

# Identification of Structural Parameters Using Damped Transfer Matrix and State Vector

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**Abstract**—A new method for identification of structural parameters is proposed using Damped Transfer Matrices (DTM) and state vectors. A new transfer matrix is derived for continuous mass systems including the damping parameters. The state vector at a location is the sum of the internal and external contributions of displacements, forces and moments at that point, when it is multiplied with the transfer matrix, state vector at the adjacent location is obtained. The structural identification algorithm proposed here involves prediction of displacement responses at selected locations of the structure using Damped Transfer Matrix and compares them with the measured responses at the respective locations. The mean square deviations between the measured and predicted responses at all locations are minimized using a non-classical optimization algorithm, and the optimization variables are the unknown stiffness and damping parameters in the DTM. A non-classical heuristic Particle Swarm Optimization algorithm (PSO) is used, since it is especially suited for global search. This DTM algorithm with successive identification strategy is applied on one element or substructure of a structure at a time and identifies all the parameters of adjacent elements successively. The algorithm is applied on numerically simulated experiments of structures such as a cantilever and one sub-structure of a nine member frame structure. Also this algorithm is verified experimentally on a sub-structure of a fixed beam. The main advantage of this algorithm is that it can be used for the local identification in a zone in a structure without modelling the entire global structure.

**Keywords**— Damped Transfer Matrix; State Vectors; Successive Identification; Particle Swarm Optimization

## I. INTRODUCTION

Structural identification (SI) problems typically deal with the estimation of mass, stiffness and damping properties of a structure from input/output measurements. It plays an important role in model updating and structural health monitoring. From a computational point of view, structural identification presents a challenging problem particularly when the system involves a large number of unknown parameters. SI algorithms are generally classified into frequency domain and time domain algorithms. Frequency domain SI algorithms have been developed more widely. Maia and Silva [1] presented some modal analysis techniques for identification. Ge and Lui [2] identified damage on structures like cantilever, ten story steel frame and plates by comparing natural frequencies of the undamaged and damaged structures.

Time domain algorithms are usually categorized as Classical or Non-classical methods. Ghanem and Shinozuka [3] reported few classical SI time domain algorithms such as Recursive Least Square method (RLS), Extended Kalman Filter method (EKF), maximum likelihood method, recursive instrumental variable method. Juang and Pappa [4] presented a deterministic SI algorithm based on state space model of second order system using Observer Kalman Filter Identification and Eigen Realisation Algorithm (OKID/ERA) by which all the structural properties such as mass, damping coefficient, stiffness can be identified. Some of the shortcomings of the classical methods are requirement of the calculation of derivatives; difficulty of converging to the global optima, requirement of initial values, and inability to deal with large number of variables. To overcome these drawbacks, Non-classical SI algorithms are used. A non-classical method is usually based on heuristic concepts such as Evolutionary principle (GA) or behavioural principle (PSO). Koh *et al* [5], identified a maximum of 52 structural parameters including damping using GA with a hybrid local search method. GA directs the search toward the global optima and the local search improves the convergence. Kennedy and Eberhart [6] developed a new stochastic optimization algorithm PSO which was proved that much superior to GA and easy to configure [7]. Perez and Behdinan [8] also used PSO for a structural identification problem of 72 bar truss with good accuracy.

The computational effort of identifying a  $n$  DOF system is of the order of  $n^2$ . Even for a modern computer, the computational speed for solving large matrices is challenging. As an alternative for this problem, transfer matrices and state vectors are used for SI algorithm. Steidel [9] derived the transfer matrix for a spring mass system and a beam element. The transfer matrix for the beam element is derived by assuming that the mass is concentrated only at end nodes and the beam element is mass less throughout its length. Meirovitch [10] determined the natural frequencies and mode shapes of a non-uniform pinned-pinned beam with ten elements using transfer matrices. Nandakumar and Shankar

[11] used the transfer matrices derived by Steidel [9] and state vectors first in SI problem and identified successfully the parameters of cantilever and ten DOF lumped mass system. Later Tuma and Cheng [12] derived an improved transfer matrix for beam element with an assumption of the mass of the beam element is concentrated at its mass center. It is found that there is a good improvement in natural frequencies.

Nandakumar and Shankar [13] derived a transfer matrix from the consistent mass matrix of the beam element and determined higher order natural frequencies with much better accuracy than the existing lumped mass based transfer matrices. Using the same transfer matrix, stiffness parameters of structures were identified with better accuracy. However all these transfer matrices discussed have a limitation in that they can be used only for lightly damped structures/materials by ignoring its damping effect. To identify properties of highly damped structures which have significant damping, a new damped transfer matrix (DTM) including damping parameters is derived. In this paper structural parameters including damping parameters were identified using DTM by Successive Structural Identification strategy.

## II. TRANSFER MATRICES AND STATE VECTORS

A state vector at a point in the structure is the summation of the internal response vector and external force vector. The former contains the output responses such as displacement, angular displacement and the internal forces and moments and the later contains the externally applied forces and moments. The Transfer Matrix (TM) is a square matrix which contains the structural parameters. When a state vector is multiplied with the TM, internal response vector at the adjacent location is obtained.

### A. Transfer Matrix for Beam element

In this section the transfer matrix for damped vibration of beams is derived. The equilibrium equation of two noded beam element is

$$[M]\ddot{x}(t) + [C]\dot{x}(t) + [K]x(t) = F(t) \quad (1)$$

where  $\ddot{x}(t)$ ,  $\dot{x}(t)$  and  $x(t)$  are nodal acceleration, velocity and displacement responses vector respectively,  $F(t)$  is nodal force vector. The state vector for a node on the beam element is  $\{X\} = \{y(t), \theta(t), M(t), V(t)\}^T + \{0, 0, F(t), \mu(t)\}^T$ , where  $y(t)$  is translational displacement,  $\theta(t)$  is angular displacement,  $M(t)$  is bending moment,  $V(t)$  is shear force,  $F(t)$  is applied force and  $\mu(t)$  is applied moment at that node. The damping in the beam element is modelled using Rayleigh's damping model.

$$[C] = \alpha[M] + \beta[K] \quad (2)$$

Also  $\ddot{x}(t) = -\omega^2 x(t)$ ,  $\dot{x}(t) = i\omega x(t)$ . therefore, the Eq.(1) becomes,

$$F(t) = [Z]x(t) \quad (3)$$

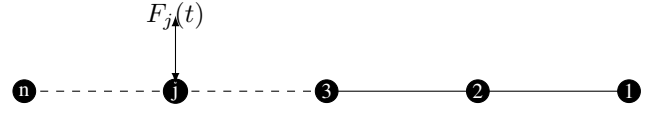


Fig. 1. Sub-structure with arbitrary point excitation

where  $[Z] = (i\omega\alpha - \omega^2)[M] + (1 + i\omega\beta)[K]$ . Since the beam element is in equilibrium, the Eq.(3) is written for one element

$$\begin{Bmatrix} -M_1(t) \\ -V_1(t) \\ - \\ M_2(t) \\ V_2(t) \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & | & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & | & Z_{23} & Z_{24} \\ - & - & + & - & - \\ Z_{31} & Z_{32} & | & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & | & Z_{43} & Z_{44} \end{bmatrix} \begin{Bmatrix} y_1(t) \\ \theta_1(t) \\ - \\ y_2(t) \\ \theta_2(t) \end{Bmatrix} \quad (4)$$

Assume the output responses and internal forces/moments at node 1 are known and external forces are zero, the force and DOF vectors in the Eq.(4) is rearranged to form state vectors  $\{X\}$  in such away that  $\{X_2\} = [T_d]\{X_1\}$ , the transfer matrix is

$$[T_d] = \begin{bmatrix} -[Q] & [0] \\ -[S] & [I] \end{bmatrix}^{-1} \begin{bmatrix} [P] & [I] \\ [R] & [0] \end{bmatrix} \quad (5)$$

where  $[P] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$ ,  $[Q] = \begin{bmatrix} Z_{13} & Z_{14} \\ Z_{23} & Z_{24} \end{bmatrix}$ ,  
 $[R] = \begin{bmatrix} Z_{31} & Z_{32} \\ Z_{41} & Z_{42} \end{bmatrix}$ ,  $[S] = \begin{bmatrix} Z_{33} & Z_{34} \\ Z_{43} & Z_{44} \end{bmatrix}$

### B. Transfer matrix and state vector for the global structure

Calculation of state vector at any node of a global structure, from one known initial state vector at a given node using transfer matrices is illustrated here. For example, a portion of a structure is considered with  $n$  nodes subjected to an arbitrary point excitation as shown in Fig.1. It is assumed that all elements in the state vector at the node 1 are known. The state vectors at other nodes can be calculated by successive multiplication of elemental transfer matrices. The equation to calculate internal response vector for  $n^{th}$  node from internal response vector at node 1 and external force vector is obtained.

$$\{X_{ni}\} = \left( \prod_{k=1}^{n-1} [T_{(n-k), (n+1-k)}] \right) \{X_{1i}\} + \sum_{j=1}^{n-1} \left( \prod_{k=1}^{n-j} [T_{(n-k), (n+1-k)}] \right) \{X_{je}\} \quad (6)$$

For free damped vibration i.e without any external forces, as a special case the above equation can be deduced as

$$\{X_n\} = \left( \prod_{k=1}^{n-1} [T_{(n-k), (n+1-k)}] \right) \{X_1\} \quad (7)$$

In the above equation,  $[T_{1,n}^G] = \prod_{k=1}^{n-1} [T_{(n-k), (n+1-k)}]$  is known as global transfer matrix.

### III. PARAMETER IDENTIFICATION BY DAMPED TRANSFER MATRIX METHOD (DTM)

The proposed DTM algorithm is used for identifying the unknown flexural rigidities ( $EI$ ) of the structure assuming the masses are known. The beam is excited with a known force at a point. The elements of initial state vector is measured at one location, from which it is possible to predict the displacement at any location in the structure using successive multiplication of the DTMs, as discussed in section II-B. The mean square deviation between the predicted and measured displacements at measured locations in the structure can be minimized by Particle Swarm Optimization algorithm (PSO) with the unknown elemental  $EI$  values,  $\alpha$  and  $\beta$  in the DTM as the optimization variables. The Successive identification strategy [13] is adopted to identify unknown parameters since it is superior in speed and accuracy for the identification of a few adjacent elements.

Since the elements of DTM are complex numbers, the elements of all predicted state vectors are complex responses except the initial state vector. Its elements are all real signals since they are measured directly. The imaginary part of the responses in the predicted state vectors is proportional to the contribution of damping property of the structure. For the lightly damped structures/materials the imaginary part of the responses is very small and hence may be neglected and only real part of the response is considered for identification. But for significantly damped structures the imaginary part of the responses is considerable, hence the complex responses are converted from Cartesian form into Polar form. Let the polar form of the responses contain magnitude ( $u_e$ ) and phase shift ( $\phi$ ). The number of time steps are to be shifted is calculated by the relation  $t_s = \frac{\phi f_s}{\omega}$  where, the  $f_s$  is sampling frequency in Hz and  $\omega$  is circular frequency of excitation in r/s. The error function between measured and predicted responses is given by

$$\varepsilon = \frac{\sum_{j=1}^L |u_m(j) - u_e(j + t_s)|^2}{L} \quad (8)$$

where  $u_m(j)$  and  $u_e(j)$  are measured and estimated displacement responses respectively at  $j^{th}$  time step.  $L$  is the number of time steps. Then the cycle is repeated for all the pairs of adjacent measured responses and identify all unknown parameters successively. Since, the number of unknown parameters to be identified is one or few for one identification cycle, the convergence is very fast, the overall computational time is very small. This strategy is promising in the identification of local parameters in a structural member.

### IV. NUMERICAL EXAMPLES AND RESULTS

The SI algorithm using damped transfer matrix (DTM) is applied on two numerically simulated experiments. i.e a uniform cantilever and a sub-structure of a nine member frame structure. The structure is excited by a harmonic force at a node and the acceleration responses are measured at

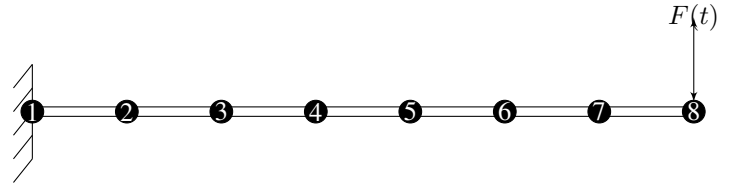


Fig. 2. Finite Element model of cantilever

selected nodes and converted to displacement responses by numerical integration. In all examples, measured responses are numerically simulated using Newmark's constant acceleration method. The unknown stiffnesses and damping parameters are searched by PSO algorithm within the search range of 50% to 150% of the exact values. In order to simulate the effect of noise in experiments, Gaussian random noise of 3% is added to the measured signals.

#### A. Example-I: Cantilever

A steel cantilever of dimension  $24.6 \times 5.7 \times 350$  mm is fixed at its one of the end as shown in Fig.2. The Young's modulus of cantilever material is 200 GPa and its density is  $7691 \text{ kg/m}^3$ . It is divided into seven finite elements of length 50mm each. The flexural rigidity ( $EI$ ) of each element is  $75.93 \text{ N.m}^2$ . The damping constants  $\alpha$  and  $\beta$  are 20.77 and  $5.71 \times 10^{-5}$  respectively. The free end is excited by a harmonic input force of  $1.5 \sin(2\pi 10t)$  N. The first natural frequency of the beam is 38.33 Hz. The effect of the damping was accounted by Rayleigh's damping with modal damping ratio of 5% at its first two modes. Since the bending moment and shear force responses are zero at the free end, the initial state vector is formed at the free end. The translational responses are measured at all nodes and angular response is measured at the free end only. From the initial state vector the unknown parameters are identified using DTM successively with PSO parameters of swarm size 50 and 50 iterations in each identification cycle with variable inertia weight varies from 0.9 to 0.4. The identification algorithm is repeated with

TABLE I  
PERCENTAGE OF ERROR IN IDENTIFIED RESULTS OF CANTILEVER

Element	% of Error			
	Complete Measurement		Incomplete Measurement	
	Noise free	3% Noise	Noise free	3% Noise
1	-0.19	0.98	3.57	4.95
2	0.17	1.88	-1.19	-1.54
3	0.01	-0.45	-0.15	0.79
4	-0.01	-1.96	3.68	-3.21
5	-0.02	-0.56	-1.06	4.99
6	-0.04	0.64	0.52	1.19
7	-0.05	2.51	-0.05	0.27
MAE	0.07	1.28	1.46	2.42
$\alpha$	-0.71	5.83	-1.56	7.40
$\beta$	-18.21	-21.43	-21.35	24.41

translational responses measured at nodes 3, 5, 7 and 8. The cantilever is divided into four substructures from nodes (1-3), (3-5), (5-7) and (7-8) and parameters are identified in each substructure successively. This problem was repeated with 3%

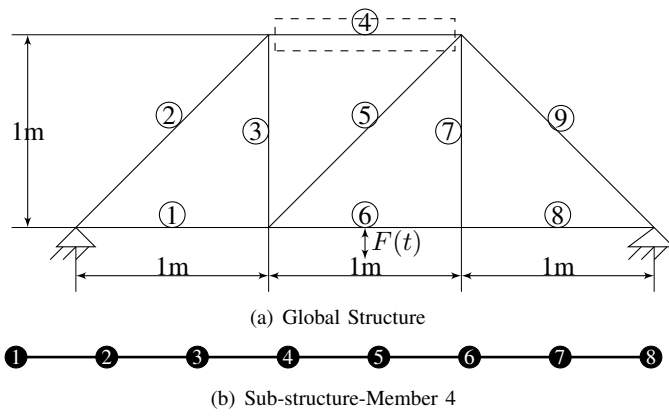


Fig. 3. Frame Structure

noise at all measured data. The total computational time is 48.5s with complete measurements and the computational time is 27.7s with incomplete measurements. The percentage of mean absolute error in identified values of  $EI$  with complete and incomplete measurements is tabulated in Table.I. In this example also the mean absolute error in identified  $EI$  values with complete measurements (0.07%) is less than that with incomplete measurement (1.46%) of responses. Similarly the percentage of error in identified damping constants ( $\alpha$  and  $\beta$ ) is also more in the case of incomplete measurement (-0.71% and -18.21%) than that of complete measurement case (-1.56% and -21.35%). Hence the DTM derived for beam element is satisfactorily identifying stiffness and damping properties of the beam.

1) *Comparison of results with other time domain methods:* Sandesh and Shankar [14] identified stiffness parameters of a substructure in a similar cantilever with six elements using a time domain Least Square technique with damping included with mean absolute error of 4.51% at 3% noise level which requires measurement at all DOF including rotation. Whereas the DTM algorithm identified results with mean absolute error of 2.42% at 3% noise level with only four translational response measurement and only one angular response measurement. This shows that the DTM algorithm performs well when compared to other SI algorithms.

#### B. Example-2: Sub-structural Identification of frame structure

A frame steel structure made of nine members is fixed at two supports as shown in Fig.3(a) and it has taken from [15]. Each member has a cross section of  $12 \times 6$ mm and a flexural rigidity ( $EI$ ) of  $43.2 \text{ N.m}^2$ . The first natural frequency is 11.9 Hz. The damping effect is taken into account by adopting Rayleigh damping with the modal damping ratio of 5%. The damping constants  $\alpha$  and  $\beta$  are calculated as 3.919 and  $6.36 \times 10^{-4}$ . It is proposed to identify the properties of the top horizontal member 4, which has a length of 1m. The properties of substructure to be identified is indicated by box in Fig.3(a), which has a length of 0.875 m. It is divided into seven elements as shown in Fig.3(b). The structure is excited by a sinusoidal input force of  $10 \sin(2\pi 10t)$  N at the midpoint of the member

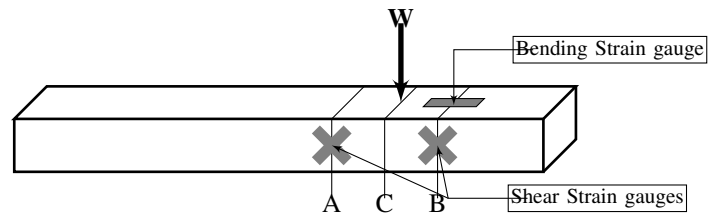


Fig. 4. Strain gauge arrangements

6. Since the boundary conditions are unknown, it is necessary to measure translational and angular responses, shear force and bending moment responses of any arbitrary node to define initial state vector. The first two responses can be measured directly by accelerometers and last two responses have to be measured by strain gauges. The initial state vector is formed at the node 8 which is  $\{X_8\} = \{y_8(t), \theta_8(t), M_8(t), V_8(t)\}^T$ . Since the external excitation force is not applied on the substructure, it is not necessary to measure.

1) *Measurement of shear force and bending moment responses:* For a rectangular section beam, the bending moment response is given by

$$M(t) = \frac{2EI\epsilon_B(t)}{h} \quad (9)$$

The shear force in the section is given by

$$V(t) = \frac{4EI\epsilon_S(t)}{h^2(1 + \nu)} \quad (10)$$

where  $EI$  is flexural rigidity of the section and  $y = h/2$ ,  $h$  is thickness of the section,  $\nu$  is Poisson's ratio. The bending strain response can be measured using strain gauge, From the above formulae, to calculate the bending moment and shear force responses at a node, the knowledge of the flexural rigidity ( $EI$ ) at that node is required. The estimation of the  $EI$  value at the starting node using a simple shear strain test as is presented here. The strain gauges are fixed to measure bending and shear strain as shown in Fig.4. At a point C in between the nodes A and B, a static load of  $W=10 \text{ kN}$  is applied and the corresponding strain at the nodes A and B are measured. Let the static strain measured at the nodes A and B be  $\epsilon_{SA}$  and  $\epsilon_{SB}$  respectively. The change in shear force at the nodes A and B is equal to the applied load  $W$  at C since the self weight of the portion AB is very small. The  $EI$  at the initial node B is given by

$$EI = \frac{Wh^2(1 + \nu)}{4(\epsilon_{SA} - \epsilon_{SB})} \quad (11)$$

for the measured values  $\epsilon_{SA}=9.8571 \times 10^{-5}$ ,  $\epsilon_{SB}=-0.0013$ ,  $\nu=0.37$  and  $h=6 \text{ mm}$ , the  $EI$  at the node 8 is obtained as  $EI_8=43.2 \text{ N.m}^2$ . The bending strain gauges and shear strain gauges fixed at only B are further required for the dynamic strain measurement. The bending moment and shear force responses at the initial node 8 is calculated from measured strain responses using Eq.(9) and Eq.(10).

2) *Parameter Identification:* The initial state vector is formed at the node 8. In case of complete measurement, translational responses are measured at all eight nodes and angular response is measured at initial node only. The parameters were identified by PSO with parameters of swarm size 50 and 50 iterations in each cycle. The total time of identification for seven EI values with complete measurement was 43.84s. The same example is identified with incomplete

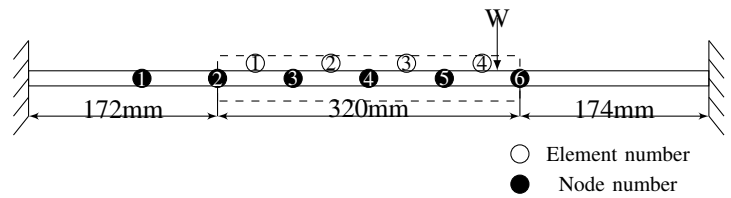


Fig. 5. Substructure of fixed beam

TABLE II  
PERCENTAGE OF ERROR IN IDENTIFIED RESULTS OF SUBSTRUCTURE OF FRAME

Element	% of Error			
	Complete Measurement		Incomplete Measurement	
	Noise free	3% Noise	Noise free	3% Noise
1	1.34	-1.40	4.99	9.99
2	0.29	2.99	-1.84	-7.71
3	0.96	0.05	5.02	4.99
4	-0.16	-0.33	4.11	-4.38
5	-0.41	1.53	-4.54	-3.26
6	-0.14	-0.74	-0.58	-1.84
7	-0.01	-0.04	0.23	0.74
MAE	0.47	1.01	3.05	4.07
$\alpha$	-1.84	-6.97	-3.18	-9.45
$\beta$	-5.56	-7.14	9.98	-12.84

measurements also. Translational displacement measurements at nodes 1, 3, 5 and 8 are used here. The structure is divided into three portions between nodes (1-3), (3-5) and (5-8). The SI algorithm starts from the initial state vector at the node 8 and identifies parameters of each portion successively. PSO used a swarm size of 50 and 100 iterations in each cycle. The total computational time for convergence was 40.98s. The percentage of error variation in identification of parameters is shown in Table.II. The mean absolute error in identified results of EI is 1.01% with complete measurements and is 4.07% with only four sensors. The DTM performs satisfactorily at 3% Noise level at all measured responses. Also the damping constants are identified with maximum percentage error of -12.84% at incomplete noisy measurement. The input force response is not needed for computation and no need to measure the same. The main advantage of the transfer matrix method is that it is more suitable for the identification of local parameters of complex structure without analysing the entire structure.

3) *Comparison of results with other Time domain methods:* It may be noted that Prashanth and Shankar [15] had identified this problem with a 2 stage neural network trained with time domain acceleration signals at two nodes. The error of identification there is expressed as a non-dimensional damage index based on the change in modulus of elasticity of an element. The mean error incurred in that method. was about 0.99% for non-noisy signal and 2.1% for signal with 5% noise but the computational effort of training the network and the complexity of using a two stage network has to be contrasted with the simplicity of the DTM method.

#### V. EXPERIMENTAL VERIFICATION OF SUB-STRUCTURE OF A FIXED BEAM

A beam made of acrylic material with cross sectional dimension of 25 × 12 mm and length of 660 mm was fixed

at both ends as shown in Fig 5. The modulus of Elasticity ( $E$ ) and density were estimated as 3.7GPa and 1190kg/m<sup>3</sup> respectively by simple experiments. The actual flexural rigidity ( $EI$ ) of the beam is 13.32 N.m<sup>2</sup>. The damping ratio( $\zeta$ ) was calculated from a simple free vibration decay test and estimated as 7%. The natural frequencies for the first two modes are 49.04Hz and 135.45Hz. Assuming Rayleigh's damping model, the exact values of damping constants  $\alpha$  and  $\beta$  were calculated as 31.67 and  $1.21 \times 10^{-4}$  respectively. The beam was divided into seven elements. A substructure of length 320mm, is shown inside the dotted rectangle in Fig.5 was considered for structural identification. The sub-structure has four elements of length 80 mm each. The node 6 was taken as starting node and the  $EI$  value at that node is required to form state vector at node 6. The flexural rigidity ( $EI_6$ ) at the starting node was identified by conducting a static experiment as explained in the section IV-B1. A static load of  $W=5.045\text{kgf}$  (49.49N) was applied at a point C in between nodes 5 and 6 at a distance 20 mm from the node 6. Strain gauges were fixed as shown in Fig 4. Five sets of readings were taken and the mean values of the shear strains at the points A and B are  $\epsilon_{SA} = 9.625\mu$  strain and  $\epsilon_{SB} = -170\mu$  strain respectively. The Poisson's ratio ( $\nu$ ) of the beam material is 0.37. Substituting the values in the Eq.(11), the flexural rigidity at the starting node  $EI_6$  was obtained which is 13.48 N.m<sup>2</sup>.

To measure dynamic response, one DYTRAN miniature accelerometer of 2gm mass with sensitivity of 107 mV/g and acceleration range of 50g was fixed at each node to measure translational acceleration. Two accelerometers were fixed very close to each other at a distance of  $dx=7$  mm at the starting node 6. The experimental set up is shown in Fig.6. The structure is excited by a sinusoidal force of  $3.4\sin(2\pi 80t)$  N at the node 1 by a LDS permanent magnet 20 N modal shaker with a maximum displacement of 5 mm with an operating frequency range of 5 Hz-13 kHz. The measurement of input force was not required for this problem, since it was applied outside of the substructure. The strain and acceleration responses were acquired by 16 channel DEWE 1201 data acquisition card(DAC) at a sampling frequency of 1000 Hz. The angular acceleration at the starting node 6 ( $\alpha_6$ ) was calculated by central difference formula. The translational acceleration at the starting node is the mean value of acceleration measured by two accelerometers. Both translational and angular accelerations were converted into respective displacement responses.

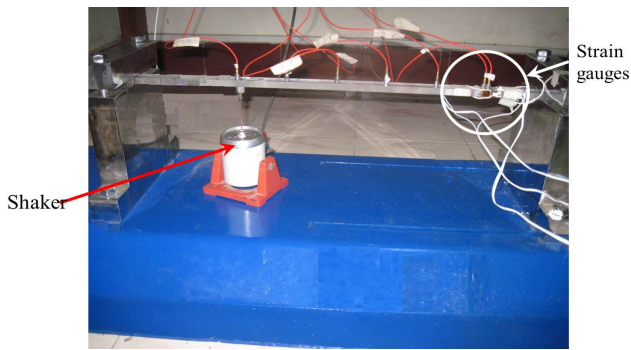


Fig. 6. Experimental set up of Fixed beam

The bending moment and shear force responses at the starting node 6 were calculated from the measured bending and shear strain responses using Eq.(9) and Eq.(10) and state vector at the node 6 was formed. The responses at the other nodes were determined from the starting state vector using DTM with predicted values of structural parameters by PSO. The mean square error between the measured and predicted values of responses were minimized using the Eq.(8). Since there are uncertainty in the experimental data, the parameters were searched between 50% and 200% of their exact value by PSO using damped transfer matrix. The PSO parameters are 50 swarm size, 100 iterations for each identification cycle. The identified parameters are shown in the Table.III. The

TABLE III  
IDENTIFIED PARAMETERS OF SUB-STRUCTURE OF FIXED BEAM

Parameter	Exact N.m <sup>2</sup>	Identified N.m <sup>2</sup>	% of Error
$EI_1$	13.32	11.05	-17.04
$EI_2$	13.32	12.90	-3.16
$EI_3$	13.32	14.43	8.39
$EI_4$	13.32	12.33	-7.42
$\alpha$	31.67	39.49	24.7
$\beta$	$1.21 \times 10^{-4}$	$1.37 \times 10^{-4}$	13.50

total computational time taken for convergence is 80.4s. The farthest element 1 was identified with least accuracy, since the error in each identification cycle is accumulated in the state vectors of the succeeding nodes. The DTM algorithm identified the damping properties of the beam with 24.7% and 13.5% mean absolute error. Hence it is clear that the DTM algorithm works on any sub-structure without considering the global model of the complete structure and identifies its local parameters with good accuracy.

## VI. CONCLUSION

A new SI method based on damped transfer matrix is presented here. The initial state vector has to be provided, and displacement at any point in the structure is predicted using transfer matrix which contains all the structural properties. Using PSO algorithm, the mean square error between measured and predicted responses can be minimized with the unknown structural parameters as the optimization variables. Since the size of the transfer matrix does not increase whatever be the

model size, computational effort is reduced. However successive operations are required to identify the unknown parameters. The successive identification method of this algorithm works fast and identifies the structural parameters with good accuracy. Numerical and experimental studies have been made on both global structures with known boundary conditions and sub-structures of unknown boundary conditions. Since the responses measured in the experimental study contains realistic noise and errors, the results have less accuracy when compared with numerically simulated study with artificially added noise. The accuracy of the method is good compared to existing time domain SI methods. A main advantage of this algorithm is the local identification of parameters of structures without the need to model the entire global structure.

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