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Robust MPC algorithms using alternative parameterisations

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***Abstract*—This paper demonstrates the efficacy of alternative parameterisations of the degrees of freedom within a robust MPC algorithm. Alternative parameterisations have been shown to improve the feasible region when the number of degrees of freedom is limited for the nominal case. This paper extends that work to the robust scenario and shows that similar benefits accrue and moreover, the increase in complexity for the robust case as compared to the nominal case is much less than might be expected. The improvements, with respect to a conventional algorithm, are demonstrated by numerical examples.**

Keywords: Robust MPC, Alternative parameterisations, Feasibility.

I. Introduction

Model Based Predictive Control (MPC) or receding horizon control (RHC) or embedded optimisation or moving horizon or predictive control [1], [2], [3], are general names for different computer control algorithms that use predictions as a basis for forming a control law. MPC has now reached a high level of maturity in academia and is widely used in industry. There is substantial interest in how to develop algorithms for the stochastic case and to deal with nonlinearity or uncertainty, in particular because formal consideration of these issues can lead to substantial computation and/or complexity. The main aim of this paper is to contribute to research which improves feasibility while tackling the robust case, perhaps at some small loss of optimality. Specifically, the focus is on the potential of alternative parameterisations of the degrees of freedom (e.g. [20]) to enable enlargement of feasible regions in the uncertain case without too much detriment to performance, optimality and the computational burden.

Two common approaches to robustness have been considered in MPC literature. The first makes use of the inherent robustness of nominal MPC algorithm design, i.e. MPC algorithm that is not specially designed for robust aspects (like stability and performance) [5], [6]. The second approach is the explicit inclusion of robustness requirements into the design of MPC algorithm and this has received considerable attention in the MPC literature. An MPC algorithm design based on including the uncertainty information in the model will be referred to as robust MPC [7], [8].

Traditionally robust MPC requires the solution of min-max optimisation problem, where optimisation is performed in order to minimize a worst-case cost over all possible uncertainty realisations [9]. Furthermore, this also guaranteed constraint satisfaction for all possible future trajectories. For a min-max optimisation problem, stability was proved in [10] by considering feedback within the horizon. In general, solving a min-max problem subject to constraints has to optimise over a sequence of control strategies rather than a sequence of fixed control moves, as the latter is computationally too demanding for practical implementation [11]. All these aspects contribute to make several variants of robust MPC intractable for on-line optimisation [7], [11].

However, various alternative robust MPC algorithms have been proposed to approximate solutions of the max-min problem but with a reduced computational burden. In [12] a classical result is presented by directly incorporating the plant uncertainty into the MPC formulation. The existence of a feedback law minimising an upper bound on the infinite horizon objective function and satisfying constraints is reduced to a convex optimisation using linear matrix inequalities (LMI). The main disadvantages are the use of LMI-based optimisation that can still be computationally demanding and moreover the methods use conservative constrained handling. Several authors reduced the online computational complexity [11], [13], [14] by performing more offline analysis using invariant ellipsoidal sets, however with a significant restriction to the volume of the feasible region. A robust triple mode MPC algorithm was proposed in [15] by introducing an additional mode in dual mode MPC with a large feasible region and good performance; in essence, the objective is to find suitable and fixed linear time varying (LTV) control law.

Of the numerous robust MPC algorithms, quite a few incorporate the notion of feedback in the prediction sequence over which the on-line optimisations take place as this reduces the divergence of the predictions which occurs in open-loop. This closed-loop paradigm improves the control performance but does not necessarily lead to an inexpensive optimisation strategy [7], [8], [9]. Reduced-complexity invariant sets are introduced in [7] for the case of quasi-infinite horizon closed-loop MPC. The reduced-complexity invariant sets may result in a decrease in the number of on-line optimisation variables [7]. This invariant set structure is used in the design of robust MPC and this paper pursues this type of approach to including uncertainty information in the model ([8], [16]).

In the nominal case, Laguerre, Kautz and generalised parameterisations are able to achieve large feasible regions while maintaining local optimality and a relatively low computational complexity [17], [21], [18], [19], [20]. This paper extends the earlier studies in [18], [20] to the case of parameter uncertainty by developing the algorithm of [7] for constructing polyhedral robust positive invariant sets; this enables the online robust MPC algorithm to be based on a standard quadratic programme while adding the benefits of improved feasibility due to the change in parameterisation.

Section II gives the necessary background about nominal MPC, generalised parameterisations for an optimal MPC and robust MPC. Section III discusses alternative parameterisations within Robust MPC using a generalised parameterisation. An algorithm is proposed for Robust MPC using the generalised parameterisation. Comparisons between the existing Robust MPC (RMPC) and the new proposed algorithms are given in section IV using numerical examples. Finally conclusions and future work are in section V.

II. Problem formulation for Robust MPC

Assume discrete time linear parameter varying (LPV) state space models of the form [8]

$$\begin{aligned} x_{k+1} &= A(k)x_k + B(k)u_k, \\ (A(k), B(k)) &\in Co\{[A_1 B_1], ..., [A_m B_m]\}, \end{aligned} \tag{1}$$

with the nominal model being (A, B) and $x_k \in \mathbb{R}^{n_x}$ and $u_k \in \mathbb{R}^{n_u}$ being the state vectors and the plant input respectively. Assume that the states and inputs at all time instants should fulfill the following constraints:

$$L_x x + L_u u \leq l. \tag{2}$$

A. Nominal MPC algorithm

The performance index [3], [22] to be minimised, at each sample instant, with respect to $u_k, u_{k+1}, \dots$ is:

$$\begin{aligned} J = \sum_{i=0}^{\infty} &(x_{k+i+1})^T Q (x_{k+i+1}) + (u_{k+i})^T R (u_{k+i}) \\ s.t. &\left\{ \begin{array}{ll} x_{k+1} = Ax_k + Bu_k, (2) & \forall k \geq 0, \\ u_k = -Kx_k & \forall k \geq n_c, \end{array} \right. \end{aligned} \tag{3}$$

with Q and R positive definite state and input cost weighting matrices and where K is the optimal feedback gain minimizing J in the absence of constraints (2). Practical limitations imply that only a finite number, that is n_c, of free control moves can be used [3]. For these cases, $u_k = -Kx_k$ is implemented by ensuring that the state x_{n_c} must be contained in a polytopic control invariant set (often denoted as the maximal admissible set or MAS):

$$\begin{aligned} \chi_0 = \{x_0 \in R^{n_x} : L_x x - L_u K x_k \leq l, \\ x_{k+1} = Ax_k + Bu_k, \forall k \geq 0\}. \end{aligned} \tag{4}$$

For simplicity of notation, the MAS is described in the form $S_0 = \{x : Mx \leq b\}$ for suitable M and b and the d.o.f. can be reformulated in terms of a new variable c_k:

$$\begin{aligned} u_k &= -Kx_k + c_k, & k = 0, ..., n_c - 1, \\ u_k &= -Kx_k, & k \geq n_c. \end{aligned} \tag{5}$$

The MCAS (maximal controlled admissible set) is given as

$$\chi_c = \{x_k : \exists C, Mx_k + NC \leq b\}, \tag{6}$$

where $C = [c_k^T, \dots, c_{k+n_c-1}^T]^T$ and hence the equivalent optimisation to (3) is:

$$\min_{C} \; J_c = C^T SC \; s.t. \; Mx_k + NC \leq b. \tag{7}$$

The optimal MPC (OMPC) algorithm is given by solving the QP optimisation (7) at every sampling instant then implementing the first component of C, that is c_k in the control law of (5). When the unconstrained control law is not predicted to violate constraints (i.e. $x_k \in \chi_0$), the optimising C is zero so the control law is $u_k = -Kx_k$. The optimisation of (7) can require a large n_c d.o.f. to obtain both good performance and a large feasible region.

B. Generalised parameterisation for Optimal MPC

More recently different alternative parameterisation techniques have been developed to improve the feasible region in nominal case. Laguerre, Kautz and generalised parameterisations have been proposed in [17], [18], [20] as effective alternatives to the standard basis set for parameterising the d.o.f. within an optimal MPC. This section will discuss briefly Laguerre, Kautz and generalised optimal MPC.

The generalised parameterisation [20] is defined using a higher order discrete network such as:

$$\begin{aligned} \mathcal{G}_i(z) = \mathcal{G}_{i-1}(z) \frac{(z^{-1} - a_1) \dots (z^{-1} - a_n)}{(1 - a_1 z^{-1}) \dots (1 - a_n z^{-1})}, \\ 0 \leq a_k < 1, \qquad k = 1 \dots n. \end{aligned} \tag{8}$$

With $\mathcal{G}_1(z) = \frac{\sqrt{(1-a_1^2)\dots(1-a_n^2)}}{(1-a_1 z^{-1})\dots(1-a_n z^{-1})}$. The generalised function with $a_k, \forall k = 1, \dots, n$ gives [20]

$$\begin{aligned} &Laguerre\ network: \quad \mathcal{G}_i = \mathcal{L}_i, \quad if\ \ a_k = [a]. \\ &Kautz\ network: \quad \mathcal{G}_i = \mathcal{K}_i, \quad if\ \ a_k = [a, b]. \end{aligned} \tag{9}$$

The generalised sequence can be computed using the following state-space model (this example is 3rd order):

$$\begin{aligned} \mathcal{G}_{k+1} = \underbrace{\begin{bmatrix} b & 0 & 0 & \dots \\ b & c & 0 & \dots \\ -ab & (1 - ac) & a & \dots \\ ab^2 & -b(1 - ac) & (1 - ac) & \dots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}}_{A_G} \mathcal{G}_k, \\ \mathcal{G}(0) = \gamma[1, 1, 1, -a, ab, -abc, a^2bc, \dots]^T, \\ \gamma = \sqrt{(1 - a^2)(1 - b^2)(1 - c^2)}. \end{aligned} \tag{10}$$

The prediction using d.o.f. and generalised functions [20] are given by

$$C = \begin{pmatrix} c_k \\ c_{k+1} \\ \vdots \end{pmatrix} = \begin{pmatrix} \mathcal{G}(0)^T \\ \mathcal{G}(1)^T \\ \vdots \end{pmatrix} \rho = H_G \rho, \tag{11}$$

where ρ is the n_G dimension decision variable when one uses the first n_G column of H_G. The only difference between Laguerre OMPC and Kautz or generalised OMPC is that of H_G matrix. For further details readers are referred to [18] and [20].

Algorithm 2.1: GOMPC

$$\rho^* = arg \min_{\rho} \; J_G = \rho^T [\sum_{i=0}^{\infty} A_G^i \mathcal{G}(0) S \mathcal{G}(0)^T (A_G^i)^T] \rho$$
$$s.t. \; Mx_k + NH_G \, \rho \; \leq \; b. \quad (12)$$

Define $c_k^* = H_G \rho_k^*$ and implement $u_k = -Kx_k + e_1^T c_k^*$. Where $e_1^T = [I, 0, \ldots, 0]$.

Remark 2.1: It is straightforward to show, with conventional arguments, that all algorithms (i.e. LOMPC, KOMPC, GOMPC) using terminal constraints within MPC problem formulation provides recursive feasibility and Lyapunov stability.

C. Robust MPC (RMPC): Dual mode MPC for LPV case

The robust MPC algorithm, here denoted RMPC, developed in [8] is very similar to the nominal optimal MPC algorithm in (7), but it is applicable to the LPV system in (1). RMPC is designed to minimise the nominal predicted performance cost subject to robust constraint satisfaction by the whole class of possible predictions.

Define an augmented system model which incorporates the 'd.o.f.' as follows:

$$X_{k+1} = \Psi_i X_k, \; \Psi_i = \begin{bmatrix} \Phi_i & B_i E \\ 0 & I_L \end{bmatrix}, \; X_k = \begin{bmatrix} x_k \\ c_k \end{bmatrix}. \quad (13)$$

with $\Phi_i = A_i - B_i K$, $E = [I, 0, \ldots, 0]$ and I_L as shift matrix. The associated constraints (2) are represented as:

$$\alpha X \leq \delta, \; \alpha = \begin{bmatrix} L_x & 0 \\ -L_u K & L_u E \end{bmatrix}, \; \delta = l. \quad (14)$$

RMPC algorithm differs from OMPC only in the inequalities that need to be satisfied. It has been shown [8] that RMPC has recursive feasibility and guaranteed stability, under the mild condition that the uncertainty is small enough to allow such an invariant set to exist for all the different dynamics Ψ_i. The robust invariant set MAS for LPV system subject to linear constraints is introduced in [7]. The MAS is calculated using an invariant condition which avoids the combinatorial explosion of the number of constraints. The MAS for robust MPC χ_{r_0} in [8] is defined in compact form as:

$$\chi_{r_0} = \{x_k : M_r x_k \leq br\}. \quad (15)$$

The associated MCAS in [8] is defined in compact form as:

$$\chi_r = \{x_k : \exists C, M_r x_k + N_r C \leq br\}. \quad (16)$$

The key success of RMPC is the definition of inequalities, i.e. M_r, N_r and b_r; the precise details of how to compute these matrices are omitted and are available in [8]. The performance cost can be (one can make improvements to this but such discussions are beyond the remit of this paper) based on a nominal performance cost but it in particular is phrased in terms of perturbations c_k to the nominal control law.

For ease of reference, we summarise RMPC below and the reader is referred to [8] for further details.

Algorithm 2.2: RMPC
At each sampling instant, solve the following optimisation problem:

$$\min_{C} J = C^T S C \;\; s.t. \;\; M_r x_k + N_r C \leq b_r,$$

where only the first block element of C is implemented in the control law of (5).

III. Using Alternative parameterisation within RMPC

A fundamental weakness of conventional MPC algorithms is poor feasibility with a small number of d.o.f. to move the initial states into the MAS. This weakness may be overcome by increasing the number of d.o.f., but this compromises the computational burden. However, the alternative parameterisations proposed in [17], [21], [18], [20] for the nominal case should significant feasibility improvements, for the same number of d.o.f.. Therefore, this section seeks to extend the use of alternative parameterisation to the robust case and thus explore whether similar feasibility benefits are possible or likely. This section will show how such parameterisation can be used to form robust invariant sets and thus deployed in appropriate robust MPC algorithm. Examples in the next section are used to demonstrate the impact on feasibility.

A. Alternative parameterisation within RMPC

The alternative parameterisation (i.e. Laguerre, Kautz and generalised functions) may be used to improve the feasible region of RMPC. Robust generalised MPC (RGMPC) is a robust MPC algorithm where the d.o.f. or input perturbations c_k are parameterised using generalised functions. As in the conventional case, the prediction cost can be represented in terms of the perturbations ρ about the nominal control law, hence:

$$J_G = \rho^T \left[\sum_{i=0}^{\infty} \mathcal{G}(i) S \mathcal{G}(i)^T\right] \rho, \quad (17)$$

with $c_{k+i} = \mathcal{G}(i)^T \rho$ and $\mathcal{G}(i) = A_G \mathcal{G}(i-1)$.

From (14), we drive the autonomous formulation using generalised parameterisation as:

$$X_{k+1} = \Psi_i X_k, \; \Psi_i = \begin{bmatrix} \phi_i & B_i \mathcal{G}_0^T \\ 0 & A_G^T \end{bmatrix}, \; X_k = \begin{bmatrix} x_k \\ \rho_k \end{bmatrix}. \quad (18)$$

These dynamics should full fill the constraints (2),

$$\alpha X \leq \delta; \; \alpha = \begin{bmatrix} L_x & 0 \\ -L_u K & L_u \mathcal{G}_0^T \end{bmatrix}, \; \delta = l. \quad (19)$$

Robust constraint handling is represented by an MCAS or χ_r which is calculated offline with the the methodology of [7], [8], but deploying alternative functions, that is equations (18,19) within the update model; this is illustrated in the following algorithm.

Algorithm 3.1: RGMPC Given a LPV system (18) subject to linear constraints (19).

1) Set the initial values for A_S and b_S to

$$A_S := \alpha; \quad b_S := l. \tag{20}$$

2) Initialise the index $i := 1$.
3) Repeat until i is not strictly larger than number of rows in A_S.
   a) Select row i from (20), check whether adding any of the constraints $\alpha_i \Psi_i X \leq l_i,\ j = 1, \ldots, m$ to A_S, b_S would decrease the size of χ_r, by solving the following linear programming (LP) for $j = 1, \ldots, m$

$$\begin{aligned} c_j = \max_{X} \quad & \alpha_i \Psi_i X - l_i \\ s.t. \quad & A_S X \leq b_S. \end{aligned} \tag{21}$$

   If $c_j > 0$, then add the constraint to A_S, b_S as follows:

$$A_S := \begin{bmatrix} A_S \\ \alpha_i \Psi_i \end{bmatrix}; \quad b_S := \begin{bmatrix} b_S \\ l_i \end{bmatrix}. \tag{22}$$

   b) Increment i.
4) End.

After calculating the inequalities for invariant set the following RGMPC algorithm can be defined.

Algorithm 3.2: RGMPC
At each sampling instant, solve the following optimisation problem:

$$\min_{\rho} J_G \ \ s.t. \quad A_S X \leq b_S.$$

Define $c_k^* = H_G \rho_k^*$ and implement $u_k = -K x_k + e_1^T c_k^*$.

Remark 3.1: Algorithm 3.1 will terminate in finite steps and only adds constraints and never removes constraints. It is clear that the resulting $\chi_r = \{X : A_S X \leq b_S\}$ will satisfies a robust positive invariant set. Algorithm convergence and invariance of the resulting set is proved similarly as in [7]. After terminating, it is recommended to remove any redundant constraints.

Remark 3.2: MAS or χ_{r_0} is calculated using above algorithm with $[x, c] = [x, 0]$ or using algorithm defined in [7].

B. Order selection of the generalised parameterisation dynamic

In generalised parameterisations with higher order prediction dynamics have more flexibility to improve feasible region with a limited number of d.o.f.. So there is a clear choice of selecting the order of the parameterisation dynamics.

The prediction dynamics for the $3rd$ order parameterisation from (10) (which can easily be extended to nth order) is quite generic with distinct eigenvalues. From the autonomous formulation using generalised parameterisation in (18), to fulfill the algebraic relations the dimension of the A_G in (10) must be the same as n_c. Moreover the key observation from the augmented model in (18) is that $dim(A_G) = n_c$ is an upper bound on maximum parameterisation dynamics order.

IV. Numerical Examples

The section will illustrate the efficacy of the alternative parameterisation within robust MPC algorithm by several numerical examples given next. The aim is to compare two aspects: (i) the size of the feasible regions; (ii) the number of inequalities required to describe the robust MCAS. For the purposes of visualisation, figures are restricted to second order systems for which it is possible to plot regions of attraction. The comparisons are based on 4 systems with $x \in \mathbb{R}^2, x \in \mathbb{R}^3$ and $x \in \mathbb{R}^4$. The alternative parameterisations are based on Laguerre, Kautz and generalised functions with $3rd$ order dynamics.

A. Example 1

$$\begin{aligned} &A_1 = \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ &A_2 = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 1.5 \end{bmatrix}, \\ &C = \begin{bmatrix} 1 & 0 \end{bmatrix},\ Q = C^T C, R = 0.5,\ n_c = 2,\ a = 0.5, \\ &(a, b) = (0.5, 0.58)\ and\ (a, b, c) = (0.65, 0.67, 0.64). \end{aligned} \tag{23}$$

B. Example 2

$$\begin{aligned} &A_1 = \begin{bmatrix} 0.6 & -0.4 \\ 1 & 1.3 \end{bmatrix},\ B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ &A_2 = \begin{bmatrix} 0.7 & -0.5 \\ 1 & 1.4 \end{bmatrix},\ B_2 = \begin{bmatrix} 1.5 \\ 0 \end{bmatrix}, \\ &C = \begin{bmatrix} 1 & 0 \end{bmatrix},\ Q = C^T C, R = 0.5,\ n_c = 2,\ a = 0.2, \\ &(a, b) = (0.3, 0.35)\ and\ (a, b, c) = (0.41, 0.42, 0.43). \end{aligned} \tag{24}$$

C. Example 3

$$\begin{aligned} &A_1 = \begin{bmatrix} 1 & 0.1 & 0.1 \\ 0 & 1 & 0.1 \\ 0 & 0 & 1 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \\ &A_2 = \begin{bmatrix} 1 & 0.2 & 0.2 \\ 0 & 1 & 0.2 \\ 0 & 0 & 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 0 \\ 1.5 \end{bmatrix}, \\ &C = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix},\ Q = C^T C, R = 0.5,\ n_c = 2,\ a = 0.4, \\ &(a, b) = (0.55, 0.58)\ and\ (a, b, c) = (0.4, 0.6, 0.58). \end{aligned} \tag{25}$$

D. Example 4

$$A_1 = \begin{bmatrix} 1 & 0.1 & 0.1 & 0.1 \\ 0 & 1 & 0.1 & 0.1 \\ 0 & 0 & 1 & 0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1 & 0.2 & 0.2 & 0.2 \\ 0 & 1 & 0.2 & 0.2 \\ 0 & 0 & 1 & 0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1.5 \end{bmatrix}, \tag{26}$$

$$C = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}, \; Q = C^T C, R = 0.5, \; n_c = 2, \; a = 0.4,$$
$$(a, b) = (0.5, 0.56) \; and \; (a, b, c) = (0.55, 0.57, 0.6).$$

with constraints

$$-1 \leq u_k \leq 1; \quad -10 \leq x_{i_k} \leq 10; \quad i = 1, \ldots, 4. \tag{27}$$

E. Feasible region comparisons

The region of attraction for examples 1 and 2 are plotted in figure 1 and 2 respectively. It is clear from both figures that the use of alternative (Laguerre, Kautz and generalised (3rd order)) parameterisation techniques within robust MPC algorithms improve the volume of the feasible region or the volume of the maximum control admissible set (MCAS). Table I shows the volume comparisons for examples 1-4 using RMPC, RLMPC, RKMPC and RGMPC (3rd order) algorithms. Alternative parameterisation improves feasible region and thus, based solely on volume considerations and as expected, alternative parameterisation based algorithms to be preferred in robust scenario.

F. Number of constraints

For completeness, it is important to compare the number of inequalities required to describe the robust MCAS as the complexity of these set descriptions has an impact on the online computational burden, the more inequalities the higher the computational burden in solving the associated QP optimisation (this paper does not discuss issues linked to the exploitation of structure and efficient QP optimisers). The number of inequalities to define χ_r is compared with same number of d.o.f. in Table II. The number of inequalities with parameterised based algorithms are slightly more in comparison with RMPC algorithms. It is clear that higher order parameterisation improves feasibility needs more d.o.f. which will compromise the complexity benefits.

G. Summary

The result shown in Figure 1 and 2, it is clear that RGMPC ($3rd$ order) has a large feasible region as compare with other algorithms, but having $n_c = 3$. This is due to the structure of $3rd$ order prediction dynamics which compromises the computational burden. Moreover for Table I and II, an interesting observation is that RGMPC ($3rd$ order) improves feasible region with higher number of inequalities.

RKMPC improves feasible region with slightly higher inequalities as compare with both RLMPC and RMPC for the same number of d.o.f. (i.e. $n_c = 2$). It is also clear from both Table I and II that GKMPC is a better choice with $n_c = 2$.

These results are based on arbitrary choices for the parameters in RGMPC, RKMPC and RLMPC. Further improvements both in feasible region and number of inequalities are possible by tailoring these parameters to the context.

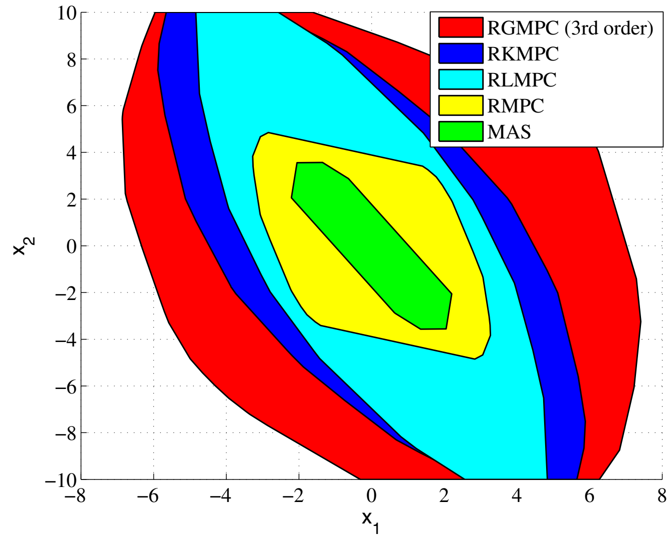

Fig. 1. Feasible regions for model (23) with $n_c = 3$

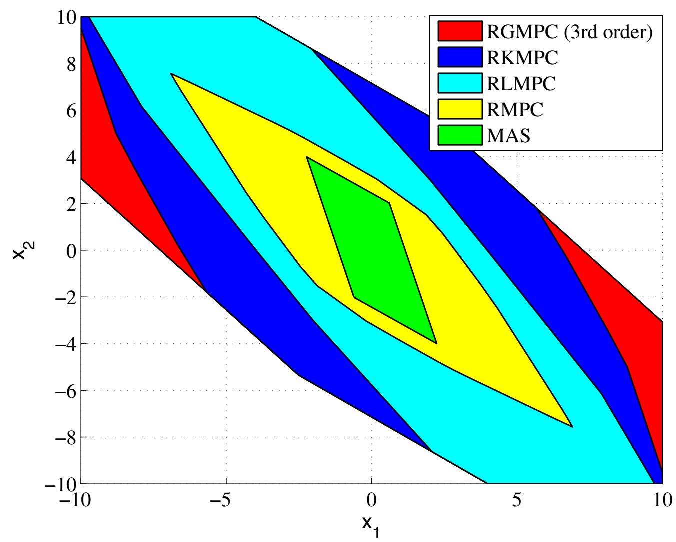

Fig. 2. Feasible regions for model (24) with $n_c = 3$

V. CONCLUSIONS AND FUTURE WORKS

The main contribution of this paper was to extend robust MPC algorithm to make use of alternative parameterisations of the d.o.f. and to consider the impact of doing so. Different alternative parameterisation functions including Laguerre, Kautz and higher order functions can be embedded within the robust MPC approach; the main requirement for this is to

TABLE I
COMPARISON OF FEASIBLE VOLUME

$n_c = 2$				
Algorithm	Example 1	Example 2	Example 3	Example 4
RMPC	29	38.1	1.5 $\times 10^3$	1.7 $\times 10^4$
RLMPC	66.9	47	4.1 $\times 10^3$	5.7 $\times 10^4$
RKMPC	71.3	52.6	4.4$\times 10^3$	6.1 $\times 10^4$
$n_c = 3$				
Algorithm	Example 1	Example 2	Example 3	Example 4
RMPC	43	56.5	2.5 $\times 10^3$	2.8 $\times 10^4$
RLMPC	108.4	150.6	5.4 $\times 10^3$	6.9 $\times 10^4$
RKMPC	141.7	215.6	6.1 $\times 10^3$	9.4 $\times 10^4$
RGMPC (3rd order)	225.4	237	6.2 $\times 10^3$	9.5 $\times 10^4$

TABLE II
COMPARISON OF INEQUALITIES

$n_c = 2$				
Algorithm	Example 1	Example 2	Example 3	Example 4
RMPC	28	20	36	55
RLMPC	50	36	57	113
RKMPC	46	54	65	127
$n_c = 3$				
Algorithm	Example 1	Example 2	Example 3	Example 4
RMPC	48	40	57	93
RLMPC	74	38	74	109
RKMPC	105	48	88	153
RGMPC (3rd order)	141	60	67	163

show how a robust invariant set can be computed with different parameterisations of the d.o.f.. Examples based on alternative parameterisation demonstrate that, for a fixed number of d.o.f., in many cases such parameterisations may improve the feasible region without a large change to the number of inequalities required to describe the robust invariant set.

Consequently this approach is worth further investigation. Moreover, there is a need to further investigate in parallel issues such as: which alternative parameterisation is best for particular problem and what choice of parameter(s) within parameterisation will lead to an efficient online optimisation QP structure? From Table II, it is observed that parameter selection effects the number of inequalities. The parameter selection will be based on both number of inequalities and feasible region. Another interesting avenue is to consider the computational efficiency for multi-parameteric quadratic programming (mp-QP) solution to RMPC. Finally, this paper consider parameter uncertainty only and thus extensions to consider disturbances are also required.

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