UKACC International Conference on Control 2012
Cardiff, UK, 3-5 September 2012

Robust H_∞ Control of Singular Systems over Networks with Data Packet Dropouts

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Abstract—**The problems of stochastic stability and H_∞ control for a class of discrete time singular networked control systems (SNCSs) with data packet dropouts and nonlinear perturbation are investigated in this paper. By modeling the sensor-to-controller and controller-to-actuator with random data packet dropouts as Markov chains, the closed-loop system can be expressed as a jump discrete singular system with four modes. A sufficient condition for the existence of a controller is established in terms of linear matrix inequalities (LMIs), the controller gain can be solvable via the cone complementary linearization method, and the designed controller guarantees the systems to be regular, causal and stochastically stable and satisfies H_∞ performance. In addition, a numerical example is given to illustrate the effectiveness of the proposed approach.**

Keywords-singular systems; networked control systems; data dropout; H_∞ control ; Markov chain

I. INTRODUCTION

Networked control systems (NCSs) have been widely used in various industrial areas, with the rapid development of computer, network and communication technology. The primary advantages of NCSs are simplified system structure, lower cost of system integration, ease of diagnosis, remote distributed control, and increasing system agility. In an NCS, several important issues need to be treated, which include the network induced delay and data packet loss, more recently, the analysis and synthesis problem of networked systems with data dropout and network induced time delays have attracted many research interests [1-7]. Due to bandwidth limited communica -tion channels, packed loss and time delays are the most important and special two issues of NCSs. Generally speaking, there were three main approaches for modeling packed loss and time delays in the NCSs. The first one is to model the data loss and induced delays as a binary switches sequence which obeys the Bernoulli process with certain probability, the NCSs in both continuous time case and discrete time case with packet loss and delays were studied in [5,6]. The second approach is to model the process as discrete time Markov jumping systems, in which transmission times are varying within a time interval or driven by a stochastic process with Markov chain, a class of methods for stabilization analysis for an NCS were proposed [3,4]. The third model is to view data packet loss as a special time delay system, which deals with the stability and controllability [2]. However, it can only be used to treat the systems with sense-to-controller packet loss or controller-to-actuator packet loss cases.

In [15], the stabilization problem for a class of NCSs in the discrete time domain is studied by modeling the sensor-to controller packet loss and controller-to-actuator packet loss as Markov chains. It is worth noting that the mentioned literature results are still very limited for they are all concerned with a nonsingular controlled plant. In real projects, a certain linear NCS model provides just an approximate singular of the real facts. Recently a few results have been reported for the singular NCSs [7, 16].

In actual physical networked transmission field, there coexist data packed loss and nonlinear perturbation, however, the aforementioned articles do not consider the two problems simultaneously. The H_∞ control problem for discrete time singular markov jump systems with data loss and nonlinear perturbation is also an important problem, and it is not simple extension to stability for the systems over networks. Motivated by recent research on singular perturbed systems and networked control systems [8-14, 18], this paper considers the problems of stochastic stability and H_∞ control for a class of discrete time singular networked control systems with data packet dropouts and nonlinear perturbation. Firstly, we consider the case that modeling the sensor-to-controller and controller-to-actuator with random data packet dropouts as Markov chains, and the nonlinear perturbation satisfying Lipschitz condition, the closed-loop system can be expressed as a jump discrete singular system with four modes in Section II; Next, the state feedback controller design and solvable problem are proposed in section III and section IV; Finally, an example is given to illustrate the effectiveness of the proposed approach in section V.

Notations: Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is semi-positive definite(respectively, positive definite); $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all n×m real matrices, respectively; I is the identity matrix with appropriate dimension; the superscript T represents the transpose of a matrix; $\|X\|$ refers to Euclidean norm of the vector X; $\mathbb{Z}$ denotes the set of non-negative integer numbers;

This work was supported by the National Natural Science Foundation of China under Grant No.61174066, also by the Jiangsu Overseas Research & Training Program for University Prominent Young & Middle aged Teachers & Presidents.

$\mathcal{E}\{\bullet\}$ denotes the mathematical expectation, and $*$ denotes the matrix entries implied by symmetry of a matrix.

II. PROBLEM STATEMENTS AND PRELIMINARIES

In this paper, considering a typical NCS model with data packet dropouts exist in the communication links from sensor to controller and controller to actuator as shown in Fig. 1.

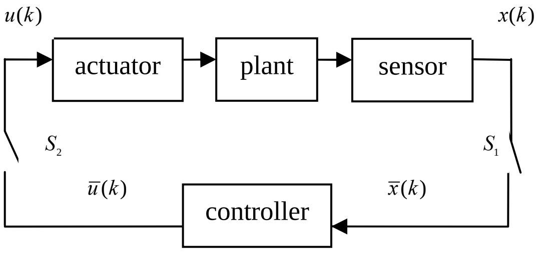

Fig.1 Framework of networked control system

The physical process to be controlled is the following singular discrete-time nonlinear model:

$$\begin{cases} Ex(k+1) = Ax(k) + Bu(k) + B_{\omega}\omega(k) + f(x,u,k) \\ \quad z(k) = Cx(k) + Du(k) + D_{\omega}\omega(k) \end{cases} \tag{1}$$

Where $x(k) \in \mathbb{R}^n$ is the system state, the matrix $E \in \mathbb{R}^{n\times n}$ may be singular, we shall assume that $rank(E) = r \le n$, $u(k) \in \mathbb{R}^p$ is the control input, $\omega(k) \in \mathbb{R}^q$ is the disturbance input which belongs to $L_2[0,\infty)$, and $z(k) \in \mathbb{R}^m$ is the system controlled output; $f(\bullet) \in \mathbb{R}^n$ is nonlinear uncertain perturbation, with $f(0,0,k) = 0$ satisfies Lipschitz condition; $A, B, B_{\omega}, C, D, D_{\omega}$ are known real constant matrices with appropriate dimensions.

Assume the system state can be measured and the data are transmitted in a single packet at each time step, the data loss information is important for controller design, it is desirable that the state feedback controller is

$$\bar{u}(k) = K\bar{x}(k) \tag{2}$$

which K is the controller gain to be determined.

In Fig.1, S_1, S_2 are networked switches, and α, β $[\alpha, \beta \in (0,1)]$ are the states, respectively, when $\alpha = \beta = 0$, there is no data packet loss, then $\bar{x}(k) = x(k)$, $\bar{u}(k) = u(k)$; when $\alpha = \beta = 1$ there exist data packet loss, then

$\bar{x}(k) = \bar{x}(k-1)$, $u(k) = u(k-1)$.

Then $\bar{x}(k)$ and $u(k)$ can be written as:

$$\begin{cases} \bar{x}(k) = (1-\alpha)x(k) + \alpha\bar{x}(k-1) \\ u(k) = (1-\beta)\bar{u}(k) + \beta u(k-1) \end{cases} \tag{3}$$

By introducing new state vectors

$\xi(k) = [x^{\mathrm{T}}(k), \bar{x}^{\mathrm{T}}(k-1), u^{\mathrm{T}}(k-1)]^{\mathrm{T}}$, then the closed-loop system resulting from equations (1), (2) and (3) can be expressed as ($i \in (1,2,3,4)$)

$$\begin{cases} \bar{E}\xi(k+1) = A_i\xi(k) + \bar{B}_{\omega}\omega(k) + \bar{f}(k) \\ \quad z(k) = C_i\xi(k) + D_{\omega}\omega(k) \end{cases} \tag{4}$$

Where

1) $\alpha = \beta = 0$, $A_1 = \begin{pmatrix} A+BK & 0 & 0 \\ I & 0 & 0 \\ K & 0 & 0 \end{pmatrix}, C_1 = [C+DK, 0, 0]$

2) $\alpha = 0, \beta = 1$, $A_2 = \begin{pmatrix} A & 0 & B \\ I & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, C_2 = [C, 0, D]$

3) $\alpha = 1, \beta = 0$, $A_3 = \begin{pmatrix} A & BK & 0 \\ 0 & I & 0 \\ 0 & K & 0 \end{pmatrix}, C_3 = [C, DK, 0]$

4) $\alpha = 1, \beta = 1$, $A_4 = \begin{pmatrix} A & 0 & B \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}, C_4 = [C, 0, D]$

$\bar{B}_{\omega} = [B_{\omega}^T \quad 0 \quad 0]^T$, $\bar{E} = \begin{bmatrix} E & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\bar{f}(k) = [f^T(\bullet) \quad 0 \quad 0]^T$

It is noted from above analysis, the closed-loop system (4) is component by four sub-systems, so it can be constructed as Markov jump systems

$$\begin{cases} \bar{E}\xi(k+1) = A_{\theta(k)}\xi(k) + \bar{B}_{\omega}\omega(k) + \bar{f}(k) \\ \quad z(k) = C_{\theta(k)}\xi(k) + D_{\omega}\omega(k) \end{cases} \tag{5}$$

where $\{\theta(k), k \in \mathbb{Z}\}$ is discrete Markov chains that takes values in $l = \{1,2,3,4\}$. Its transition probability matrix is $\Pi = \{\lambda_{ij}\}$, which is defined $\lambda_{ij} = P(\theta(k+1) = j \mid \theta(k) = i)$ with $\lambda_{ij} \ge 0$ and $\sum_{j=1}^{4}\lambda_{ij} = 1$ for all $i \in l$.

Assume the nonlinear $\bar{f}(k)$ of system (4) satisfies

$$\bar{f}^{\mathrm{T}}(k)\bar{f}(k) \le \xi^{\mathrm{T}}(k)H\xi(k) \tag{6}$$

Hear H is known real constant matrix.

Remark 2.1 When $E = I, f = 0$, system(5) reduces to

$$\begin{cases} \xi(k+1) = A_{\theta(k)}\xi(k) + \bar{B}_{\omega}\omega(k) \\ \quad z(k) = C_{\theta(k)}\xi(k) + D_{\omega}\omega(k) \end{cases} \tag{7}$$

is a special case of this paper, which discussed in [18].

The following definitions will be used in the sequel.

Definition 2.1 [12] The discrete singular system (5) under without disturbance ($\omega(k) \equiv 0$) is said to be stochastically stable if for any $x_0 \in \mathbb{R}^n$, there exist $\delta(x_0, \theta_0) > 0$ and a scalar ρ, such that

$$\lim_{N\to\infty} \mathcal{E}\left\{\sum_{k=0}^{N} \|x\|^2 \,\middle|\, x_0, \theta_0\right\} \le \rho\delta(x_0, \theta_0) \tag{8}$$

Definition 2.2 [10] For all $i \in l$, the discrete singular system (5) is said to be

i . regular if $\det(z\bar{E} - A_i)$ is not identically zero.

ii. causal if $\deg(\det(z\bar{E} - A_i)) = rank(\bar{E})$.

iii. stochastically admissible if it is regular, causal and stochastically stable.

Definition 2.3 [11] System (5) with $u(k)\equiv 0$ is said to be robustly mean square quadratic stability, if there exists a scalar $\gamma>0$ such that $\|z(k)\|_2 \le \gamma\|\omega(k)\|_2$, for any nonzero disturbance $\omega(k)\in L_2[0,\infty)$, where

$$\|z(k)\|_2=\sum_{k=0}^{\infty}\mathcal{E}\left\{z^T(k)z(k)\right\},\|\omega(k)\|_2=\sum_{k=0}^{\infty}\mathcal{E}\left\{\omega^T(k)\omega(k)\right\}$$

The objective of this paper is to establish a sufficient condition such that the closed-loop system (5) is stochastically stable and to design a state-feedback controller in the form of (2), to satisfy H_∞ performance.

III. Stability Analysis

In this section, we analyze the stochastic stability and H_∞ performance of system (5). Before presenting the main results, we introduce the following lemma.

Lemma 3.1 [10] The system (5) with $\omega(k)\equiv 0$ is stochastically admissible if and only if there exists $P_i>0$ such that the following matrix inequalities holds:

$$\begin{cases} \bar{E}^T P_i \bar{E}\ge 0 \\ A_i^T \bar{P}_i A_i-\bar{E}^T P_i\bar{E}<0 \end{cases} \tag{9}$$

where $\bar{P}_i=\sum_{j=1}^{N}\lambda_{ij}P_i$

Now, we propose the results of this section as follows.

Theorem 3.1 Given scalar $\gamma>0$ for each $i\in l$, if and only if there exist matrices $P_i=P_i^T$ and $\varepsilon>0$ such that the following matrix inequalities (10) and (11) hold

$$\bar{E}^T P_i\bar{E}\ge 0 \tag{10}$$

$$\begin{bmatrix} A_i^{\mathrm{T}}\bar{P}_iA_i-\bar{E}^{\mathrm{T}}P_i\bar{E} & A_i^{\mathrm{T}}\bar{P}_i\bar{B}_\omega & C_i^{\mathrm{T}} & H^{\mathrm{T}} \\ * & \bar{B}_\omega^{\mathrm{T}}\bar{P}_i\bar{B}_\omega-\gamma^2 I & D^T & 0 \\ * & * & -\gamma I & 0 \\ * & * & * & -\varepsilon^{-1}I \end{bmatrix}<0 \tag{11}$$

then the SNCs (5) is stochastically admissible and robust asymptotically stable , moreover satisfies H_∞ performance γ norm.

Where $\bar{P}_i=\sum_{j=1}^{4}\lambda_{ij}P_i$

Proof: stochastically admissible and asymptotically stable analysis.

construct the following lyapunov function

$$V(\xi(k),\theta(k))=\xi^T(k)\bar{E}^T P_{\theta(k)}\bar{E}\xi(k) \tag{12}$$

when $\omega(k)\equiv 0$, for simplicity $\theta(k)=i$, calculating the difference of $\Delta V(k,\theta(k))$ along the trajectory of system (5) and taking the mathematical expectation, we have

$$\mathcal{E}\Delta V(\xi(k),\theta(k))=\mathcal{E}[V(\xi(k+1),\theta(k+1))]-V(\xi(k),\theta(k))$$
$$=\mathcal{E}\left\{\xi^T(k+1)\bar{E}^T P_{i+1}\bar{E}\xi(k+1)\right\}-\xi^T(k)\bar{E}^T P_i\bar{E}\xi(k)$$
$$=\begin{bmatrix}\xi(k)\\ \bar{f}(k)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix} A_i^{\mathrm{T}}\bar{P}_iA_i-\bar{E}^{\mathrm{T}}P_i\bar{E} & H^{\mathrm{T}} \\ H & -\varepsilon^{-1}I\end{bmatrix}\begin{bmatrix}\xi(k)\\ \bar{f}(k)\end{bmatrix}$$
$$\le-\lambda_{\min}\left\{-\begin{bmatrix} (A_i^{\mathrm{T}}\bar{P}_iA_i-\bar{E}^{\mathrm{T}}P_i\bar{E}) & H^{\mathrm{T}} \\ H & -\varepsilon^{-1}I\end{bmatrix}\right\}\bar{\xi}^{\mathrm{T}}(k)\bar{\xi}(k)$$
$$\le-\rho x^{\mathrm{T}}(k)x(k) \tag{13}$$

where $\lambda_{\min}(\bullet)$ denotes the minimum eigenvalue of matrix $(\bullet)$, $\bar{\xi}(k)=\begin{bmatrix}\xi^{\mathrm{T}}(k) & \bar{f}(k)^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$ and $0<\rho<\min\{\lambda_{\min}(\bullet)\}$.

Inequality (13) implies that

$$\mathcal{E}[V((k+1),\theta(k+1))]-\mathcal{E}[V(0,\theta(0))]\le-\rho\sum_{k=0}^{N}\mathcal{E}[x^{\mathrm{T}}(k)x(k)] \tag{14}$$

So $\sum_{k=0}^{N}\mathcal{E}[x^{\mathrm{T}}(k)x(k)]\le\frac{1}{\rho}\mathcal{E}[V(\xi(0),\theta(0))]$,

let $N\to\infty$ then

$$\sum_{k=0}^{\infty}\mathcal{E}[x^{\mathrm{T}}(k)x(k)]\le\frac{1}{\rho}\mathcal{E}[V(\xi(0),\theta(0))]<\infty \tag{15}$$

applying definition 2.1 and lemma 3.1, we can obtain the system is stochastically admissible and asymptotically stable.

Then we prove system (5) with zero initial condition, the output $z(k)$ satisfies H_∞ performance.

Definition $J=\sum_{k=0}^{N}\mathcal{E}\left\{\gamma^{-1}z^T(k)z(k)-\gamma\omega^T(k)\omega(k)\right\}$ (16)

Through computation, we have

$$J\le\mathcal{E}\sum_{k=0}^{N}\left\{\Delta V(k,\theta(k))+\gamma^{-1}z^T(k)z(k)-\gamma\omega^T(k)\omega(k)\right\}$$
$$=\mathcal{E}\sum_{k=0}^{N}\left\{\begin{bmatrix}\xi(k)\\ \bar{f}(k)\\ \omega(k)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\tilde{A} & A_i^{\mathrm{T}}\bar{P}_i\bar{B}_\omega & C_i^{\mathrm{T}} \\ * & \bar{B}_\omega^{\mathrm{T}}\bar{P}_i\bar{B}_\omega-\gamma^2 I & D^T \\ * & * & -\gamma I\end{bmatrix}\begin{bmatrix}\xi(k)\\ \bar{f}(k)\\ \omega(k)\end{bmatrix}\right\} \tag{17}$$

where $\tilde{A}=A_i^{\mathrm{T}}\bar{P}_iA_i-\bar{E}^{\mathrm{T}}P_i\bar{E}+\varepsilon H^{\mathrm{T}}H$

According to the theorem 3.1, applying Schur complements formula

$\lim_{N\to\infty}\mathcal{E}\sum_{k=0}^{N}\{Inequality(17)\}<0$,

then $\sum_{k=0}^{\infty}\mathcal{E}\left\{z^T(k)z(k)\right\}<\gamma\sum_{k=0}^{\infty}\mathcal{E}\left\{\omega^T(k)\omega(k)\right\}$,

so the system has H_∞ performance γ norm.

This completes the proof.

Remark 3.1 For systems (7), considered the random data packet dropout as Bernoulli process ($\alpha=\beta=1$ or $\alpha=\beta=0$) for state feedback controller designing in [18]. Without disturbance ($\omega(k)\equiv 0$) for systems (7), a question of robust mean square stability of networked control systems with packed dropout was investigated in [15], in the theorem 3.1, we consider the system with external and nonlinear disturbance simultaneously, and establish a sufficient stochastic stability condition for state feedback controller designing, which is more generality in actual networked data transmission and application.

IV. STATE FEEDBACK H_∞ CONTROLLER DESIGN

In theorem 3.1, not given out the controller gain K solve method, in this section, we will present a design method for the state feedback controller (2) in terms of LMIs.

Theorem 4.1 Given scalars $\gamma > 0$, $\varepsilon > 0$ for each $i \in l$, if there exist matrices $X_i > 0$, $P_i > 0$ and K, such that the inequalities (18) and (19) are feasible, then the SNCs (5) is stochastically admissible and asymptotically stable, satisfies H_∞ performance norm γ.

$$\bar{E}^T X_i \bar{E} \geq 0 \tag{18}$$

$$\begin{bmatrix} \Omega_1 & \Omega_2 & \Omega_3 & \Omega_4 & \Omega_5 \\ * & -\sum_{j\in l}\lambda_{ij}P_i & 0 & C_{0i}+DKE_i & 0 \\ * & * & -\gamma I & D_\omega^T & 0 \\ * & * & * & -\gamma I & 0 \\ * & * & * & * & -\varepsilon H^{-1} \end{bmatrix} < 0 \tag{19}$$

where

$$\Omega_2 = \begin{bmatrix} \sqrt{\lambda_{i1}}(A_{0i}+M_iKN_i) \\ \sqrt{\lambda_{i2}}(A_{0i}+M_iKN_i) \\ \sqrt{\lambda_{i3}}(A_{0i}+M_iKN_i) \\ \sqrt{\lambda_{i4}}(A_{0i}+M_iKN_i) \end{bmatrix} \quad \Omega_3 = \begin{bmatrix} \sqrt{\lambda_{i1}}\bar{B}_\omega \\ \sqrt{\lambda_{i2}}\bar{B}_\omega \\ \sqrt{\lambda_{i3}}\bar{B}_\omega \\ \sqrt{\lambda_{i4}}\bar{B}_\omega \end{bmatrix}$$

$$\Omega_1 = -diag\left[X_i, X_i, X_i, X_i\right] \quad \Omega_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$\Omega_5 = \begin{bmatrix} X_i & 0 & 0 & 0 \end{bmatrix}^T$$

Proof: First, in theorem 3.1, we rewrite A_i and C_i as

$$A_i = A_{0i} + M_iKN_i,\quad C_i = C_{0i} + DKE_i$$

where

$$A_{01} = \begin{bmatrix} A & 0 & 0 \\ I & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, C_{01} = \begin{bmatrix} C & 0 & 0 \end{bmatrix}$$

$$A_{02} = \begin{bmatrix} A & 0 & B \\ I & 0 & 0 \\ 0 & 0 & I \end{bmatrix}, C_{02} = \begin{bmatrix} C & 0 & D \end{bmatrix}$$

$$A_{03} = \begin{bmatrix} A & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}, C_{03} = \begin{bmatrix} C & 0 & 0 \end{bmatrix}$$

$$A_{04} = \begin{bmatrix} A & 0 & B \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, C_{04} = \begin{bmatrix} C & 0 & D \end{bmatrix}$$

$$M_1 = M_2 = \begin{bmatrix} B^T & 0 & I \end{bmatrix}^T, M_3 = M_4 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

$$N_1 = \begin{bmatrix} I & 0 & 0 \end{bmatrix}, N_2 = \begin{bmatrix} 0 & I & 0 \end{bmatrix} \quad N_3 = N_4 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} I & 0 & 0 \end{bmatrix}, E_3 = \begin{bmatrix} 0 & I & 0 \end{bmatrix} \quad E_2 = E_4 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

Then applying Schur complements formula, inequality (11) can be rewritten as an equivalent form (20)

$$\begin{bmatrix} \bar{\Omega}_1 & \bar{\Omega}_2 & \bar{\Omega}_3 & \bar{\Omega}_4 & \bar{\Omega}_5 \\ * & -\sum_{j\in l}\lambda_{ij}P_i & 0 & C_{0i}+DKE_i & 0 \\ * & * & -\gamma I & D_\omega^T & 0 \\ * & * & * & -\gamma I & 0 \\ * & * & * & * & -\varepsilon H^{-1} \end{bmatrix} < 0 \tag{20}$$

where

$$\bar{\Omega}_1 = -diag\left[P_i^{-1}, P_i^{-1}, P_i^{-1}, P_i^{-1}\right],\ \bar{\Omega}_2 = \Omega_2, \bar{\Omega}_3 = \Omega_3, \bar{\Omega}_4 = \Omega_4$$

$$\bar{\Omega}_5 = \begin{bmatrix} P_i^{-1} & 0 & 0 & 0 \end{bmatrix}^T$$

Let $P_i^{-1} = X_i$ in inequality (20), it yields (19).

This completes the proof.

Remark 4.1 It should be noted that inequality (20) are non-convex due to P_i^{-1}, which can be solved via the cone complementary linearization method [17], the controller gain solve method converted into the following nonlinear optimization problem involving LMI conditions.

$$\min_{\gamma, P_i, X_i} tr(\sum_{i=1}^{4} P_iX_i)$$

$$\text{subject to} \quad i)(18) \text{ and } (19),\ ii)\begin{bmatrix} P_i & I \\ I & X_i \end{bmatrix} > 0 \tag{21}$$

The γ sub-optimal H_∞ controller gain K can be found using the following algorithm:

Algorithm 4.1

Step 1 Find an initial feasible set (P^0, X^0, K^0) satisfying (21), let $k = 0$.

Step 2 Solve the following LMI problem

$$\begin{cases} \min_{\gamma, P_i, X_i} tr(\sum_{i=1}^{4} P_i^k X_i + P_i X_i^k) \\ \text{subject to} \quad i)(18) \text{ and } (19), ii)\begin{bmatrix} P_i & I \\ I & X_i \end{bmatrix} > 0 \end{cases}$$

Step 3 let $P_i^{k+1} = P_i, X_i^{k+1} = X_i, K^{k+1} = K^k$

Step 4 If $k > N$, where N is the maximum numbers of iterations allowed, give up and stop.

Step 5 If (P, X, K) satisfy (21), then stop, if not, let $k = k+1$ go to step2.

V. ILLUSTRATIVE EXAMPLE

In this section, we present a numerical example to illustrate the theoretical results developed earlier. Consider the system (1) with the following matrices borrowed from [18] example 3, we have the parameters

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ -1 & -0.5 \end{bmatrix},\ B = \begin{bmatrix} 6 \\ 2 \end{bmatrix}, B_\omega = \begin{bmatrix} -0.03 & 0.01 \\ 0 & 0.01 \end{bmatrix}$$

$$C = \begin{bmatrix} -0.1 & -0.05 \end{bmatrix}, D_\omega = \begin{bmatrix} 0.01 & 0.01 \end{bmatrix},\ D = 0.5, H = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Applying augmentation matrix method, it can be obtained parameters of system (5)

$$A_{01} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -0.5 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{02} = \begin{bmatrix} 1 & 1 & 0 & 0 & 6 & 0 \\ -1 & -0.5 & 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_{03} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & -0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{04} = \begin{bmatrix} 1 & 1 & 0 & 0 & 6 & 0 \\ -1 & -0.5 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_{01} = C_{03} = \begin{bmatrix} -0.1 & -0.05 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C_{02} = C_{04} = \begin{bmatrix} -0.1 & -0.05 & 0 & 0 & 0.5 & 0 \end{bmatrix}$$

$$M_1 = M_2 = \begin{bmatrix} 6 & 2 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_3 = M_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad N_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$N_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad N_3 = N_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E_3 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, E_2 = E_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The transition probability matrix is given as

$$\Pi = \begin{bmatrix} 0.3 & 0.5 & 0.1 & 0.1 \\ 0.2 & 0.3 & 0.5 & 0.0 \\ 0.1 & 0.7 & 0.0 & 0.2 \\ 0.6 & 0.1 & 0.2 & 0.1 \end{bmatrix}$$

applying algorithm 4.1 to this example, it can be obtained $\gamma_{\min} = 0.175$, the controller gain matrix

$$K = \begin{bmatrix} 0.2490 & -0.4263 \\ -0.2328 & 0.4256 \end{bmatrix}$$

In the initial condition $x(0) = [1 \quad -1]^{\mathrm{T}}$ and $\theta(0) = 1$, the external disturbance $\omega(k)$ is assumed to be,

$$\omega(k) = \begin{cases} 0.2, & 10 \le k \le 15 \\ -0.2, & 15 \le k \le 25 \\ 0, & else \end{cases}$$

the system state response and control output trajectory are given in Fig.2 and Fig.3, respectively.

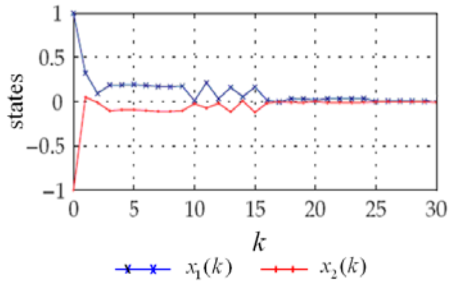

Fig.2 The state response of system

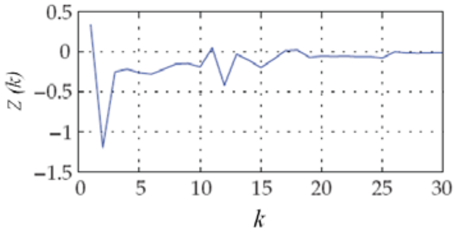

Fig.3 The control output of system

from the simulation, it shows the asymptotical stability of the closed-loop system.

For system (1), when $E = I, f = 0$, in [14] only considered the networked control systems are modeled as a Markov jump linear system with two operation modes, compared with references [15, 16, 18], our result has more generality in practical application.

CONCLUSION

In this paper, the problem of robust H_∞ control of discrete time singular systems over networks has been investigated, both the data packed dropout and nonlinear disturbance are considered, the system communication link failure is modeled as markovian jump singular systems with four models, based on the Lyapunov function method and LMI technique, an approach has been developed to design a state feedback H_∞ controller, such that the closed-loop system is stochastically stable and preserves a guaranteed H_∞ performance. An example is shown that the approach presented in this paper is effective.

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