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Global Bounded Controlled Consensus of Multi-Agents Systems with Non-Identical Nodes and Communication Time-Delay Topology

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***Abstract*—This paper investigates the global bounded consensus problem of Networked Multi-Agent Systems exhibiting nonlinear, non-identical agent dynamics with communication time-varying delay. Globally bounded controlled consensus conditions based on pinning control method and adaptive pinning control method are derived. The proposed consensus criteria ensures that all agents eventually move along desired trajectories in terms of boundedness. The proposed controlled consensus criteria generalizes the case of identical agent dynamics to the case of non-identical agent dynamics, and many related results of other researches in this area can be viewed as special cases of the above results. We finally demonstrate the effectiveness of the theoretical results by means of a numerical simulation.**

I. INTRODUCTION

Networked Multi-Agent Systems (NMAS) analysis involves the study of how the network architectures and interactions between network components influence global control goals and some important contributions have been made in recent years [1], [2], [3], [4].

The consensus problem has been studied across many fields of science and engineering [5], [6], [7], [8], [9], [10], [11], [12], [13]. The controlled consensus problem of NMAS with non-identical agent dynamics is much more complicated than the identical case and few results have been reported to date [14].

The present paper will focus on the global consensus problems of NMAS based on pinning control methods [15], [16], [17], and the proposed controlled consensus property is formulated in terms of certain boundedness of state errors. In this paper, we'll generalize many existing results for the case of identical agent dynamics to the case of non-identical agent dynamics based on the pinning control method.

The rest of this paper is organized as follows. A controlled continuous-time NMAS model with communication time-delay is presented in Section II. The main results including pinning control and adaptive pinning control bounded consensus criterion are derived in Section III and V respectively. Section IV gives a numerical simulation example to verify the effectiveness of the proposed results, followed by conclusions in Section VI.

II. PROBLEM DESCRIPTION

Let $G = (\mathcal{V}, \mathcal{A})$ be a graph of order N consisting of a set of vertices $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ and a set of edges $\mathcal{A} \subseteq \mathcal{V} \times \mathcal{V}$. An edge (v_j, v_i) in graph G means that agent v_i sends some information to agent v_j. The set of neighbors of agent v_i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{A}\}$.

We consider a MAS consisting of N non-identical agents with communication delay:

$$\dot{x}_i = f_i(x_i) + c \sum_{j \in \mathcal{N}_i}^{N} a_{ij} \Gamma x_j(t - \tau), i = 1, 2, \cdots, N, \qquad (1)$$

where $x_i = (x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t))^T \in R^n$ are the state variables of the agent v_i, $f_i(x_i) : R^n \to R^n$ are continuously differentiable mappings with Jacobian Df_i, representing the self-dynamics of the agent v_i, $c > 0$ denotes the coupling strength, $\Gamma = (\gamma_{ij}) \in R^{n \times n}$ is the inner coupling matrix, and where $\gamma_{ij} \neq 0$ means two connected agents are linked via their ith and jth state variables, respectively. The adjacency matrix $A = (a_{ij}) \in R^{N \times N}$ (which is symmetric and irreducible) represents the communication topology relation of the MAS, and is defined by $a_{ij} = a_{ji} = 1 (v_j \in \mathcal{N}_i)$, $a_{ij} = 0 (v_j \notin \mathcal{N}_i)$ and $a_{ii} = -\sum_{j \neq i} a_{ij}$. τ is a constant coupling delay which reflects the reality that the agent v_i can't obtain information from agent v_j instantaneously.

The average dynamic of all agents is defined by the vector field $\bar{f}(x(t)) = \frac{1}{N} \sum_{k=1}^{N} f_k(x(t))$ with Jacobian $D\bar{f}_i(x(t))$.

The average state trajectory is chosen as the desired moving trajectory

$$s(t) = \frac{1}{N} \sum_{k=1}^{N} x_k(t). \qquad (2)$$

We now discuss the problem of global consensus for the system (1). The consensus problem formulation in the present paper is quite different from many others, where the consensus problem is solvable if the states of all agents satisfy $x_i(t) \to x_j(t)$, $\forall i, j = 1, 2, \cdots, N$ as $t \to \infty$. The consensus problem here will be depicted instead via certain boundedness of $x_i(t) - x_j(t)$, $\forall i, j = 1, 2, \cdots, N$ as $t \to \infty$. This better reflects reality as it is impossible for MAS (1) to achieve exact consensus. To address this case we will focus on making the states of all agents converge to a bounded set.

We denote $x(t)$, $s(t)$, $u(t)$, $e(t)$, $w(t)$ and $V(w(t), t)$ as x, s, u, e, w and V respectively.

III. Linear feedback pinning controller

To achieve the goal, we apply the feedback control strategy on a small fraction δ $(0 < \delta \leq 1)$ of the agents in system (1). Suppose that nodes $i_1, i_2, \cdots, i_l$ are selected to be under control, where $l = [\delta N]$ stands for the smaller but nearest integer to the real number δN. This controlled MAS can be described as

$$\begin{cases} \dot{x}_{i_k} = f_{i_k}(x_{i_k}) + c\sum_{j=1}^{N} a_{i_k j}\Gamma x_j(t-\tau) + u_{i_k}, 1 \leq k \leq l, \\ \dot{x}_{i_k} = f_{i_k}(x_{i_k}) + c\sum_{j=1}^{N} a_{i_k j}\Gamma x_j(t-\tau), l+1 \leq k \leq N. \end{cases} \tag{3}$$

The local linear negative feedback control law is chosen as follows:

$$\begin{cases} u_{i_k} = -d_{i_k}(x_{i_k} - s), & 1 \leq k \leq l, \\ u_{i_k} = 0, & l+1 \leq k \leq N, \end{cases} \tag{4}$$

where the feedback gain $d_{i_k} > 0$.

Combine (3) and (4) and rearrange the order of the nodes in the network. Let the first l nodes be controlled, and $e_i = x_i - s$, $i = 1, 2, \cdots, N$. It's obvious that $\frac{c}{N}\sum_{k=1}^{N}\sum_{j=1}^{N} a_{kj}\Gamma x_j(t-\tau) = 0$ and $\sum_{i=1}^{N} e_i = 0$. Then by applying the Newton-Leibniz formula, error systems can be written as

$$\begin{cases} \dot{e}_i = D\bar{f}(s)e_i + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t-\tau) \\ \quad + \int_0^1 (Df_i(s+\tau e_i) - D\bar{f}(s))e_i d\tau \\ \quad - \frac{1}{N}\sum_{k=1}^{N}\int_0^1 Df_k(s+\tau e_k)e_k d\tau \\ \quad + f_i(s) - \bar{f}(s) - d_i e_i, & 1 \leq i \leq l, \\ \dot{e}_i = D\bar{f}(s)e_i + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t-\tau) \\ \quad + \int_0^1 (Df_i + \tau e_i) - D\bar{f}(s))e_i d\tau \\ \quad - \frac{1}{N}\sum_{k=1}^{N}\int_0^1 Df_k(s+\tau e_k)e_k d\tau \\ \quad + f_i(s) - \bar{f}(s), & l+1 \leq i \leq N. \end{cases} \tag{5}$$

The following work will focus on simplifying the error systems (5) by means of a series of transformations using a procedure similar to [14].

Define the following matrix

$$D = diag(D_1, D_2, \cdots, D_N) \in R^{nN \times nN},$$

where $D_i = diag\{-d_i, -d_i, \cdots, -d_i\} \in R^{n \times n}$.

Let $e = (e_1^T, e_2^T, \cdots, e_N^T)^T$, then (5) becomes

$$\dot{e} = \bar{\Sigma}(t)e + cA \otimes \Gamma e(t-\tau) + I(t)e - \frac{1}{N}H(t)e + F(t), \tag{6}$$

where $I(t) = diag\{\int_0^1 (Df_1(s+\tau e_1) - D\bar{f}(s))d\tau \cdots \int_0^1 (Df_N(s+\tau e_N) - D\bar{f}(s))d\tau\}$, $\bar{\Sigma}(t) = I_N \otimes D\bar{f}(s) + D$, $H^T(t) = (H_1^T(t), \cdots, H_N^T(t))$, $H_i(t) = (\int_0^1 Df_1(s+\tau e_1)d\tau, \cdots, \int_0^1 Df_N(s+\tau e_N)d\tau)$, $F_i^T(t) = (f_1^T(s) - \bar{f}^T(s), \cdots, f_N^T(s) - \bar{f}^T(s))$.

Since A is symmetric and irreducible, according to [14], there exists a unitary matrix $\Phi = (\varphi_{ij})_{N\times N} = (\Phi_1, \Phi_2, \cdots, \Phi_N)$. This together with $w(t) = (\Phi^T \otimes I_n)e$ gives

$$\begin{aligned} \dot{w} = &(\Phi^T \otimes I_n)\bar{\Sigma}(t)(\Phi \otimes I_n)w \\ &+ (\Phi^T \otimes I_n)(cA \otimes \Gamma)(\Phi \otimes I_n)w(t-\tau) \\ &+ (\Phi^T \otimes I_n)I(t)(\Phi \otimes I_n)w \\ &- \frac{1}{N}(\Phi^T \otimes I_n)H(t)(\Phi \otimes I_n)w + (\Phi^T \otimes I_n)F(t). \end{aligned} \tag{7}$$

Note that $H(t) = \sqrt{N}\sum_{k=1}^{N}(\mathbf{0} \ \cdots \ \mathbf{0} \ \bar{\Phi}_k \ \mathbf{0} \ \cdots \ \mathbf{0}) \otimes \int_0^1 Df_k(s+\tau e_k)d\tau$, where $\bar{\Phi}_k$ stands for the matrix with its k-th column equal to Φ_1 and the remaining elements are zero. Then we have $\frac{1}{N}(\Phi^T \otimes I_n)H(t)(\Phi \otimes I_n) = \frac{1}{\sqrt{N}}\sum_{k=1}^{N}(\mathbf{0} \cdots \mathbf{0} \, I_k \, \mathbf{0} \cdots \mathbf{0}) \otimes \int_0^1 Df_k(s+\tau e_k)d\tau(\Phi \otimes I_n)$, where I_k stands for the matrix with its k-th column equals $(1 \ 0 \ \cdots \ 0)^T$ and the remaining of its elements are zero.

Thus, a simple calculation gives $\frac{1}{N}(\Phi^T \otimes I_n)H(t)(\Phi \otimes I_n) = \frac{1}{\sqrt{N}}\sum_{k=1}^{N}\begin{pmatrix} \Upsilon_k \\ 0 \end{pmatrix} \otimes \int_0^1 Df_k(s(t)+\tau e_k(t))d\tau$, where $\Upsilon_k \in R^{1\times N}$ and $0 \in R^{(N-1)\times N}$. Therefore, $\dot{w} = \bar{\Sigma}(t)w + c\Lambda \otimes \Gamma w(t-\tau) + (\Phi^T \otimes I_n)I(t)(\Phi \otimes I_n)w - \begin{pmatrix} * \\ 0 \end{pmatrix} w + (\Phi^T \otimes I_n)F(t)$. Since $w_1 \equiv 0$, we only need to consider $w_2, w_3, \cdots, w_N$. Rewriting in the component form we have

$$\begin{aligned} \dot{w}_i = &\Sigma_i(t)w_i + c\lambda_i\Gamma w_i(t-\tau) + (\Phi_i^T \otimes I_n)F(t) \\ &+ (\Phi_i^T \otimes I_n)I(t)(\Phi \otimes I_n)w, \ i = 2, 3, \cdots, N, \end{aligned} \tag{8}$$

where $\Sigma_i = \bar{D}f(s) + D_i$.

So far, we have transferred the consensus problem of system (1) to the stability problem of the $N-1$ of $n-$dimensional systems.

Theorem 1 Suppose that $\|I(t)\| \leq \gamma$ is satisfied. If there exist matrices $P_i(t) \in \mathcal{PC}^1_{n\times n}$, $Q_i > 0$, $\Theta_i > 0$, $\Pi_i > 0$, X_i, Y_i and Z_i of appropriate dimensions such that

$$B = \begin{pmatrix} B_1 & B_2 \\ B_2^T & B_3 \end{pmatrix} < 0, \ \begin{pmatrix} X_i & Y_i \\ Y_i^T & Z_i \end{pmatrix} \geq 0, \tag{9}$$

for $i = 2, 3, \cdots, N$, where $B_1 = \dot{P}_i(t) + P_i(t)\Sigma_i(t) + \Sigma_i^T(t)P_i(t) + hX_i + Y_i^T + Y_i + Q_i + h\Sigma_i^T(t)Z_i\Sigma_i(t)$, $B_2 = c\lambda_i P_i(t)\Gamma - Y_i + hc\lambda_i\Sigma_i^T(t)Z_i\Gamma$ and $B_3 = \Pi_i^{-1} + \Theta_i^{-1} - Q_i + hc^2\lambda_i^2\Gamma^T Z_i\Gamma$, then the MAS (1) will achieve bounded consensus for the time-invariant delay $\tau \in [0, h]$ for some $h < \infty$.

Proof. Construct the following Lyapunov-Krasovskii functional as

$$V = \sum_{i=2}^{N}\sum_{k=1}^{3} V_k, \tag{10}$$

where

$$
\begin{aligned}
V_1 &= w_i^T P_i(t) w_i,\\
V_2 &= \int_{-\tau}^{0}\int_{t+\beta}^{t} \dot{w}_i^T(\alpha) Z_i \dot{w}_i(\alpha) d\alpha d\beta,\\
V_3 &= \int_{t-\tau}^{t} w_i^T(\alpha) Q_i w_i(\alpha) d\alpha.
\end{aligned}
$$

The i-th $(i = 2, 3, \cdots, N)$ equation in system (8) can be written as

$$
\begin{aligned}
\dot{w}_i &= (\Sigma_i(t) + c\lambda_i\Gamma)w_i - c\lambda_i\Gamma \int_{t-\tau}^{t} \dot{w}_i(\alpha)d\alpha\\
&\quad + (\Phi_i^T \otimes I_n)I(t)(\Phi \otimes I_n)w + (\Phi_i^T \otimes I_n)F(t). \quad (11)
\end{aligned}
$$

Defining $a(.)$, $b(.)$ and M in [18] as $a(\alpha) = w_i(t)$, $b(\alpha) = \dot{w}_i(\alpha)$ and $M = c\lambda_i P_i(t)\Gamma$ for all $\alpha \in [t-\tau, t]$ then we have

$$
\begin{aligned}
\dot{V}_1 \leq\; & w_i^T[\dot{P}_i(t) + P_i(t)\Sigma_i(t) + \Sigma_i^T(t)P_i(t)\\
& + hX_i + Y_i^T + Y_i]w_i + \int_{t-\tau}^{t} \dot{w}_i^T(\alpha) Z_i \dot{w}_i(\alpha) d\alpha\\
& + 2w_i^T(c\lambda_i P_i(t)\Gamma - Y_i)w_i(t-\tau)\\
& + 2w_i^T P_i(t)(\Phi_i^T \otimes I_n)I(t)(\Phi_i \otimes I_n)w\\
& + 2w_i^T P_i(t)(\Phi_i^T \otimes I_n)F(t). \quad (12)
\end{aligned}
$$

Moreover, $\dot{V}_2$ can be enlarged as

$$
\begin{aligned}
\dot{V}_2 \leq\; & h[\Sigma_i(t)w_i + c\lambda_i\Gamma w_i(t-\tau)]^T Z_i[\Sigma_i(t)w_i\\
& + c\lambda_i\Gamma w_i(t-\tau)] + 2h(\Sigma_i(t)w_i)^T Z_i(\Phi_i^T \otimes I_n)I(t)\\
& (\Phi \otimes I_n)w + 2h(\Sigma_i(t)w_i)^T Z_i(\Phi_i^T \otimes I_n)F(t)\\
& + 2h(c\lambda_i\Gamma w_i(t-\tau))^T Z_i(\Phi_i^T \otimes I_n)I(t)(\Phi \otimes I_n)w\\
& + 2h(c\lambda_i\Gamma w_i(t-\tau))^T Z_i(\Phi_i^T \otimes I_n)F(t)\\
& + 2h((\Phi_i^T \otimes I_n)I(t)(\Phi \otimes I_n)w)^T Z_i(\Phi_i^T \otimes I_n)F(t)\\
& + h((\Phi_i^T \otimes I_n)F(t))^T Z_i((\Phi_i^T \otimes I_n)F(t))\\
& + h((\Phi_i^T \otimes I_n)I(t)(\Phi \otimes I_n)w)^T Z_i((\Phi_i^T \otimes I_n)I(t)\\
& (\Phi \otimes I_n)w) - \int_{t-\tau}^{t} \dot{w}_i^T(\alpha) Z_i \dot{w}_i(\alpha) d\alpha. \quad (13)
\end{aligned}
$$

and

$$
\dot{V}_3 = w_i^T Q_i w_i - w_i^T(t-\tau)Q_i w_i(t-\tau). \quad (14)
$$

Applying the Young Inequality, then we have $2h(c\lambda_i\Gamma w_i(t-\tau))^T Z_i(\Phi_i^T \otimes I_n)I(t)(\Phi \otimes I_n)w \leq w_i^T(t-\tau)\Pi_i^{-1}w_i(t-\tau) + h^2c^2\lambda_i^2 w^T((\Phi \otimes I_n)^T I(t)(\Phi_i^T \otimes I_n)^T Z_i\Gamma\Pi_i\Gamma^T Z_i(\Phi_i^T \otimes I_n)I(t)(\Phi \otimes I_n))w(t)$, and $2h(c\lambda_i\Gamma w_i(t-\tau))^T Z_i(\Phi_i^T \otimes I_n)F(t) \leq w_i^T(t-\tau)\Theta_i^{-1}w_i(t-\tau) + h^2c^2\lambda_i^2 F^T(t)(\Phi_i^T \otimes I_n)^T Z_i\Gamma\Theta_i\Gamma^T Z_i(\Phi_i^T \otimes I_n)F(t)$. Applying these two inequalities and the conditions of the theorem results

$$
\begin{aligned}
\dot{V} \leq\; & \sum_{i=2}^{N} \begin{pmatrix} w_i \\ w_i(t-\tau) \end{pmatrix}^T B \begin{pmatrix} w_i \\ w_i(t-\tau) \end{pmatrix}\\
& + 2\mu(t)\beta + (\|w\|(2\gamma\beta + 2h\gamma\|\Sigma_i(t)\| \sum_{i=2}^{N} \lambda_{max}(Z_i)\\
& + 2h\mu(t)\|\Sigma_i(t)\| \sum_{i=2}^{N} \lambda_{max}(Z_i) + h\gamma^2 \sum_{i=2}^{N} \lambda_{max}(Z_i)\\
& + h^2c^2\gamma^2\lambda_{max}^{\frac{1}{2}}(\Gamma\Gamma^T) \sum_{i=2}^{N} \lambda_{max}(\Pi_i)\lambda_i^2\lambda_{max}^2(Z_i)\\
& + h^2c^2\mu^2(t)\lambda_{max}^{\frac{1}{2}}(\Gamma\Gamma^T) \sum_{i=2}^{N} \lambda_{max}(\Theta_i)\lambda_i^2\lambda_{max}^2(Z_i))\|w\|\\
& + 2h\gamma \sum_{i=2}^{N} \lambda_{max}(Z_i)\mu(t)) + h\mu^2(t) \sum_{i=2}^{N} \lambda_i^2\lambda_{max}(Z_i). \quad (15)
\end{aligned}
$$

Thus when

$$
\|w\| \geq \frac{2\mu(t)\beta + 2h\gamma \sum_{i=2}^{N} \lambda_{max}(Z_i)\mu(t)}{\varpi(t)},
$$

we have

$$
\dot{V} \leq -\delta\|w\|^2 + h\mu^2(t) \sum_{i=2}^{N} \lambda_{max}(Z_i)\lambda_i^2, \quad (16)
$$

where $\varpi(t) = -(2\gamma\beta + 2h\gamma\|\Sigma_i(t)\| \sum_{i=2}^{N} \lambda_{max}(Z_i) + 2h\mu(t)\|\Sigma_i(t)\| \sum_{i=2}^{N} \lambda_{max}(Z_i) + h\gamma^2 \sum_{i=2}^{N} \lambda_{max}(Z_i) + h^2c^2\gamma^2\lambda_{max}^{\frac{1}{2}}(\Gamma\Gamma^T) \sum_{i=2}^{N} \lambda_{max}(\Pi_i)\lambda_i^2\lambda_{max}^2(Z_i) + h^2c^2\mu^2(t) \lambda_{max}^{\frac{1}{2}}(\Gamma\Gamma^T) \sum_{i=2}^{N} \lambda_{max}(\Theta_i)\lambda_i^2\lambda_{max}^2(Z_i)) - \delta$. Thus, according to [19] and Lyapunov stability theory, bounded consensus is ultimately achieved. This completes the proof.

IV. Adaptive pinning controller

In this section, we will derive globally consensus criteria via direct adaptive pinning control method. Without loss of generality, we still assume that the first l agents are selected as pinned agents with the adaptive controllers:

$$
\begin{cases}
u_i = -d_i(t)(x_i - s), & 1 \leq i \leq l,\\
\dot{d}_i(t) = h_i e_i^T P_i(t) e_i, & \\
u_i = 0, & l+1 \leq i \leq N,
\end{cases} \quad (17)
$$

where constant $h_i > 0$ and positive definite matrix $P_i(t) \in R^{n\times n}$. Applying Newton-Leibniz formula, then the error MAS can be rewritten as

$$\begin{cases}\dot{e}_i = D\bar{f}(s)e_i + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t-\tau) \\ \qquad + \int_0^1 (Df_i(s+\tau e_i) - D\bar{f}(s))e_i d\tau \\ \qquad - \frac{1}{N}\sum_{k=1}^{N}\int_0^1 Df_k(s+\tau e_k)e_k d\tau \\ \qquad + f_i(s) - \bar{f}(s) - d_i(t)e_i, \qquad 1 \le i \le l, \\ \dot{d}_i(t) = h_i e_i^T P_i(t) e_i, \\ \dot{e}_i = D\bar{f}(s)e_i + c\sum_{j=1}^{N} a_{ij}\Gamma e_j(t-\tau) \\ \qquad + \int_0^1 (Df_i(s+\tau e_i) - D\bar{f}(s))e_i d\tau \\ \qquad - \frac{1}{N}\sum_{k=1}^{N}\int_0^1 Df_k(s+\tau e_k)e_k d\tau \\ \qquad + f_i(s) - \bar{f}(s), \qquad l+1 \le i \le N. \end{cases} \tag{18}$$

Repeating a similar procedure to the previous subsection, the controlled consensus problem of system (1) is equivalent to the stability problem of the following $N-1$ of n-dimensional systems.

$$\begin{cases}\dot{w}_i = D\bar{f}(s(t))w_i - d_i(t)w_i + c\lambda_i\Gamma w_i(t-\tau) \\ \qquad + (\Phi_i^T \otimes I_n)I(t)(\Phi \otimes I_n)w \\ \qquad + (\Phi_i^T \otimes I_n)F(t), \qquad 2 \le i \le l, \\ \dot{d}_i(t) = h_i w_i^T P_i(t) w_i, \\ \dot{w}_i = D\bar{f}(s)w_i + c\lambda_i\Gamma w_i(t-\tau) \\ \qquad + (\Phi_i^T \otimes I_n)I(t)(\Phi \otimes I_n)w \\ \qquad + (\Phi_i^T \otimes I_n)F(t), \qquad l+1 \le i \le N, \end{cases} \tag{19}$$

where w_i, w, Φ, Φ_i, $I(t)$ and $F(t)$ are the same as the previous subsection.

Theorem 2 Suppose that $\|I(t)\| \le \gamma$ is satisfied. If there exist matrices $P_i(t) \in \mathcal{PC}^1_{n\times n}$, $Q_i > 0$, $\Theta_i > 0$, $\Pi_i > 0$, X_i, Y_i and Z_i of appropriate dimensions and constant $d > 0$ such that

$$B = \begin{pmatrix} B_1 & B_2 \\ B_2^T & B_3 \end{pmatrix} < 0, \quad \begin{pmatrix} X_i & Y_i \\ Y_i^T & Z_i \end{pmatrix} \ge 0, \tag{20}$$

for $i = 2, 3, \cdots, N$, where $B_1 = \dot{P}_i(t) + P_i(t)(Df(s)) + (Df(s))^T P_i(t) - 2dP_i(t) + hX_i + Y_i^T + Y_i + Q_i + h\Sigma_i^T(t)Z_i\Sigma_i(t)$, $B_2 = c\lambda_i P_i(t)\Gamma - Y_i + hc\lambda_i\Sigma_i^T(t)Z_i\Gamma$ and $B_3 = \Pi_i^{-1} + \Theta_i^{-1} - Q_i + hc^2\lambda_i^2\Gamma^T Z_i \Gamma$, then the system (1) will achieve bounded consensus for the time-invariant delay $\tau \in [0, h]$ for some $h < \infty$.

Proof. Construct the following Lyapunov-Krasovskii functional as

$$V = \sum_{i=2}^{N}\sum_{k=1}^{3} V_k + \sum_{i=2}^{l}\frac{(d_i(t)-d)^2}{h_i}, \tag{21}$$

where

$$V_1 = w_i^T P_i(t) w_i,$$
$$V_2 = \int_{-\tau}^{0}\int_{t+\beta}^{t} \dot{w}_i^T(\alpha) Z_i \dot{w}_i(\alpha) d\alpha d\beta,$$
$$V_3 = \int_{t-\tau}^{t} w_i^T(\alpha) Q_i w_i(\alpha) d\alpha.$$

The remaining part of the proof is similar to that of Theorem 1 and is therefore omitted here. This completes the proof.

V. Example

To demonstrate the theoretical results obtained above, we construct a MAS consisting of 11 agents described as follows

$$\dot{x}_i(t) = f_i(x_i(t)) + c\sum_{j\in\mathcal{N}_i}^{N} a_{ij}\Gamma x_j(t-\tau), \tag{22}$$

where $f_i(x_i(t)) = B_i x_i(t) + g(x_i(t))$, $B_i (i = 1, 2, \cdots, 6)$ and $B_i (i = 7, 8, \cdots, 11)$ are chosen as follows:

$$\begin{pmatrix} -10+0.1\times(i-1) & 10-0.1\times(i-1) & 0 \\ 1 & -1 & 1 \\ 0 & -15-0.1\times(i-1) & 0 \end{pmatrix},$$
$$\begin{pmatrix} -10-0.1\times(i-6) & 10+0.1\times(i-6) & 0 \\ 1 & -1 & 1 \\ 0 & -15+0.1\times(i-6) & 0 \end{pmatrix},$$

and

$$g(x_i(t)) = (-9.5sin(\frac{\pi x_{i1}(t)}{3.2} + \pi)\ 0\ 0)^T, \quad i = 1, 2, \cdots, 11.$$

The communication coupling matrix $C = (C_1^T C_2^T \cdots C_{11}^T)$, $C_1 = (-8\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 1\ 1\ 1)$, $C_2 = (1\ -8\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ 1)$, $C_3 = (1\ 1\ -6\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 1)$, $C_4 = (0\ 1\ 1\ -5\ 0\ 1\ 1\ 0\ 0\ 1\ 0)$, $C_5 = (1\ 1\ 0\ 0\ -6\ 0\ 1\ 1\ 1\ 1\ 0)$, $C_6 = (1\ 0\ 0\ 1\ 0\ -5\ 1\ 0\ 1\ 1\ 0)$, $C_7 = (1\ 1\ 0\ 1\ 1\ 1\ -7\ 1\ 0\ 1\ 0)$, $C_8 = (0\ 0\ 1\ 0\ 1\ 0\ 1\ -5\ 0\ 1\ 1)$, $C_9 = (1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ -6\ 1\ 1)$, $C_{10} = (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ -10\ 1)$, $C_{11} = (1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ -6)$. $\Gamma = diag\{2, 2, 2\}$, respectively, where the matrix A is produced by means of the Scale-Free network program.

Design the following controllers

$$\begin{cases} u_{i_k} = -d_{i_k}(x_{i_k}(t) - s(t)), & i_k = 1, 2 \text{ and } 10, \\ u_{i_k} = 0, & \text{else}, \end{cases}$$

with $d_1 = 0.5$, $d_2 = 0.5$, $d_{10} = 0.5$ and

$$\begin{cases} u_{i_k} = -d_{i_k}(t)(x_{i_k}(t) - s(t)), & i_k = 1, 2 \text{ and } 10, \\ \dot{d}_{i_k}(t) = h_{i_k} e_{i_k}^T P_{i_k}(t) e_{i_k}, \\ u_{i_k} = 0, & \text{else}, \end{cases}$$

with $h_1 = 0.1$, $h_2 = 0.2$, $h_{10} = 0.3$, $s(t)$ can then be evaluated by simulation.

Given the initial values of 11 agents as $(10\ 5\ -10)^T$, $(12\ 6\ -12)^T$, $(14\ 7\ -14)^T$, $(16\ 8\ -16)^T$, $(18\ 9\ -18)^T$, $(20\ 10\ -20)^T$, $(-18\ 11\ 18)^T$, $(-16\ 12\ 16)^T$, $(-14\ 13\ 14)^T$, $(-12\ 14\ 12)^T$, $(-10\ 15\ 10)^T$ respectively and $P_{i_k}(t) = I_3$. We may verify the conditions of Theorem 1 and Theorem 2 readily. This demonstrates the bounded consensus of the MAS is achieved for any time delay $0 < \tau \le 0.061$. Simulation results are depicted in Fig.1 to Fig.4 for $\tau = 0.061$ and $c = 1$.

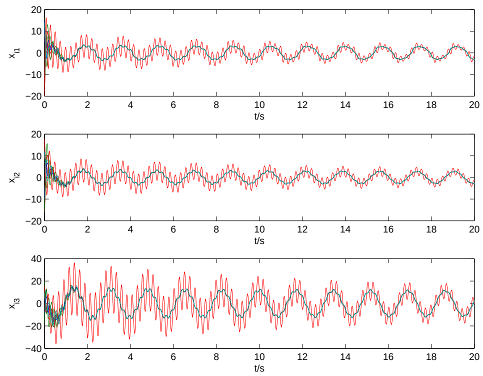

Fig.1. All agent dynamics under pinning control.

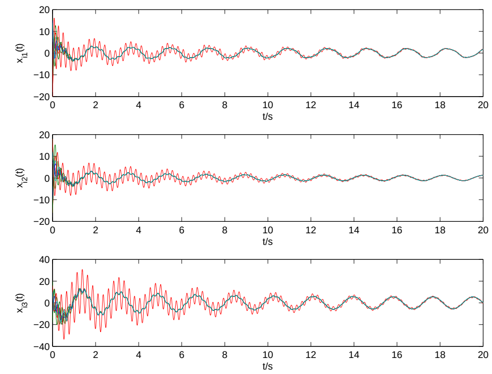

Fig.2. All agent dynamics under adaptive pinning control.

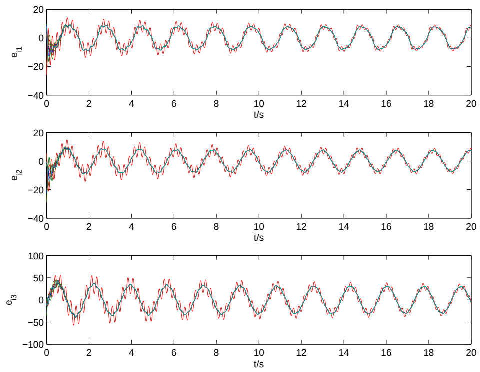

Fig.3. All agent error dynamics under pinning control.

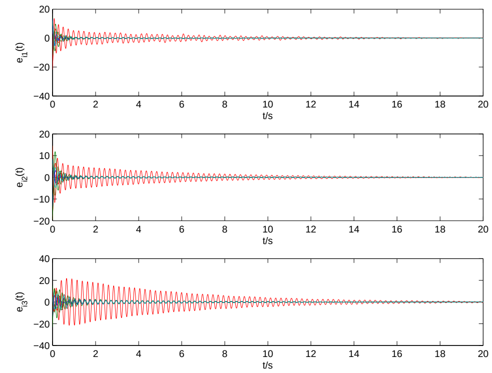

Fig.4. All agent error dynamics under adaptive pinning control.

VI. Conclusion

In this paper, we've investigated the controlled consensus problems of NMAS with different agent dynamics. The derived criteria are verified via theoretical analysis and numerical simulation. The consensus for the NMAS is achieved based on pinning control and adaptive pinning control methods. It should be noted that the conditions are still restrictive and all the delays are the same. Further investigations will focus on relaxing these limitations.

References

[1] Sandro Zampieri. Trends in networked control systems, *Proceedings of 17th World Congress of IFAC*, Seoul, Korea, July 6-11, 2008, pp. 2886 – 2894.

[2] J.P. Desai, J.P. Ostrowski and V. Kumar. Modeling and control of formation of nonholonomic mobile robust. *IEEE Int. Trans. on Robotics and Automation*, 2001, 17(6): 905 – 908.

[3] M. Porfiri, D.G. Roberson and D.J. Stilwell. Tracking and formation control of multiple autonomous agents: a two-level consensus approach. *Automatica*, 2007, 43: 1318 – 1328.

[4] J. Cortés. Global formation-shape stabilization of relative sensing networks. *2009 American Control Systems*, Hyatt Regency Riverfront, St. Louis, USA, June 10-12, 2009, pp. 1460 – 1465.

[5] Y.G. Hong, J.P. Hu and L.X. Gao. Tracking control for multi-agent consensus with an active leader and variable topology. *Automatica*, 2006, 42: 1177 – 1182.

[6] F. Xiao and L. Wang. Asynchronous consensus in continuous-time multi-agent systems with switching topology and time-varying delays. *IEEE Transactions on Automatic Control*, 2008, 53(8): 1804 – 1816.

[7] E.S. Kazerooni and K. Khorasani. Optimal consensus algorithms for cooperative team of agents subject to partial information. *Automatica*, 2008, 44: 2766 – 2777.

[8] R. Olfati-Saber and R.M. Murray. Consensus problmes in networks with switching topology and time-delays. *IEEE Transactions on Automatic Control*, 2004, 49(9): 1520 – 1533.

[9] M. Arack. Passivity as a design tool for group coordination. *IEEE Transactions on Automatic Control*, 2007, 52(8): 1380 – 1390.

[10] F. Chen, Z.Q. Chen, L.Y. Xiang, Z.X. Liu and Z.Z. Yuan. Reaching a consensus via pinning control. *Automatica*, 2009, 45: 1215 – 1220.

[11] Y.P. Tian and C.L Liu. Robust consensus of multi-agent systems with diverse input delays and asymmetric interconnection perturbations. *Automatica*, 2009, 45: 1347 – 1353.

[12] P. Lin, Y.M. Jia and L. Li. Distributed robust H_∞ Consensus control in directed networks of agents with time-delay. *Systems and Control Letters*, 2008, 57(8): 643 – 653.

[13] P.A. Bliman and G.F. Trecate. Average consensus problems in networks of agents with delayed communications. *Automatica*, 2008, 44: 1985 – 1995.

[14] D. J.Hill and J. Zhao. Global synchronization of complex dynamical networks with non-identical nodes, *Proceedings of the 47th IEEE Conference on Decision and Control*, Cancun, Mexico, Dec. 9-11, 2008, 817 – 822.

[15] J.C. Zhao, J.A. Lu and Q.J. Zhang. Pinning a complex dynamical network to a homogenous trajectory. *IEEE Transactions on Circuits and Systems-II: Express Briefs*, 2009, 56(6): 514 – 518.

[16] W. Wu, W.J. Zhou and T.P. Chen. Cluster Synchronization of Linearly Coupled Complex Networks Under Pinning Control. *IEEE Transactions on Circuits and Systems-I*, 2009, 56(4): 829 – 839.

[17] Y.Y. Wu, W. Wei, G.Y. Li and J. Xiang. Pinning Control of Uncertain Complex Networks to a Homogeneous Orbit. *IEEE Transactions on Circuits and Systems-II: Express Briefs*, 2009, 56(3): 235 – 239.

[18] Y.M. Moon, P. Park, W.H. Kwon, Y.S. Lee. Delay-dependent robust stabilization of uncertain state-delayed systems. *Int J. Control*, 2001, 74: 1447 – 1455.

[19] C.C. Hua, X.P. Guan and P. Shi. Decentralized robust model reference adaptive cotnrol for interconnected time-delay systems. *J.Comput.Appl.Math.*, 2006, 193: 383 – 396.