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Dynamic Optimization of Polymer Flooding with Free Terminal Time based on Maximum Principle

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Abstract—**Polymer flooding is an important technology for enhanced oil recovery (EOR). In this paper, an optimal control model of distributed parameter systems (DPS) for polymer flooding is established, which involves the performance index as maximum of the profit, the governing equations as the seepage equations of polymer flooding, and some inequality constraints as polymer concentration and injection amount limitation. The injection polymer concentration and the terminal time of polymer flooding are chosen as control variables. For this distributed parameter optimal control problem (OCP) with free terminal time, a solution method based on maximum principle is proposed. Firstly, the free terminal time OCP of polymer flooding is transformed into a fixed final time problem by introducing a normalized time variable. Then through application of the maximum principle, adjoint equations and gradients of the objective functional are obtained to optimize the injection polymer concentration and the terminal time simultaneously. Finally, the numerical results of an example illustrate the effectiveness of the proposed method.**

Keywords-optimal control; maximum principle; distributed parameter system; polymer flooding; free terminal time.

I. INTRODUCTION

The optimal control method has been researched in EOR techniques in recent years. Ramirez and Fathi firstly applied it to optimize the injection process of surfactant flooding [1], [2]. Then the optimal control method was used to other enhanced oil recovery techniques [3], such as steam flooding, caustic flooding and gas injection etc. The optimal gas-cycling decision problem of a condensate reservoir has been studied by Ye [4]. The dynamic optimization of water flooding with smart wells has been studied before by Brouwer [5], [6], Sarma [7], [8] and Zhang [9].

Polymer flooding is one of the most effective EOR techniques [10]. Because of the high costs associated with polymer flooding projects, optimal control method must be developed to reduce producing costs while increasing the profit of oil recovered. In this paper, the OCP of a polymer flooding process with free terminal time is considered. The performance index of the OCP is expressed by maximizing the economic benefit. The governing equations are a set of partial differential equations (PDEs), which are a pressure equation, a water saturation equation and a polymer concentration equation respectively. The constraint conditions include the polymer concentration constraint and other inequality constraints. The control variables are chosen as the injection concentrations and the free terminal time. Then the determination of polymer injection strategies turns to solve this OCP of DPS. A normalized time variable is introduced to transform the free terminal time OCP of polymer flooding into a fixed final time problem. Then the necessary conditions for optimality are obtained by Pontryagin's maximum principle. A gradient numerical method is presented for solving the transformed OCP. Finally, an example of polymer flooding project involving a heterogeneous reservoir case is investigated and the results show the efficiency of the proposed method.

The rest of this article is organized as follows: In section II the optimal control model of polymer flooding with free terminal time is built. In section III the original OCP with free terminal time is transformed into a fixed terminal time problem and the necessary conditions for optimality are obtained. In section IV a gradient numerical method is proposed for solving the transformed OCP. In section V an example of polymer flooding accompanied with the optimal results is given. And in section VI some conclusions are derived.

II. MATHEMATICAL FORMULATION OF OPTIMAL CONTROL

A. Performance Index

Let $\Omega \in R^2$ denote the domain of reservoir with boundary $\partial\Omega$, n be the unit outward normal on $\partial\Omega$, and $(x, y) \in \Omega$ be the coordinate of a point in the reservoir. We suppose that there exist N_w injection wells and N_o production wells in the oilfield. The injection wells are located at $L_w = \{(x_{wi}, y_{wi}) \mid i = 1, 2, \ldots, N_w\}$ and the production wells are located at $L_o = \{(x_{oj}, y_{oj}) \mid j = 1, 2, \ldots, N_o\}$, respectively. For polymer flooding, we might wish to increase the profit and reduce the producing cost. Given a free terminal time t_f, the performance index is given mathematically by

$$\max\ J = \int_0^{t_f} \iint_{\Omega} \left[\xi_o (1 - f_w) q_{out} - \xi_p q_{in} c_{pin} \right] d\sigma dt, \qquad (1)$$

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where ξ_p is the cost coefficient of polymer ($10^4\$/m^3$), ξ_o is the price coefficient of oil ($10^4\$/m^3$), f_w is the fractional flow of water, q_{in} is the velocity of polymer injection (m/day), q_{out} is the velocity of fluid production (m/day) and c_{pin} is the injection concentration of polymer (g/L).

B. Governing Equations

Let $p(x,y,t)$, $S_w(x,y,t)$ and $c_p(x,y,t)$ denote the pressure, water saturation and polymer concentration of the reservoir, respectively, at a point $(x,y)\in\Omega$ and a time $t\in[0,t_f]$, then $p(x,y,t)$, $S_w(x,y,t)$ and $c_p(x,y,t)$ satisfy the following partial differential equations (PDEs):

- Pressure equation

$$\frac{\partial}{\partial x}\left(k_p r_o \frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_p r_o \frac{\partial p}{\partial y}\right)-(1-f_w)q_{out}=h\frac{\partial a_o}{\partial t}, \quad (2)$$

- Water saturation equation

$$\frac{\partial}{\partial x}\left(k_p r_w \frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_p r_w \frac{\partial p}{\partial y}\right)+q_{in}-f_w q_{out}=h\frac{\partial a_w}{\partial t}, \quad (3)$$

- Polymer concentration equation

$$\begin{aligned}&\frac{\partial}{\partial x}\left(k_d r_d \frac{\partial c_p}{\partial x}\right)+\frac{\partial}{\partial x}\left(k_p r_c \frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_d r_d \frac{\partial c_p}{\partial y}\right)+\\ &\frac{\partial}{\partial y}\left(k_p r_c \frac{\partial p}{\partial y}\right)+q_{in}c_{pin}-f_w q_{out}c_p=h\frac{\partial a_c}{\partial t}.\end{aligned} \quad (4)$$

The boundary conditions and initial conditions are

$$\left.\frac{\partial p}{\partial n}\right|_{\partial\Omega}=0,\ \left.\frac{\partial S_w}{\partial n}\right|_{\partial\Omega}=0,\ \left.\frac{\partial c_p}{\partial n}\right|_{\partial\Omega}=0, \quad (5)$$

$$\begin{aligned}&p(x,y,0)=p^0(x,y),\ S_w(x,y,0)=S_w^0(x,y),\\ &c_p(x,y,0)=c_p^0(x,y),\end{aligned} \quad (6)$$

The corresponding parameters in (2)–(4) are defined as

$$k_p = K h\,,\ k_d = D h\,, \quad (7)$$

$$r_o=\frac{k_{ro}}{B_o\mu_o},\ r_w=\frac{k_{rw}}{B_w R_k \mu_w},\ r_c=\frac{k_{rw}c_p}{B_w R_k \mu_p},\ r_d=\frac{\phi_p S_w}{B_w}, \quad (8)$$

$$a_o=\frac{\phi(1-S_w)}{B_o},\ a_w=\frac{\phi S_w}{B_w},\ a_c=\frac{\phi_p S_w c_p}{B_w}+\rho_r(1-\phi)C_{rp}, \quad (9)$$

where $K(x,y)$ is the absolute permeability (μm^2), h is the thickness of the reservoir bed (m), D is the diffusion coefficient of polymer (m^2/s), ρ_r (kg/m^3) is the rock density, and μ_o ($mPa\cdot s$) is the oil viscosity. Other parameters definition can refer to [11] for details.

C. Constraints

The performance index (1) should be subject to the polymer concentration constraint

$$0\le c_{pin}\le c_{\max}, \quad (10)$$

the injection amount constraint

$$\int_0^{t_f}\iint_\Omega q_{in}c_{pin}\,d\sigma dt\le m_{p\max}, \quad (11)$$

and the terminal state constraint

$$f_w|_{t=t_f}=98\%, \quad (12)$$

where $c_{\max}$ is the maximum injection concentration and $m_{p\max}$ is the maximum polymer amount.

III. Necessary Conditions of Optimal Control

A. Problem Transformation

For the orginal OCP of polymer flooding with free terminal time, a normalized time variable is introduced,

$$\tau = t/t_f, \quad (13)$$

Since $t\in[0,\ t_f]$, we have $\tau\in[0,\ 1]$. By using the definite integral by substitution, the performance index (1) is expressed as

$$\max\ J=\int_0^1\iint_\Omega t_f\left[\xi_o(1-f_w)q_{out}-\xi_p q_{in}c_{pin}\right]d\sigma d\tau. \quad (14)$$

The system state vector is denoted by

$$\boldsymbol{u}(x,y,t)=[p,S_w,c_p]^T. \quad (15)$$

The control for the process is the polymer concentration of injected fluid

$$v(x,y,t)=c_{pin},\ (x,y)\in L_w. \quad (16)$$

Then the governing equations (2)–(4) which can be expressed by

$$\frac{\partial\boldsymbol{a}}{\partial t}=\tilde{\boldsymbol{f}}(\boldsymbol{u},\boldsymbol{u}_x,\boldsymbol{u}_y,\boldsymbol{u}_{xx},\boldsymbol{u}_{yy},v,t), \quad (17)$$

are normalized as

$$\frac{\partial\boldsymbol{a}}{\partial\tau}=t_f\tilde{\boldsymbol{f}}(\boldsymbol{u},\boldsymbol{u}_x,\boldsymbol{u}_y,\boldsymbol{u}_{xx},\boldsymbol{u}_{yy},v,\tau), \quad (18)$$

where $\boldsymbol{u}_l=\partial\boldsymbol{u}/\partial l$, $l=x,y$.

If t_f is treated as a new optimization variable and $\boldsymbol{v}=[v,\ t_f]^T$ is denoted as control vector, the orginal OCP of polymer flooding is transformed into the following fixed terminal time problem,

$$\max\ J=\int_0^1\iint_\Omega F(\boldsymbol{u},\boldsymbol{v},\tau)\,d\sigma d\tau, \quad (19)$$

$$s.t.\quad \boldsymbol{f}(\dot{\boldsymbol{u}},\boldsymbol{u},\boldsymbol{u}_x,\boldsymbol{u}_y,\boldsymbol{u}_{xx},\boldsymbol{u}_{yy},\boldsymbol{v},\tau)=0 \tag{20}$$

$$\boldsymbol{g}(\boldsymbol{u},\boldsymbol{u}_x,\boldsymbol{u}_y,\boldsymbol{u}_{xx},\boldsymbol{u}_{yy},\tau)=0\,, \tag{21}$$

$$\boldsymbol{u}(x,y,0)=\boldsymbol{u}^0(x,y), \tag{22}$$

$$\int_0^1\iint_\Omega c_1(\boldsymbol{v})d\sigma d\tau \le 0, \tag{23}$$

$$c_2(\boldsymbol{u}|_{\tau=1})=0, \tag{24}$$

$$0\le \boldsymbol{v}\le \boldsymbol{v}_{\max}. \tag{25}$$

where $\dot{\boldsymbol{u}}=\partial\boldsymbol{u}/\partial\tau$. With this transformation, at $t=t_f$, $\tau_f=1$, and in the dimensionless time domain the terminal time is fixed.

B. Maximum Principle of DPS

A convenient way to cope with such an OCP of DPS (19)–(25) is through the use of distributed adjoint variables. We define the Hamiltonian as

$$H=F+\boldsymbol{\lambda}^T\boldsymbol{f}, \tag{26}$$

where $\boldsymbol{\lambda}(x,y,\tau)$ is the adjoint vector. Then the argument functional is given by,

$$\begin{aligned} J_A &= J+\int_0^1\iint_\Omega \boldsymbol{\lambda}^T\boldsymbol{f}(\dot{\boldsymbol{u}},\boldsymbol{u},\boldsymbol{u}_x,\boldsymbol{u}_y,\boldsymbol{u}_{xx},\boldsymbol{u}_{yy},\boldsymbol{v},\tau)d\sigma d\tau \\ &= \int_0^1\iint_\Omega H(\dot{\boldsymbol{u}},\boldsymbol{u},\boldsymbol{u}_x,\boldsymbol{u}_y,\boldsymbol{u}_{xx},\boldsymbol{u}_{yy},\boldsymbol{v},\tau)d\sigma d\tau. \end{aligned} \tag{27}$$

The increment of J_A, denoted by ΔJ_A, is formed by introducing variations $\delta\boldsymbol{u}$, $\delta\boldsymbol{u}_x$, $\delta\boldsymbol{u}_y$, $\delta\boldsymbol{u}_{xx}$, $\delta\boldsymbol{u}_{yy}$, $\delta\dot{\boldsymbol{u}}$, and δv giving

$$\begin{aligned} \Delta J_A = J_A(&\boldsymbol{u}+\delta\boldsymbol{u},\boldsymbol{u}_x+\delta\boldsymbol{u}_x,\boldsymbol{u}_y+\delta\boldsymbol{u}_y,\boldsymbol{u}_{xx}+\delta\boldsymbol{u}_{xx},\boldsymbol{u}_{yy}+\delta\boldsymbol{u}_{yy}, \\ &\dot{\boldsymbol{u}}+\delta\dot{\boldsymbol{u}},\boldsymbol{v}+\delta\boldsymbol{v})-J_A(\boldsymbol{u},\boldsymbol{u}_x,\boldsymbol{u}_y,\boldsymbol{u}_{xx},\boldsymbol{u}_{yy},\dot{\boldsymbol{u}},\boldsymbol{v}). \end{aligned} \tag{28}$$

Expanding (28) in a Taylor series and retaining only the linear terms gives the variation of the functional, δJ_A,

$$\begin{aligned} \delta J_A = \int_0^1\iint_\Omega \Bigg[&\left(\frac{\partial H}{\partial \boldsymbol{u}}\right)^T\delta\boldsymbol{u}+\left(\frac{\partial H}{\partial \boldsymbol{u}_x}\right)^T\delta\boldsymbol{u}_x+\left(\frac{\partial H}{\partial \boldsymbol{u}_{xx}}\right)^T\delta\boldsymbol{u}_{xx}+ \\ &\left(\frac{\partial H}{\partial \boldsymbol{u}_y}\right)^T\delta\boldsymbol{u}_y+\left(\frac{\partial H}{\partial \boldsymbol{u}_{yy}}\right)^T\delta\boldsymbol{u}_{yy}+\left(\frac{\partial H}{\partial \dot{\boldsymbol{u}}}\right)^T\delta\dot{\boldsymbol{u}}+ \\ &\left(\frac{\partial H}{\partial \boldsymbol{v}}\right)^T\delta\boldsymbol{v}\Bigg]d\sigma d\tau. \end{aligned} \tag{29}$$

Since the variations $\delta\boldsymbol{u}$, $\delta\boldsymbol{u}_l$, $\delta\boldsymbol{u}_{ll}$ ($l=x,y$) and $\delta\dot{\boldsymbol{u}}$ are not independent can be expressed in terms of the variations $\delta\boldsymbol{u}$ by integrating the following three terms by parts

$$\begin{aligned} \iint_\Omega\left[\left(\frac{\partial H}{\partial \boldsymbol{u}_l}\right)^T\delta\boldsymbol{u}_l\right]d\sigma = &\iint_\Omega\frac{\partial}{\partial l}\left[\left(\frac{\partial H}{\partial \boldsymbol{u}_l}\right)^T\delta\boldsymbol{u}\right]d\sigma- \\ &\iint_\Omega\left[\frac{\partial}{\partial l}\left(\frac{\partial H}{\partial \boldsymbol{u}_l}\right)^T\delta\boldsymbol{u}\right]d\sigma, \end{aligned} \tag{30}$$

$$\begin{aligned} \iint_\Omega\left[\left(\frac{\partial H}{\partial \boldsymbol{u}_{ll}}\right)^T\delta\boldsymbol{u}_{ll}\right]d\sigma = &\iint_\Omega\left[\frac{\partial^2}{\partial l^2}\left(\frac{\partial H}{\partial \boldsymbol{u}_{ll}}\right)\right]^T\delta\boldsymbol{u}d\sigma+ \\ &\iint_\Omega\frac{\partial}{\partial l}\left[\left(\frac{\partial H}{\partial \boldsymbol{u}_{ll}}\right)^T\delta\boldsymbol{u}_l-\frac{\partial}{\partial l}\left(\frac{\partial H}{\partial \boldsymbol{u}_{ll}}\right)^T\delta\boldsymbol{u}\right]d\sigma, \end{aligned} \tag{31}$$

$$\int_0^1\left(\frac{\partial H}{\partial \dot{\boldsymbol{u}}}\right)^T\delta\dot{\boldsymbol{u}}=\left[\left(\frac{\partial H}{\partial \dot{\boldsymbol{u}}}\right)^T\delta\boldsymbol{u}\right]\Bigg|_0^1-\int_0^1\frac{\partial}{\partial\tau}\left(\frac{\partial H}{\partial \dot{\boldsymbol{u}}}\right)^T\delta\boldsymbol{u}d\tau. \tag{32}$$

By using the above expressions (30)–(32), the first variation δJ_A is written as

$$\begin{aligned} \delta J_A = &\int_0^1\iint_\Omega\Bigg(\frac{\partial H}{\partial \boldsymbol{u}}-\sum_{l=x,y}\frac{\partial}{\partial l}\frac{\partial H}{\partial \boldsymbol{u}_l}+\sum_{l=x,y}\frac{\partial^2}{\partial l^2}\frac{\partial H}{\partial \boldsymbol{u}_{ll}}+ \\ &-\frac{\partial}{\partial\tau}\frac{\partial H}{\partial \dot{\boldsymbol{u}}}\Bigg)^T\delta\boldsymbol{u}\,d\sigma d\tau+\int_0^1\iint_\Omega\sum_{l=x,y}\frac{\partial}{\partial l}\Bigg\{\left(\frac{\partial H}{\partial \boldsymbol{u}_{ll}}\right)^T\delta\boldsymbol{u}_l+ \\ &\left[\frac{\partial H}{\partial \boldsymbol{u}_l}-\frac{\partial}{\partial l}\left(\frac{\partial H}{\partial \boldsymbol{u}_{ll}}\right)^T\right]\delta\boldsymbol{u}\Bigg\}d\sigma d\tau+ \\ &\iint_\Omega\left[\left(\frac{\partial H}{\partial \dot{\boldsymbol{u}}}\right)^T\delta\boldsymbol{u}\right]\Bigg|_0^1 d\sigma+\int_0^1\iint_\Omega\left(\frac{\partial H}{\partial \boldsymbol{v}}\right)\delta v d\sigma\, d\tau. \end{aligned} \tag{33}$$

Applying Pontryagin's Maximum Principle, the necessary conditions for an extremum of J_A are given by

- Adjoint Equations

$$\frac{\partial H}{\partial \boldsymbol{u}}-\sum_{l=x,y}\left(\frac{\partial}{\partial l}\frac{\partial H}{\partial \boldsymbol{u}_l}+\frac{\partial^2}{\partial l^2}\frac{\partial H}{\partial \boldsymbol{u}_{ll}}\right)-\frac{\partial}{\partial\tau}\frac{\partial H}{\partial \dot{\boldsymbol{u}}}=0. \tag{34}$$

- Transversality Boundary Conditions

$$\iint_\Omega\sum_{l=x,y}\frac{\partial}{\partial l}\left\{\left(\frac{\partial H}{\partial \boldsymbol{u}_{ll}}\right)^T\delta\boldsymbol{u}_l+\left[\frac{\partial H}{\partial \boldsymbol{u}_l}-\frac{\partial}{\partial l}\left(\frac{\partial H}{\partial \boldsymbol{u}_{ll}}\right)^T\right]\delta\boldsymbol{u}\right\}d\sigma=0. \tag{35}$$

- Transversality Terminal Conditions

$$\frac{\partial H}{\partial \dot{\boldsymbol{u}}}=0\,,\ \text{at}\ \ \tau=1. \tag{36}$$

- Optimal Control

With the first three necessary conditions being satisfied, the first variation becomes

$$\delta J_A = \int_0^1 \iint_\Omega \left(\frac{\partial H}{\partial \boldsymbol{v}}\right) \delta \boldsymbol{v} d\sigma d\tau. \quad (37)$$

If the variation $\delta\boldsymbol{v}$ is not constrained, then the necessary condition for an extremum is $\partial H / \partial \boldsymbol{v} = 0$.

If the variation $\delta\boldsymbol{v}$ is constrained, which means that the control is at a constraint boundary, then the necessary condition for maximizing the performance functional is

$$\max_{\boldsymbol{v}} H. \quad (38)$$

C. Necessary Conditions of OCP for Polymer Flooding

Let $\lambda(x,y,\tau) = (\lambda_1, \lambda_2, \lambda_3)^T$ denote the adjoint vector of OCP for polymer flooding. Applying the theory developed and substituting the governing equations (2)–(4) into (34), the adjoint equations reduce for the polymer flooding under consideration as given in,

$$\begin{aligned}&\sum_{l=x,y}\left\{\frac{\partial}{\partial l}\left(k_p r_o \frac{\partial \lambda_1}{\partial l}\right)+\frac{\partial}{\partial l}\left(k_p r_w \frac{\partial \lambda_2}{\partial l}\right)+\frac{\partial}{\partial l}\left(k_p r_c \frac{\partial \lambda_3}{\partial l}\right)-\right.\\&\left[k_p \frac{\partial r_o}{\partial p}\frac{\partial p}{\partial l}\frac{\partial \lambda_1}{\partial l}+k_d \frac{\partial r_w}{\partial p}\frac{\partial p}{\partial l}\frac{\partial \lambda_2}{\partial l}+\left(k_p \frac{\partial r_c}{\partial p}\frac{\partial p}{\partial l}+\right.\right.\\&\left.\left.\left.k_d \frac{\partial r_d}{\partial p}\frac{\partial c_p}{\partial l}\right)\frac{\partial \lambda_3}{\partial l}\right]\right\}-q_{out}\left(\xi_o \frac{\partial f_w}{\partial p}-\frac{\partial f_w}{\partial p}\lambda_1+\frac{\partial f_w}{\partial p}\lambda_2+\right.\\&\left.c_p \frac{\partial f_w}{\partial p}\lambda_3\right)+\frac{\partial a_o}{\partial p}\frac{\partial \lambda_1}{\partial \tau}+\frac{\partial a_w}{\partial p}\frac{\partial \lambda_2}{\partial \tau}+\frac{\partial a_c}{\partial p}\frac{\partial \lambda_3}{\partial \tau}=0,\end{aligned} \quad (39)$$

$$\begin{aligned}&\sum_{l=x,y}\left[-k_p \frac{\partial p}{\partial l}\left(\frac{\partial r_o}{\partial S_w}\frac{\partial \lambda_1}{\partial l}+\frac{\partial r_w}{\partial S_w}\frac{\partial \lambda_2}{\partial l}+\frac{\partial r_c}{\partial S_w}\frac{\partial \lambda_3}{\partial l}\right)-\right.\\&\left.k_d \frac{\partial r_d}{\partial S_w}\frac{\partial c_p}{\partial l}\frac{\partial \lambda_3}{\partial l}\right]-q_{out}\left(\xi_o \frac{\partial f_w}{\partial S_w}-\frac{\partial f_w}{\partial S_w}\lambda_1+\frac{\partial f_w}{\partial S_w}\lambda_2+\right.\\&\left.c_p \frac{\partial f_w}{\partial S_w}\lambda_3\right)+\frac{\partial a_o}{\partial S_w}\frac{\partial \lambda_1}{\partial \tau}+\frac{\partial a_w}{\partial S_w}\frac{\partial \lambda_2}{\partial \tau}+\frac{\partial a_c}{\partial S_w}\frac{\partial \lambda_3}{\partial \tau}=0,\end{aligned} \quad (40)$$

$$\begin{aligned}&\sum_{l=x,y}\left[\frac{\partial}{\partial l}\left(k_d r_d \frac{\partial \lambda_3}{\partial l}\right)-k_p \frac{\partial p}{\partial l}\left(\frac{\partial r_w}{\partial c_p}\frac{\partial \lambda_2}{\partial l}+\frac{\partial r_c}{\partial c_p}\frac{\partial \lambda_3}{\partial l}\right)\right]-\\&q_{out}\left[\xi_o \frac{\partial f_w}{\partial c_p}-\frac{\partial f_w}{\partial c_p}\lambda_1+\frac{\partial f_w}{\partial c_p}\lambda_2+\left(c_p \frac{\partial f_w}{\partial c_p}+f_w\right)\lambda_3\right]+\\&\frac{\partial a_c}{\partial c_p}\frac{\partial \lambda_3}{\partial \tau}=0,\end{aligned} \quad (41)$$

The boundary conditions of adjoint equations for the OCP of polymer flooding are expressed as

$$\left(r_o \frac{\partial \lambda_1}{\partial l}+r_w \frac{\partial \lambda_2}{\partial l}\right)\bigg|_{\partial\Omega}=0,\ \frac{\partial \lambda_3}{\partial l}\bigg|_{\partial\Omega}=0,\ l=x,y. \quad (42)$$

The terminal conditions of adjoint equations can be simplified to

$$\lambda_1(x,y,\tau_f)=0,\ \lambda_2(x,y,\tau_f)=0,\ \lambda_3(x,y,\tau_f)=0. \quad (43)$$

IV. Numerical Solution

We propose an iterative numerical technique for determining the optimal injection strategies of polymer flooding. The computational procedure is based on adjusting estimates of control $\boldsymbol{v}$ to improve the value of the objective functional. If the control $\boldsymbol{v}$ is not optimal, then a correction $\delta\boldsymbol{v}$ is determined so that the functional is made lager, that is, $\delta J_A > 0$. If $\delta\boldsymbol{v}$ is selected as

$$\delta \boldsymbol{v} = w \cdot \frac{\partial H}{\partial \boldsymbol{v}}, \quad (44)$$

where w is an arbitrary positive weighting factor. Then the functional variation becomes

$$\delta J_A = \int_0^1 \iint_\Omega w \left(\frac{\partial H}{\partial \boldsymbol{v}}\right)^T \left(\frac{\partial H}{\partial \boldsymbol{v}}\right) d\sigma d\tau \geq 0. \quad (45)$$

Thus, choosing $\delta\boldsymbol{v}$ as the gradient direction ensures a local improvement in the objective functional, J_A. Substituting the governing equations into (44), we obtain the gradient of performance index with respect to the injection polymer concentration v

$$\nabla J(v) = w t_f q_{in}(\lambda_3 - \xi_p),\ (x,y) \in L_w, \quad (46)$$

and the gradient of performance index with respect to the terminal time t_f

$$\nabla J(t_f) = w \int_0^1 \iint_\Omega \left[\xi_o(1-f_w)q_{out} - \xi_p q_{in} c_{pin} + \boldsymbol{\lambda}^T \tilde{\boldsymbol{f}}\right] d\sigma d\tau, \quad (47)$$

The computational algorithm of control iteration based on gradient direction is as follows:

(1) Initialization: Make an initial guess for the control t_f and $v(x,y,\tau)$, $(x,y) \in L_w$, $\tau \in [0,\ 1]$.

(2) Resolution of Governing Equations: Using stored current value of control, integrate the governing equations forward in time with known initial governing conditions. The profit functional is evaluated, and the coefficients involved in the adjoint equations which are function of the state solution are computed and stored.

(3) Resolution of Adjoint Equations: Using the stored coefficients, integrate the adjoint equations numerically backward in time with known final time adjoint conditions by Equation (43). Compute and store $\nabla J(\boldsymbol{v})$ as defined by Equations (46) and (47).

(4) Computation of New Control: Using the evaluated $\nabla J(\boldsymbol{v})$, an improved function is computed as

$$\boldsymbol{v}^{new} = \boldsymbol{v}^{old} + \nabla J(\boldsymbol{v}). \quad (48)$$

A single variable search strategy can be used to find the value of the positive weighting factor w which maximizes the improvement in the performance functional using Equation (46) and (47).

(5) Termination: Go to Step (2) until reach the following stop criteria

$$\left|J^{new} - J^{old}\right| < \varepsilon, \qquad (49)$$

where ε is a small positive number.

It should be noted that the penalty function method is used to deal with the injection amount constraint and the terminal state constraint (23) and (24). The details of this method can refer to [12].

V. EXAMPLE

The two-phase flow of oil and water in a heterogeneous reservoir is considered. The reservoir covers an area of 421.02 × 443.8 m^2 and has a thickness of 5 m and is discretized into 90 (9×10×1) grid blocks by finite difference method. There are four injection wells and a production well in reservoir as shown in Figure 1. Polymer is injected when the fractional flow of water for the production well comes to 97% after water flooding. In the performance index calculation, we use the price of oil $\xi_o = 0.0503\ (10^4\$/m^3)$ [80 ($/bbl)], and the cost of polymer $\xi_p = 2.5\times10^{-4}\ (10^4\$/kg)$. The fluid velocity of production well q_{out} is 0.4624 m/day and the fluid velocity of injection wells q_{in} are all 0.1156 m/day. For the constraint (10), the maximum polymer concentration is $c_{\max} = 2.2\ (g/L)$. The parameters of the reservoir description and the fluid data are shown in Table I. Other parameters can refer to [11].

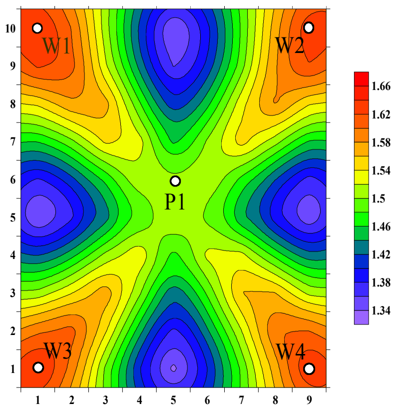

Figure 1. Permeability distribution and well position

The initial injection strategy obtained from engineering method is 1.7 (g/L). The time domain of polymer injection is 0–1500 $days$. When the water fractional flow of production well reaches 98%, the terminal time $t_f = 5498\ (days)$. The performance index is $J = \$1.572\times10^7$ with oil production 32022 m^3 and polymer injection 153000 kg. For comparison, the results obtained by engineering method are considered as the initial control strategies of the proposed iterative gradient method. The maximum injection polymer amount is $m_{p\max} = 153000\ (kg)$. A backtracking search strategy [12] is used to find the appropriate weighting term w and the stopping criterion is chosen as $\varepsilon = 1\times10^{-5}$. By using the proposed algorithm, we obtain a cumulative oil of 32750 m^3 and a cumulative polymer of 153000.02 kg yielding the profit of $J^* = \$1.609\times10^7$. The results show an increase in performance index of $ 3.7×10^5. The optimized terminal time is $t_f = 5165\ (days)$. Figure 2–5 show the injection strategies of the two methods. Figure 6 and Figure 7 show the curves of water fractional flow and accumulative oil production, respectively. The fractional flow of water obtained by proposed method is lower than that by engineering method. Therefore, with the same cumulative polymer injection, the proposed method gets more oil production and higher recovery ratio.

TABLE I. RESERVOIR DESCRIPTION AND FLUID DATA

Symbol	Data	Symbol	Data
p^0	12 MPa	S_w^0	0.35
c_p^0	0	μ_o	15 cp
μ_w	1 cp	ϕ	0.31
D	0.002	ρ_r	2000 kg/m^3
h	5 m	C_{rp}	9.38×10^{-6}

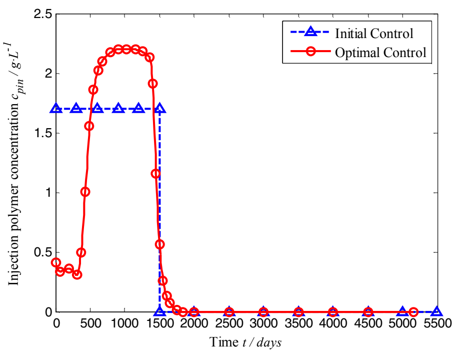

Figure 2. Injection polymer concentration of W1

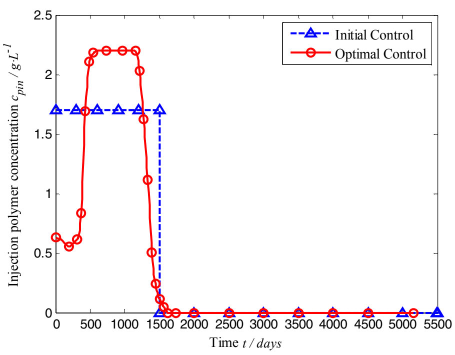

Figure 3. Injection polymer concentration of W2

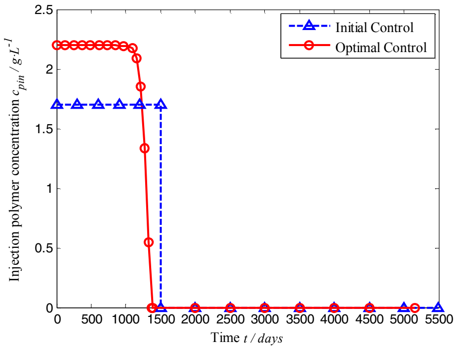

Figure 4. Injection polymer concentration of W3

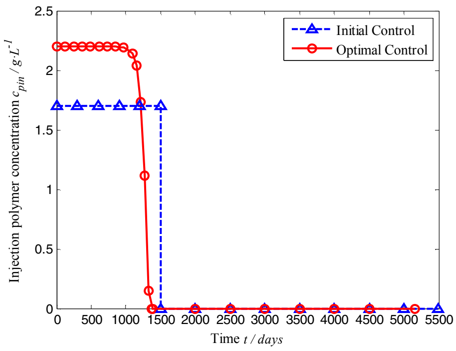

Figure 5. Injection polymer concentration of W4

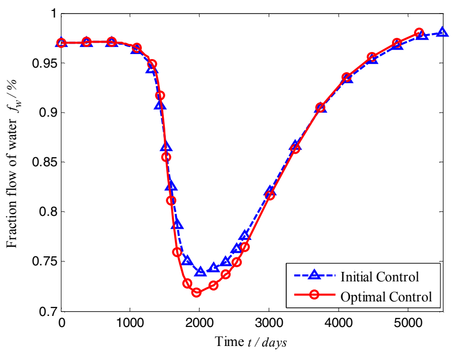

Figure 6. Water fraction flow of the production well P1

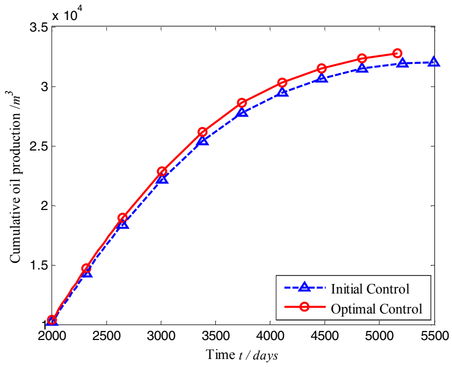

Figure 7. Cumulative oil production

VI. CONCLUSION

In this work, a new optimal control model of DPS is established for the dynamic injection strategies making of polymer flooding. The original problem with free terminal time is transformed into a fixed terminal time OCP by introducing a normalized timer variable. Necessary conditions of this OCP are obtained by using Pontryagin's maximum principle. An iterative computational algorithm is proposed for the determination of optimal injection strategies. The optimal control model of polymer flooding and the proposed method are used for a reservoir example and the optimum injection concentration profile is offered. The results show that the profit is enhanced by the proposed method. Meanwhile, more oil production and higher recovery ratio are obtained. The approach used is a powerful tool that can aid significantly in the development of operational strategies for EOR processes.

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