the type of the next element, the plots are updated and the user is shown the value of any estimated parameter. The user can then modify that value, perhaps on the basis that a subsequent element is affecting this one. The user can then accept the element and value, or cancel, which could happen if the wrong type of element had been selected.

The system estimates the gain of an element from the first value in the gain array. For a pole or zero, the user specifies whether its time constant is found using the gain or phase array. If the former, the time constant is the reciprocal of the angular frequency where the gain is a factor 0.707 away from the low frequency gain; if the latter the frequency where the phase is $\pm 45^{O}$ is used. For quadratic poles or zeros, the frequency where the phase is $\pm 90^{O}$ gives the estimate of ω_n and this value and the gain there give an estimate of ζ. If $\zeta > 1$, the result is shown as two time constants. This allows the user to identify two close poles or two close zeros as one element. When a pole and a zero are close, a lead-lag element can be identified: the frequency where the phase is a local minimum or maximum, and the value of that phase are used to find the time constants.

Fig 3 shows the GUI part way through the identification of a system with two poles. Here the gain was first estimated, as 8.9973, the actual value is 9. Then the time constant of the first pole was found, estimated at 24.895: it should have been 25. The graphs on the left show that this model accurately follows the actual plot at low frequencies. The graphs on the right indicate that a further pole needs to be identified.

At the end, the student presses the reveal button and the actual transfer function is given. The student can then see how well the process has worked by comparing the actual and estimated transfer functions and Bode plots.

Fig 4 shows part of the GUI at the end of identifying the two pole system. The second pole should have had a time constant of 5, but the estimate was 4.65. The remainder plots are not quite horizontal lines showing that there have been small (but expected) errors in the estimates of the parameters.

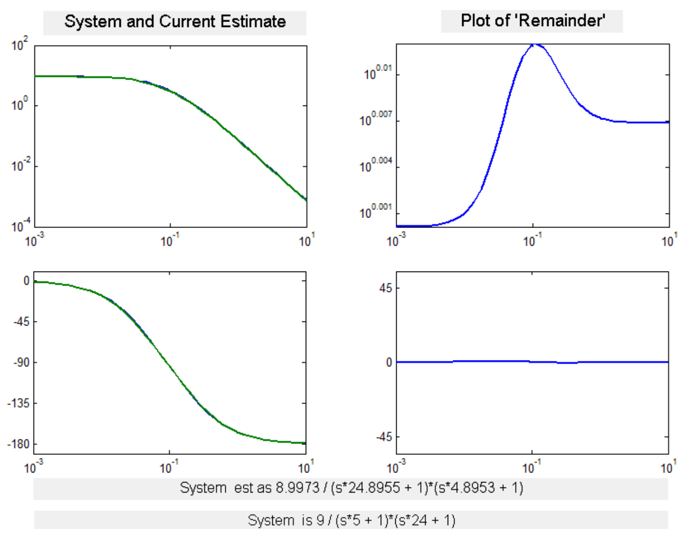

Figure 4 At the end of identification of two pole system

V. ASSIGNMENTS

These GUIs were used in two assignments. The first was aimed at familiarizing the student with frequency response plots. It comprised tasks set after lectures 2, 3 and 4, including the use of the GUI to plot various systems, and to design a simple controller and assess its performance. Formative feedback only was provided for this part.

The first task was to call the m file provided which generated a system based on the student's username: the numerator and denominator polynomials were displayed symbolically, as shown at the top of Fig 2. Students then noted the corner frequencies of that system, and used MATLAB to calculate the gain and phase at those frequencies. In the second task, students used the GUI to plot a few different systems, including one based on their username. Finally, the students used MATLAB to design a proportional controller to achieve a phase margin specification, plot the step response and calculate key values such as overshoot and time to peak.

Students were provided with a word document into which results were to be pasted, with questions asking the student to reflect on their answers, provide the lecturer with feedback on the GUI and say whether they felt it useful to teach asymptotes. Built into this document were options for formative assessment of the work – such as the section was missing, wrong, partially correct or correct. The marker could quickly circle the appropriate assessment and, with more detailed comments, provided rapid and relevant formative feedback on this part of the assignment, which are known to be beneficial [7], [8].

The second assignment included more plotting, of systems with second order poles, the use of the identification GUI, and the design of Phase Lead, P+I and PID controllers using frequency domain data. For the design, students were provided with an m file with functions provided for defining systems, for plotting graphs, and empty functions for controller design which the students had to complete. Again results, and code, were pasted into a word document, but this now had a marking scheme instead of comments like 'wrong'. Students were also asked to comment on the GUI.

VI. EVALUATION

The 20 students on the module generally did the first assignment well: the two who did not cited difficulties understanding the GUI. Most answered the questions asked.

The first question asked was: 'Has the GUI helped/hindered you to understand asymptotes/Bode plots". Almost all students said it had. Some specific quotes are given below:

"At first I felt the GUI was hindering me as I could not work out how to get the correct asymptotes, however the GUI started to encourage me to research and learn the theory … Looking back on the learning process I feel it was very useful for understanding the concepts of asymptotes and Bode plots".

"The GUI has vastly improved my understanding of Bode plots, mainly due to it being a focused exercise towards improving sketching Bodes that includes being able to check answers against the actual plots of the system".

The second question asked if it would have been better if they had been asked to specify the gain slopes rather than the value of the asymptotic gain at the corner frequencies. Many students said that would have been easier, but they would not have learnt as much. Some specific quotes are given below.

"I have previously used the method 'slope specification' when plotting by hand so it is useful to have another method."

"I feel if I had learnt both and tried both on a similar style GUI I would have learnt far more. It would have been useful to practice both methods so that we could work out which method suited us individually."

"I don't think that I would have learnt so much if I had to only enter the slopes"

The third question was whether it was useful to teach asymptotes . All who expressed a preference believed it was.

"It is very useful teaching about asymptotes because the process of plotting them enables one to understand how the various terms of a transfer function relate to the system's gain, phase and corner frequencies. Without the manual plotting of asymptotes I don't believe that my understanding of Bode plots … would be nearly as in depth"

"[Definitely] as it offers a basis for Bode plots without having to do thousands of calculations. … It helped me a lot in understanding Bode plots."

Students were also asked to make suggestions for improving the GUI. On-line help was requested, and there were specific comments on the entry of data at each corner frequency in the plotting GUI. These will be used next year.

For the second GUI students were asked if it helped/ hindered understanding. Again almost all students felt it did.

"I found the GUI extremely helpful as I was able to see what is happening with the frequency response of the system when different terms to the transfer function are added. [Now] I definitely feel more confident about Bode plots"

"The System Id GUI has helped most of all in understanding the estimation of the composition of transfer functions …. The values of each of the elements of the transfer function, which are generally estimable via the analysis of the angular frequency at which changes in the bode plot occur, were an element … that the author had not fully appreciated in the past. Being able to, at a glance, agree or disagree with the value presented in the estimation was helpful"

VII. REFLECTION

The students believe that both GUIs have helped them understand the material, and they agreed that teaching asymptotes was beneficial. Although the GUIs were demonstrated in lectures, some difficulties were found, so more help will be given next year as regards the use of both GUIs.

It was interesting to see the comment that it would be more useful to have initially been asked to specify the slope of the gain after the corner frequencies and in later examples to give the value of the gain asymptote at the corner frequency. The first method could be used in familiarization.

Reflecting on this the author realized that for the identification GUI students specified an element type and the system automatically calculated the associated element parameter(s), whereas the plotting GUI expected the student to do relevant calculations. This was inconsistent.

Hence for next year, for the plotting GUI, initially users will enter slopes to specify gain, and later the asymptotic gain at the corner frequencies. Also, the identification GUI will have two modes, one where the parameters are found automatically, and one where a hint is given and the user has to then calculate the parameter. For instance, for a lead-lag element the hint will give data where the phase is a local maximum/minimum using which the student will calculate the two time constants.

VIII. CONCLUSION

Two useful GUIs have been produced which the students believe have helped their understanding of frequency response methods. Student feedback is also influencing the development of the GUIs so that next year there will be a better combination of automatic and student-based calculations.

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APPENDIX

This appendix contains the derivation of the range of frequencies around the corner frequency for the extra 'diagonal' phase asymptotes. Conventionally this range is recommended as 100 rad/s [5], and Fig 5 shows these, and the actual phase, for both a single pole and a quadratic pole with damping ratio 0.5.

For a single pole, this approach is quite good, as the area of the region on one side of the asymptote is approximately equal to that on the other. However for a quadratic pole the areas are quite different. A better approach, advocated here, is for the slope of the diagonal asymptote to equal that of the slope of a pole or zero at the corner frequency.

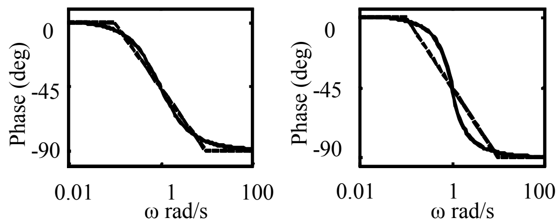

Figure 5. Phase and Asymptotes for Single and Quadratic Pole

As regards, finding the diagonal asymptote, let r be the range over which the diagonal asymptote spans, centred on the corner frequency, ω_{CF}, of a single or a quadratic pole or zero, which changes the phase by nπ. If φ is the phase of such an element, plotted against log(ω), the slope of the asymptote is set by the slope of the element evaluated at ω_{CF}, so

$$\frac{n\pi}{\log(r)} = \frac{d\varphi}{d\log(\omega)}\bigg|_{\omega_{CF}} = \frac{\omega}{\log(e)}\frac{d\varphi}{d\omega}\bigg|_{\omega_{CF}} \tag{6}$$

This can be rearranged to give

$$r = e^{n\pi \Big/ \left(\omega \frac{d\varphi}{d\omega}\big|_{\omega_{CF}}\right)} \tag{7}$$

For a single pole or zero, with time constant T, n = ± 0.5 and $\varphi = \pm\tan^{-1}(\omega T)$,

$$\omega\frac{d\varphi}{d\omega}\bigg|_{\omega_{CF}} = \pm\frac{\omega T}{1+\omega^2T^2}\bigg|_{\frac{1}{T}} = \pm\frac{1}{2} \tag{8}$$

$$r = e^{\pi} \tag{9}$$

For a quadratic pole or zero with corner frequency ω_n and damping ratio ζ, for which n = ±1, it can be shown that

$$\omega\frac{d\varphi}{d\omega}\bigg|_{\omega_{CF}} = \pm\frac{\omega\, 2\zeta\omega_n\left(\omega_n^2+\omega^2\right)}{\left(\omega_n^2-\omega^2\right)^2+4\zeta^2\omega_n^2\omega^2}\bigg|_{\omega_n} = \pm\frac{1}{\zeta} \tag{10}$$

$$r = e^{\pi\zeta} \tag{11}$$

Fig 6 shows the phase and these asymptotes for a single and quadratic pole, using these values of r. The phase is easier to sketch than the conventional approach. Instead of crossing the diagonal asymptote and then intersecting with it at the corner frequency, the actual phase moves smoothly between the horizontal asymptote before or after the corner frequency (at some integer * ½ π) and the diagonal asymptote. The gain curve moves between asymptotes in a similar manner.

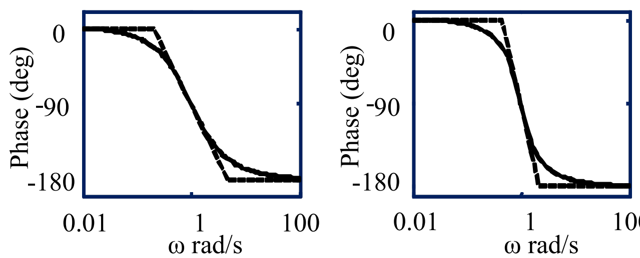

Figure 6 Better Phase Asymptotes for Single and Quadratic Pole