

# Estimation of pulmonary elastance fuzzy model by data combination of two respiration phases

M. Nakamichi\*, S. Kanae\*, Z.J. Yang<sup>†</sup> and K. Wada<sup>‡</sup>

\*Fukui University of Technology,

3-6-1 Gakuen, Fukui-city, Fukui 915-8505, Japan

<sup>†</sup>Ibaraki University,

4-12-1 Makanarusawa, Hitachi-city, Ibaraki 316-8511, Japan

<sup>‡</sup>Kyusyu University,

744 Motooka, Nishi-ku, Fukuoka-city, Fukuoka 819-0395, Japan

Email: kanae@fukui-ut.ac.jp

**Abstract**—Pulmonary characteristics differ in patients, and the suitable setting of ventilation condition is needed for every patient in the artificial respiration. The pulmonary elastance is one of the important features of lung, and it is a basis for deciding the airway pressure limit value. To get the pulmonary elastance of the patient from measurement data of the artificial respiration, the fuzzy logic technique has been proposed for estimating the pulmonary elastance and the static  $P - V$  curve in our previous works. In this paper, a new technique of fuzzy modeling based on data combination of two respiration phases is proposed to improve the estimation precision, and some estimation examples using real patient data are given to illustrate the superiority of the proposed method over the previous algorithm in the precision.

**Key Words**—modeling, estimation, fuzzy logic, artificial respiration, elastance

## I. INTRODUCTION

Artificial respirator is used for patients with little or no autonomous breathing ability. Using a respirator, particular attention should be paid to set suitable ventilation condition. Pulmonary characteristics differ in patients and change by the extent of illness progress or recovery. Now, the setting of the respirator is decided by the experience and the intuition of the doctor. This is a problem that there is no appropriate method for deciding ventilation condition. This problem may cause a medical accident. These circumstances motivate us to develop a method to estimate the pulmonary elastance of the patient and to set a ventilation condition of the artificial respirator.

In our previous work, we have presented an estimation technique of the pulmonary elastance by fuzzy logic, and have represented the static  $P - V$  curve needed for the respirator setting. However, the estimation precision of previous technique was not satisfactory, the estimated static  $P - V$  curve cannot be used for the respirator setting. In this paper, we improved the estimation precision by new technique that combine the measurement data of two phases of breath. Furthermore, this paper shows some estimation examples using real patient data

to illustrate the superiority of the proposed method over the previous algorithm in the precision.

The paper is organized as follows. Section 2 introduces the static  $P - V$  curve and nonlinear differential equation model of respiratory systems. A new method that estimating the pulmonary elastance by fuzzy logic is described in Section 3. Some examples of estimation are given in Section 4. Finally, Section 5 concludes this paper.

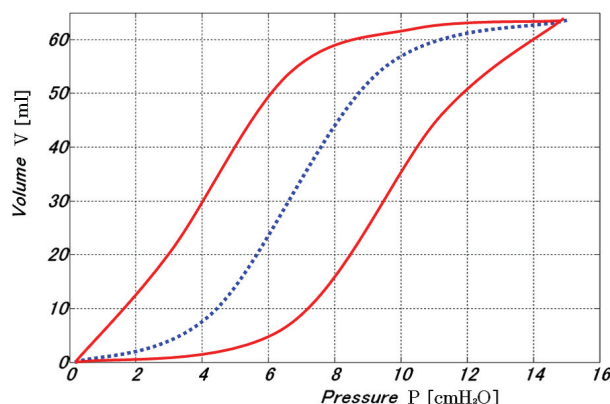


Fig.1. The dynamical  $P - V$  curve (solid line) and the static  $P - V$  curve (dotted line).

## II. NONLINEAR DIFFERENTIAL EQUATION MODEL OF RESPIRATORY SYSTEMS

The static state is a state without the flow of air in the respiratory system. In this state, the static  $P - V$  curve is drawn by the pressure ( $P_l(t)$ ) and the volume ( $V(t)$ ) inside of lung. The gradient of the static  $P - V$  curve is the compliance, and the inverse of the compliance is the elastance of the lung. The pulmonary elastance ( $f_E(V)$ ) is a nonlinear function of  $V(t)$ , and the static  $P - V$  curve is expressed by the following equation:

$$P_l(t) = f_E(V)V(t). \quad (1)$$

This static  $P - V$  curve expresses the important feature of the lung, and it is a basis for deciding the air-way pressure limit value.

Fig.1 shows the dynamic  $P - V$  curve (solid line) and the static  $P - V$  curve (dotted line) by respiratory data over one period. The dynamic  $P - V$  curve can be drawn by air-way pressure and air-volume of lung that can be measured directly. However, the static  $P - V$  curve cannot be drawn directly, because inside pressure of lung cannot be measured directly. So, we must estimate the static  $P - V$  curve by technique of the system identification.

For a rational setting of the respirator, a mathematical model is needed to describe the respiration. Some related works are addressed in the literature[2][3]. The lung of human is divided into right and left. Even if each lung is modeled by a simple first order differential equation, the synthesized overall respiratory system model becomes to second order differential equation [1]. Respiratory system model as the following equation is proposed by Kanae *et al*,

$$P_{ao}(t) + a_1 \dot{P}_{ao}(t) = f_E(V)V(t) + g_R(\dot{V})\dot{V}(t) + b_2 \ddot{V}(t) + P_{eea} + \epsilon(t), \quad (2)$$

where  $P_{ao}(t)$  is air-way opening pressure,  $F(t) = \dot{V}(t)$  is air-flow,  $P_{eea}$  is the end-expiratory alveolar pressure. In addition,  $g_R(\dot{V})$  is so-called air-way resistance. The pressure loss  $P_r$  is expressed by the following equation,

$$P_r(t) = g_R(F)F(t) = (r_1 + r_2|\dot{V}(t)|)\dot{V}(t). \quad (3)$$

The pulmonary elastance and the air-way resistance are described by polynomials of volume  $V(t)$ .  $\epsilon(t)$  contains the modeling error and the measurement noise. In the model (2), the order of equation are the same as the conventional model[2]. Therefore, the model has the capability of describing the nonlinear dynamical characteristics of respiration. We can measure air-way opening pressure, volume, air-flow in each sampling period.

The fuzzy logic is suitable for expressing a nonlinear function, and can reduce computational complexity in the estimation by the fuzzy division. After that, using this respiratory system model (2), this study presents a method to estimate the pulmonary elastance by fuzzy logic.

### III. FUZZY MODEL OF RESPIRATORY SYSTEMS

#### A. Fuzzy Logic

Mamdani's fuzzy IF-THEN rules are very famous in fuzzy logic[4]. The structure of these rules is that all inputs are set in part of the antecedent and all outputs are set in part of the consequent.

In this study, the volume ( $V(t)$ ) is defined as the part of the antecedent and pulmonary elastance of the first order function of ( $f_E(V) = k_1 + k_2V$ ) is defined as the part of the consequent. Fuzzy variables in part of the antecedent are defined as *small*, *medium*, *big*.

#### B. Previous Method

Using technique called Functional Type SIRMs (Single Input Rule Modules) fuzzy reasoning method, We estimated the pulmonary elastance in previous study. Functional Type SIRMs fuzzy reasoning method is proposed by Seki *et al* [5]. Firstly, this method defines IF-THEN fuzzy rule modules of single input type. The overall reasoning result is weighted sum of the reasoning results of each rule modules. In this study, input term  $V$  is divided into the inspiratory volume  $V_{in}$  and the expiratory volume  $V_{out}$ . Therefore, fuzzy rules is structured as follows,

*Rule -  $V_{out}$  :*

*if  $V_{out} = small_{out}$ ,*

*then  $f_{Eout(small)} = k_{1out(small)} + k_{2out(small)}V$ ;*

*if  $V_{out} = medium_{out}$ ,*

*then  $f_{Eout(medium)} = k_{1out(medium)} + k_{2out(medium)}V$ ;*

*if  $V_{out} = big_{out}$ ,*

*then  $f_{Eout(big)} = k_{1out(big)} + k_{2out(big)}V$ .*

*Rule -  $V_{in}$  :*

*if  $V_{in} = small_{in}$ ,*

*then  $f_{Ein(small)} = k_{1in(small)} + k_{2in(small)}V$ ;*

*if  $V_{in} = medium_{in}$ ,*

*then  $f_{Ein(medium)} = k_{1in(medium)} + k_{2in(medium)}V$ ;*

*if  $V_{in} = big_{in}$ ,*

*then  $f_{Ein(big)} = k_{1in(big)} + k_{2in(big)}V$ .*

(4)

The consequent part parameters of fuzzy rules (4) are estimated by the numerical integration technique and the least squares method[1][6]. The reasoning result of each rule modules is calculated by the Center of Gravity Method as follows,

*Rule -  $V_{out}$  :*

$$h_j^{V_{out}} = A_j^{V_{out}}(V^0), \quad (j = small, medium, big),$$

$$f_{Eout}^0 = \frac{\sum_{j=small}^{big} h_j^{V_{out}} f_{Eout(j)}(V^0)}{\sum_{j=small}^{big} h_j^{V_{out}}}; \quad (5)$$

Rule –  $V_{in}$  :

$$h_j^{V_{in}} = A_j^{V_{in}}(V^0), \quad (j = small, medium, big),$$

$$f_{E_{in}}^0 = \frac{\sum_{j=small}^{big} h_j^{V_{in}} f_{E_{in}(j)}(V^0)}{\sum_{j=small}^{big} h_j^{V_{in}}}, \quad (6)$$

where,  $A$  is fuzzy set of the antecedent variable  $V$ ,  $h_j$  is the conformity degree of the antecedent.

The overall reasoning result  $f_E^0$  can be calculated as follows:

$$f_E^0 = \omega_{V_{out}} f_{E_{out}}^0 + \omega_{V_{in}} f_{E_{in}}^0, \quad (7)$$

where,  $\omega_i$  ( $i = V_{in}, V_{out}$ ) is so-called the serious consideration degree for the overall reasoning result.

Finally,  $f_E^0$  is substituted for relation equation between the pressure ( $P_l$ ) and the volume ( $V$ ) inside of lung as follows:

$$P_l = f_E^0 V. \quad (8)$$

The static  $P - V$  curve is drawn by equation (8).

However, estimation precision for this technique was not satisfactory. We will explain a process to improve estimation precision later.

### C. Estimation of consequent part parameters of the fuzzy rule

This section introduces the method that estimates the function parameters of consequent part about an arbitrary rule in fuzzy rule of equation (4). Using the respiratory system model, the first order function ( $f_E(V) = k_1 + k_2V$ ) substitutes the relationship of

$$P_{ao}(t) + a_1 \dot{P}_{ao}(t) = k_1 V(t) + k_2 V^2(t) + g_R(\dot{V})\dot{V}(t) + b_2 \ddot{V}(t) + P_{eea} + \epsilon(t). \quad (9)$$

Define data vector  $\varphi(t)$  and parameter vector  $\theta$  as follows:  $\varphi^T(t) = [-\dot{P}_{ao}(t), V(t), V^2(t), \dot{V}(t), |\dot{V}(t)|\dot{V}(t), \ddot{V}(t), 1.0]$ ;  $\theta^T = [a_1, k_1, k_2, r_1, r_2, b_2, P_{eea}]$ . Then, using relationship between the volume and the flow ( $F(t) = \dot{V}(t)$ ), data vector  $\varphi(t)$  can be written as follows:  $\varphi^T(t) = [-\dot{P}_{ao}(t), V(t), V^2(t), F(t), |F(t)|F(t), \dot{F}(t), 1.0]$ .

The model equation can also be written in short form as follows:

$$P_{ao}(t) = \varphi^T(t)\theta + \epsilon(t). \quad (10)$$

Equation (9) is continuous-time model. An identification model is obtained by applying the numerical integration technique which is known as an effective approach for continuous-time model identification[7].

Measurement data are sampled data of air pressure  $P_{ao}(k)$ , flow  $F(k)$ , volume  $V(k)$ , where  $k$  ( $k = 1, 2, \dots, N$ ) denotes sampling instant.  $N$  define the data size.  $T$  define

the sampling period of data collection. Then, at time instant  $t = kT$ , integrate both sides of equation (10) over the interval  $[(k - \ell)T, kT]$ . Using the numerical integration technique that is proposed by Sagara *et al*, left side of equation (10) can be calculated as follows:

$$y(k) = \int_{(k-\ell)T}^{kT} P_{ao}(\tau) d\tau \doteq \sum_{j=0}^{\ell} g_j P_{ao}(k - j), \quad (11)$$

where,  $\ell$  is a natural number that decides the window size of numerical integration. The coefficients  $g_i$  ( $i = 1, 2, \dots, \ell$ ) are determined by formulae of numerical integration. When the trapezoidal rule is taken, they are given as follows:

$$\begin{cases} g_0 = g_\ell = T/2, \\ g_i = T, \quad i = 1, 2, \dots, \ell - 1. \end{cases} \quad (12)$$

As calculation of equation (11), data vector  $\varphi(t)$  can be calculated by

$$\phi(k) = \int_{(k-\ell)T}^{kT} \varphi(\tau) d\tau = \begin{bmatrix} -P_{ao}(k) + P_{ao}(k - \ell) \\ \sum_{j=0}^{\ell} g_j V(k - j) \\ \sum_{j=0}^{\ell} g_j V^2(k - j) \\ V(k) - V(k - \ell) \\ \sum_{j=0}^{\ell} g_j |F(k - j)|F(k - j) \\ F(k) - F(k - \ell) \\ \ell T \end{bmatrix}. \quad (13)$$

Get together the approximation error  $\Delta_E$  caused by numerical integration and the integral of original error term  $\epsilon$  in  $e(k)$ . Namely, Let  $e(k)$  be

$$e(k) = \Delta_E + \int_{(k-\ell)T}^{kT} \epsilon(\tau) d\tau. \quad (14)$$

Consequently, an identification model of discrete-time form is provided from equation (11), equation (13) and equation (14) as follows:

$$y(k) = \phi^T(k)\theta + e(k). \quad (15)$$

From the measurements of air pressure  $P_{ao}(k)$ , flow  $F(k)$ , volume  $V(k)$ , calculates  $y(k)$  and  $\phi(k)$  at each time instant  $k = \ell + 1, \dots, N$ . And the vector equation is structured by them as follows:

$$\mathbf{y} = \Phi\theta + \mathbf{e}, \quad (16)$$

where,  $\mathbf{y} = [y(N) \dots y(\ell + 1)]^T$ ,  $\Phi = [\phi(N) \dots \phi(\ell + 1)]^T$ ,  $\mathbf{e} = [e(N) \dots e(\ell + 1)]^T$ .

The least squares estimate that minimizes the criterion function  $J = \|\mathbf{y} - \Phi\theta\|^2$  is given by

$$\hat{\theta} = (\Phi^T \Phi)^{-1} \Phi^T \mathbf{y}. \quad (17)$$

Then, the function of consequent part about an arbitrary rule in fuzzy rules (4), in other words, the function of pulmonary elastance is estimated as

$$\hat{f}_E(V) = \hat{k}_1 + \hat{k}_2 V. \quad (18)$$

Executing the above-mentioned calculation on all fuzzy rules, fuzzy rules are made.

#### D. New Method

It is considered that there are two reasons to result unsatisfactory precision of estimation. One is that the design of serious consideration degree has to be performed by hand-operation. Another one is that there is relatively large error in each connection area of two fuzzy rules, and the whole precision becomes worse as the partition number of variable increased. We are going to solve these two problems here.

The idea is to combine the data of two respiratory phases. Firstly, prepares the regression data in each range of fuzzy variables according to equation (16). Then, combines the data of two phases of expiratory and inspiratory respiration in every same range. Consequently, the fuzzy rules are combined as follows:

$$\begin{aligned} & \text{if } V = \text{small}, \\ & \text{then } f_{E(\text{small})} = k_{1(\text{small})} + k_{2(\text{small})}V; \\ & \text{if } V = \text{medium}, \\ & \text{then } f_{E(\text{medium})} = k_{1(\text{medium})} + k_{2(\text{medium})}V; \\ & \text{if } V = \text{big}, \\ & \text{then } f_{E(\text{big})} = k_{1(\text{big})} + k_{2(\text{big})}V. \end{aligned} \quad (19)$$

Using combined data, the parameters of the consequent part of the fuzzy rules (19) are estimated by the least squares method as mentioned above.

Using above fuzzy rules (19), the overall reasoning result of pulmonary elastance is estimated. The reasoning result of fuzzy rules (19) is calculated by the Center of Gravity Method as follows:

$$\begin{aligned} h_j^V &= A_j^V(V^0), \quad (j = \text{small}, \text{medium}, \text{big}), \\ f_E^0 &= \frac{\sum_{j=\text{small}}^{\text{big}} h_j^V f_{E(j)}(V^0)}{\sum_{j=\text{small}}^{\text{big}} h_j^V}. \end{aligned} \quad (20)$$

where,  $A$  is fuzzy set of the antecedent variable  $V$ ,  $h_j$  is the conformity degree of the antecedent. The static  $P - V$  curve is drawn by equation (8).

In this way, problems of a calculation error and the serious consideration degree are solved by combining the data of two phases of breath.

## IV. EXAMPLES OF ESTIMATION

In this section, some experimental results are shown to estimate the static  $P - V$  curve with real clinical data of artificial respiration. Here, ranges of the fuzzy variables (*small*, *medium* and *big*) are set by manual operation and each of the serious consideration degree is assumed as degree of 0.5. Fig.2 and Fig.4 are membership functions of the fuzzy variables in each experiment. Fig.3 and Fig.5 are experimental results that estimate static  $P - V$  curve with real clinical data of artificial respiration. The circle points and the square points are quasi-static elastance  $P - V$  points for inspection. It is shown that the estimated static  $P - V$  curve passes near of inspection data similar the true static  $P - V$  curve. Table.1 is mean-square error of estimated static  $P - V$  curve by each method to inspection data. Table.1 shows that estimated static  $P - V$  curve by new method passes near of inspectional data points.

It is important that we must set suitable range of the fuzzy variables. In these experiments, we were able to find suitable values by a hand-operated design. However, the estimated precision is not satisfactory when fuzzy variables are not suitable values. Considering of the problems mentioned above and pulmonary characteristics that differ in patients, it is necessary to devise optimization algorithm for range setting of the fuzzy variables. Finally, we aim at the automation of estimation of pulmonary elastance and the improvement of estimated precision in the future.

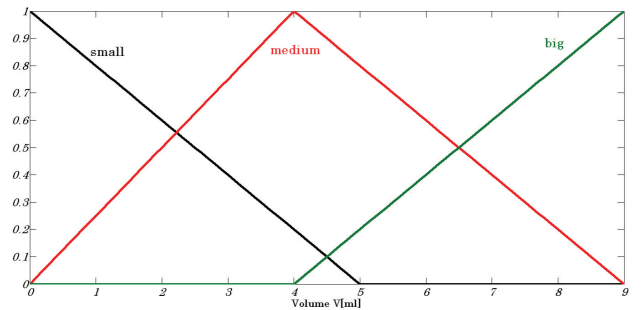


Fig.2. Data1: Membership functions of fuzzy variables.

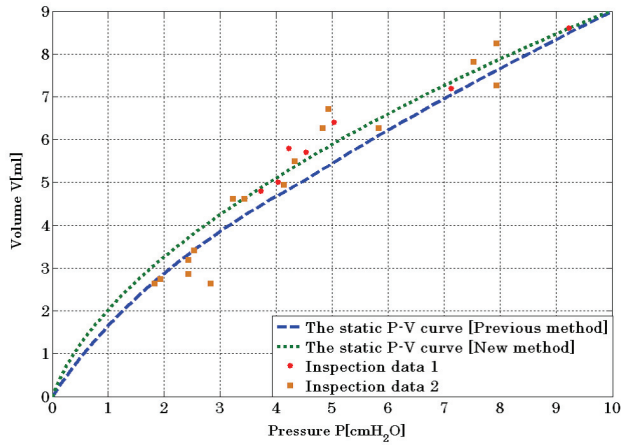


Fig.3. Data1: Estimated static  $P - V$  curve and experimental static elastic recoil pressure-volume points( $\circ$ ,  $\square$ ).

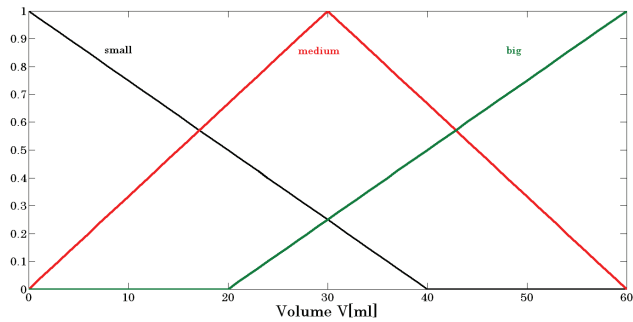


Fig.4. Data2: Membership functions of fuzzy variables.

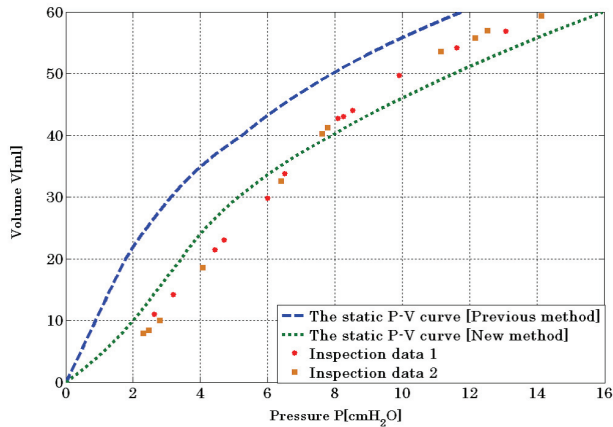


Fig.5. Data2: Estimated static  $P - V$  curve and experimental static elastic recoil pressure-volume points( $\circ$ ,  $\square$ ).

### V. CONCLUSIONS

The pulmonary elastance is one of the important features of lung, and it is a basis for deciding the air-way pressure

TABLE I  
MEAN-SQUARE ERROR OF ESTIMATED STATIC  $P - V$  CURVE BY EACH METHOD TO INSPECTION DATA.

	Previous [cmH <sub>2</sub> O]	New [cmH <sub>2</sub> O]
Data 1	0.5658	0.3525
Data 2	5.4845	1.2985

limit value. In this study, an estimation method of pulmonary elastance based on fuzzy logic is proposed. Using technique to combine the data of two phase of breath, one fuzzy rule that merged the expiratory fuzzy rule with the inspiratory fuzzy rule is made. The reasoning result of fuzzy rule is calculated by the Center of Gravity Method.

As a result of estimated pulmonary elastance with real clinical data of artificial respiration, we can draw the static  $P - V$  curve that passes near of inspectional data points. However, it is necessary to set suitable range of the fuzzy variables of the antecedent part. Therefore, by devising optimization algorithm for range setting, we aim at the automation of estimation of pulmonary elastance and the improvement of estimated precision in the future.

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