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Global Controlled Consensus of Multi-Agent Systems with Different Agent Dynamics and Time-Varying Communication Delay

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***Abstract*—This paper investigates the global bounded consensus problem of Networked Multi-Agent Systems exhibiting nonlinear, non-identical agent dynamics with communication time-varying delay. Globally bounded controlled consensus conditions based on pinning control method and adaptive pinning control method are derived. The proposed consensus criteria ensures that all agents eventually move along desired trajectories in terms of boundedness. The proposed controlled consensus criteria generalizes the case of identical agent dynamics to the case of non-identical agent dynamics, and many related results of other researches in this area can be viewed as special cases of the above results. We finally demonstrate the effectiveness of the theoretical results by means of a numerical simulation.**

I. Introduction

Networked Multi-Agent Systems (NMAS) has attracted many attention due to the broad applications of NMAS in many areas. How to design appropriate protocols and algorithms such that the set of agents can realize common objective, such as consensus, is a critical problem, especially for the case of unreliable information exchange and communication delays, and some relevant important contributions have been made in recent years [1], [2], [3], [4].

The consensus problem requires an agreement to be reached that depends on the state of all agents. The topic has been studied across many fields of science and engineering [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19], [20]. It is noted that the agent dynamics in most existing works are often restricted to linear and identical ones. Obviously, in practice, this is not always the case. The controlled consensus problem of NMAS with nonlinear agent dynamics and communication delay are more complicated and just a few results have been made [21], [22]. In addition, most research in consensus problems usually assume that the final consensus value to be a constant, which may not be the case in the sense that the information state of each agent may be dynamically evolving in time according to some inherent dynamics. It is interesting to study controlled consensus problems where the final consensus value evolves with time or as a function of environmental dynamics.

The behavior of the NMAS with non-identical agent dynamics is much more complicated than the identical case. Usually, no common equilibrium for all agents exists even if each agent has an equilibrium, neither does a consensus manifold exist in the classical sense. The NMAS with non-identical agent dynamics cannot be decoupled into a number of lower dimensional systems exactly like the identical-agent case. Yet, a NMAS with non-identical agents may still exhibit some kinds of consensus behaviors which are far from being fully understood. Certain reasonable and satisfactory boundedness of state motions errors between different agents can be taken as useful consensus properties. The present paper will focus on the global consensus problems of NMAS based on pinning control methods [23], [24], [25], [26], and the proposed controlled consensus property is formulated in terms of certain boundedness of state errors.

The rest of this paper is organized as follows. A controlled continuous-time NMAS model with communication time-delay is presented in Section II. The main results including pinning control and adaptive pinning control bounded consensus criterion are derived in Section III and IV respectively. Section V gives a numerical simulation example to verify the effectiveness of the proposed results, followed by conclusions in Section VI.

II. Problem description

Let $G = (\mathcal{V}, \mathcal{A})$ be a graph of order N consisting of a set of vertices $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$ and a set of edges $\mathcal{A} \subseteq \mathcal{V} \times \mathcal{V}$. An edge (v_j, v_i) in graph G means that agent v_i sends some information to agent v_j. The set of neighbors of agent v_i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{A}\}$.

We consider a NMAS consisting of N non-identical agents with communication delay:

$$\dot{x}_i = f_i(x_i) + c \sum_{j \in \mathcal{N}_i} a_{ij} \Gamma x_j(t - \tau(t)), i = 1, 2, \cdots, N, \quad (1)$$

where $x_i = (x_{i1}(t), x_{i2}(t), \cdots, x_{in}(t))^T \in R^n$ are the state variables of the agent v_i, $f_i(x_i) : R^n \to R^n$ are continuously differentiable mappings with Jacobian Df_i, representing the self-dynamics of the agent v_i, $c > 0$ denotes the coupling strength, $\Gamma = (\gamma_{ij}) \in R^{n\times n}$ is the inner coupling matrix, and where $\gamma_{ij} \neq 0$ means two connected agents are linked via their ith and jth state variables, respectively. The adjacency matrix $A = (a_{ij}) \in R^{N\times N}$ (which is symmetric and irreducible) represents the communication topology relation of the NMAS, and is defined by $a_{ij} = a_{ji} = 1(v_j \in \mathcal{N}_i)$, $a_{ij} = 0(v_j \notin \mathcal{N}_i)$ and $a_{ii} = -\sum_{j\neq i} a_{ij}$. $\tau(t)$ is a time-varying coupling delay which reflects the reality that the agent v_i can't obtain information from agent v_j instantaneously.

The average dynamic of all agents is defined by the vector field $\bar{f}(x(t)) = \frac{1}{N}\sum_{k=1}^{N} f_k(x(t))$ with Jacobian $D\bar{f}_i(x(t))$.

The average state trajectory is chosen as the desired moving trajectory

$$s(t) = \frac{1}{N}\sum_{k=1}^{N} x_k(t). \tag{2}$$

We now discuss the problem of global consensus for the system (1). The consensus problem here will be depicted instead via certain boundedness of $x_i(t) - x_j(t), \forall i, j = 1, 2, \cdots, N$ as $t \to \infty$. This better reflects reality as it is impossible for NMAS (1) to achieve exact consensus. To address this case we will focus on making the states of all agents converge to a bounded set.

We denote $x(t)$, $s(t)$, $u(t)$, $e(t)$, $w(t)$, $d_i(t)$ and $V(w(t), t)$ as x, s, u, e, w, d_i and V respectively.

III. Linear feedback pinning controller

To achieve the goal, we apply the feedback control strategy on a small fraction δ $(0 < \delta \leq 1)$ of the agents in system (1). Suppose that nodes $i_1, i_2, \cdots, i_l$ are selected to be under control, where $l = [\delta N]$ stands for the smaller but nearest integer to the real number δN. This controlled NMAS can be described as

$$\begin{cases} \dot{x}_{i_k} = f_{i_k}(x_{i_k}) + c\sum_{j\in\mathcal{N}_i} a_{i_k j}\Gamma x_j(t-\tau(t)) \\ \qquad +u_{i_k}, \qquad 1 \leq k \leq l, \\ \dot{x}_{i_k} = f_{i_k}(x_{i_k}) + c\sum_{j\in\mathcal{N}_i} a_{i_k j}\Gamma x_j(t-\tau(t)), \\ \qquad\qquad l+1 \leq k \leq N. \end{cases} \tag{3}$$

The local linear negative feedback control law is chosen as follows:

$$\begin{cases} u_{i_k} = -d_{i_k}(x_{i_k} - s), & 1 \leq k \leq l, \\ u_{i_k} = 0, & l+1 \leq k \leq N, \end{cases} \tag{4}$$

where the feedback gain $d_{i_k} > 0$.

Combine (3) and (4) and rearrange the order of the nodes in the network. Let the first l nodes be controlled, and $e_i = x_i - s$, $i = 1, 2, \cdots, N$. It's obvious that $\frac{c}{N}\sum_{k=1}^{N}\sum_{j\in\mathcal{N}_i} a_{kj}\Gamma x_j(t-\tau(t)) = 0$ and $\sum_{i=1}^{N} e_i = 0$. Then by applying the Newton-Leibniz formula, error systems can be written as

$$\begin{cases} \dot{e}_i = D\bar{f}(s)e_i + c\sum_{j\in\mathcal{N}_i} a_{ij}\Gamma e_j(t-\tau(t)) \\ \quad + \int_0^1 (Df_i(s+\tau e_i) - D\bar{f}(s))e_i d\tau \\ \quad - \frac{1}{N}\sum_{k=1}^{N}\int_0^1 Df_k(s+\tau e_k)e_k d\tau \\ \quad + f_i(s) - \bar{f}(s) - d_i e_i, \quad 1 \leq i \leq l, \\ \dot{e}_i = D\bar{f}(s)e_i + c\sum_{j\in\mathcal{N}_i} a_{ij}\Gamma e_j(t-\tau(t)) \\ \quad + \int_0^1 (Df_i + \tau e_i) - D\bar{f}(s))e_i d\tau \\ \quad - \frac{1}{N}\sum_{k=1}^{N}\int_0^1 Df_k(s+\tau e_k)e_k d\tau \\ \quad + f_i(s) - \bar{f}(s), \quad l+1 \leq i \leq N. \end{cases} \tag{5}$$

The following work will focus on simplifying the error systems (5) by means of a series of transformations using a procedure similar to [22].

Define the following matrix

$$D = diag(D_1, D_2, \cdots, D_N) \in R^{nN\times nN},$$

where $D_i = diag\{-d_i, -d_i, \cdots, -d_i\} \in R^{n\times n}$.

Let $e = (e_1^T, e_2^T, \cdots, e_N^T)^T$, then (5) becomes

$$\dot{e} = \bar{\Sigma}(t)e + cA\otimes\Gamma e(t-\tau(t)) + I(t)e - \frac{1}{N}H(t)e + F(t), \tag{6}$$

where $I(t) = diag\{\int_0^1 (Df_1(s+\tau e_1) - D\bar{f}(s))d\tau \cdots \int_0^1 (Df_N(s+\tau e_N) - D\bar{f}(s))d\tau\}$, $\bar{\Sigma}(t) = I_N \otimes D\bar{f}(s) + D$, $H^T(t) = (H_1^T(t), \cdots, H_N^T(t))$, $H_i(t) = (\int_0^1 Df_1(s+\tau e_1)d\tau, \cdots, \int_0^1 Df_N(s+\tau e_N)d\tau)$, $F_i^T(t) = (f_1^T(s) - \bar{f}^T(s), \cdots, f_N^T(s) - \bar{f}^T(s))$.

Since A is symmetric and irreducible, according to [22], there exists a unitary matrix $\Phi = (\varphi_{ij})_{N\times N} = (\Phi_1, \Phi_2, \cdots, \Phi_N)$. This together with $w(t) = (\Phi^T \otimes I_n)e$ gives

$$\begin{aligned} \dot{w} = &(\Phi^T\otimes I_n)\bar{\Sigma}(t)(\Phi\otimes I_n)w \\ &+ (\Phi^T\otimes I_n)(cA\otimes\Gamma)(\Phi\otimes I_n)w(t-\tau(t)) \\ &+ (\Phi^T\otimes I_n)I(t)(\Phi\otimes I_n)w + (\Phi^T\otimes I_n)F(t) \\ &- \frac{1}{N}(\Phi^T\otimes I_n)H(t)(\Phi\otimes I_n)w. \end{aligned} \tag{7}$$

Note that $H(t) = \sqrt{N}\sum_{k=1}^{N}(\mathbf{0} \ \cdots \ \mathbf{0} \ \bar{\Phi}_k \ \mathbf{0} \ \cdots \ \mathbf{0}) \otimes \int_0^1 Df_k(s+\tau e_k)d\tau$, where $\bar{\Phi}_k$ stands for the matrix with its k-th column equal to Φ_1 and the remaining elements are zero. Then we have $\frac{1}{N}(\Phi^T\otimes I_n)H(t)(\Phi\otimes I_n) = \frac{1}{\sqrt{N}}\sum_{k=1}^{N}(\mathbf{0} \ \cdots \ \mathbf{0} \ I_k \ \mathbf{0} \ \cdots \ \mathbf{0})\otimes\int_0^1 Df_k(s+\tau e_k)d\tau(\Phi\otimes I_n)$, where I_k stands for the matrix with its k-th column equals $(1 \ 0 \ \cdots \ 0)^T$ and the remaining of its elements are zero.

Thus, a simple calculation gives $\frac{1}{N}(\Phi^T\otimes I_n)H(t)(\Phi\otimes I_n) = \frac{1}{\sqrt{N}}\sum_{k=1}^{N}\begin{pmatrix} \Upsilon_k & 0 \end{pmatrix}^T \otimes \int_0^1 Df_k(s(t)+\tau e_k(t))d\tau$, where $\Upsilon_k \in R^{1\times N}$ and $0 \in R^{(N-1)\times N}$. Therefore, $\dot{w} = \bar{\Sigma}(t)w + c\Lambda\otimes\Gamma w(t-\tau(t)) + (\Phi^T\otimes I_n)I(t)(\Phi\otimes I_n)w - \begin{pmatrix} * & 0 \end{pmatrix}^T w + (\Phi^T\otimes I_n)F(t)$. Since $w_1 \equiv 0$, we only need to consider $w_2, w_3, \cdots, w_N$. Rewriting in the component form we have

$$\begin{aligned} \dot{w}_i = &\Sigma_i(t)w_i + c\lambda_i\Gamma w_i(t-\tau(t)) + (\Phi_i^T\otimes I_n)F(t) \\ &+ (\Phi_i^T\otimes I_n)I(t)(\Phi\otimes I_n)w, \ i = 2, 3, \cdots, N, \end{aligned} \tag{8}$$

where $\Sigma_i = \bar{D}f(s) + D_i$.

So far, we have transferred the consensus problem of system (1) to the stability problem of the $N-1$ of $n-$dimensional systems.

Theorem 1 Suppose there exist positive definite matrices $P_i(t) \in \mathcal{PC}^1_{n\times n}$, Q_i and constants $\zeta > 0$, $\gamma \geq 0$, $a > 0$ and $b > 0$ such that

$$a\|x\|^2 \leq x^T P_i(t)x + \int_{t-\tau(t)}^{t} w_i^T(\alpha) Q_i w_i(\alpha) d\alpha \leq b\|x\|^2, \quad \forall t \in R^+,\ x \in R^n, i = 2, 3, \cdots, N, \tag{9}$$

$$\dot{P}_i(t) + P_i(t)\Sigma_i(t) + \Sigma_i^T(t)P_i(t) + Q_i + c^2\lambda_i^2 P_i(t)\Gamma Q_i^{-1}\Gamma^T P_i(t) + \zeta I \leq 0,\ i = 1, 2, \cdots, N, \tag{10}$$

$$\|I(t)\| \leq \gamma,\ i = 1, 2, \cdots, N. \tag{11}$$

Let

$$\mu(t) = \|F(t)\| \tag{12}$$

be bounded and

$$\beta = (\sum_{i=2}^{N} \|P_i(t)\|^2)^{\frac{1}{2}}, \tag{13}$$

if $\zeta > 2\gamma\beta$, then system (6) converges to the set

$$M = \{e | \|e\| \leq \frac{2b}{a} \frac{\beta \overline{lim}_{t\to\infty}\mu(t)}{\zeta - 2\gamma\beta - \delta}\}, \tag{14}$$

for any time-varying delay $\tau(t) > 0$, namely, $e(t) = x_i(t) - \frac{1}{N}\sum_{k=1}^{N} x_k(t) \to \Omega$ as $t \to \infty$, where $\delta > 0$ is any constant satisfying $\delta < \zeta - 2\gamma\beta$, Furthermore, the NMAS (1) achieves bounded consensus for any fixed time delay $\tau(t) > 0$, $0 \leq \dot{\tau}(t) \leq 1$.

Proof. Choose the following Lyapunov-Krasovskii functional as

$$V = \sum_{i=2}^{N} V_i, \tag{15}$$

$$V_i = w_i^T P_i(t) w_i + \int_{t-\tau(t)}^{t} w_i^T(\alpha) Q_i w_i(\alpha) d\alpha. \tag{16}$$

Differentiating (16) along the trajectory of (8) gives

$$\begin{aligned} \dot{V}_i = {} & w_i^T(\dot{P}_i(t) + P_i(t)\Sigma_i(t) + \Sigma_i^T(t)P_i(t) + Q_i)w_i \\ & + 2w_i^T P_i(t)(\Phi_i^T \otimes I_n)I(t)(\Phi_i \otimes I_n)w \\ & + 2w_i^T P_i(t)(\Phi_i^T \otimes I_n)F(t) + 2w_i^T(c\lambda_i P_i(t)\Gamma)w_i(t-\tau(t)) \\ & - w_i^T(t-\tau(t))Q_i w_i(t-\tau(t)). \end{aligned} \tag{17}$$

Applying the Young Inequality to the equality (17) results in

$$\begin{aligned} \dot{V}_i \leq {} & w_i^T(\dot{P}_i(t) + P_i(t)\Sigma_i(t) + \Sigma_i^T(t)P_i(t) + Q_i \\ & + c^2\lambda_i^2 P_i(t)\Gamma Q_i^{-1}\Gamma^T P_i(t))w_i + 2w_i^T P_i(t)(\Phi_i^T \otimes I_n)F(t) \\ & + 2w_i^T P_i(t)(\Phi_i^T \otimes I_n)I(t)(\Phi \otimes I_n)w. \end{aligned} \tag{18}$$

Condition (10) implies that the first term on the right hand side of (18) satisfies

$$\begin{aligned} & w_i^T(\dot{P}_i(t) + P_i(t)\Sigma_i(t) + \Sigma_i^T(t)P_i(t) + Q_i \\ & \quad + c^2\lambda_i^2 P_i(t)\Gamma Q_i^{-1}\Gamma^T P_i(t))w_i \leq -\zeta\|w_i\|^2. \end{aligned} \tag{19}$$

The second term on the right hand side of (18) satisfies

$$2w_i^T P_i(t)(\Phi_i^T \otimes I_n)F(t) \leq 2\mu(t)\|P_i(t)\|\|w_i\|. \tag{20}$$

Applying condition (11) we know the third term on the right hand side of (18) satisfies

$$2w_i^T P_i(t)(\Phi_i^T \otimes I_n)I(t)(\Phi_i \otimes I_n)w \leq 2\gamma\|P_i(t)\|\|w_i\|\|w\|. \tag{21}$$

Since $V = \sum_{i=2}^{N} V_i$, we have

$$\begin{aligned} \dot{V} &= \sum_{i=2}^{N} \dot{V}_i \\ &= -\zeta\|w\|^2 + 2(\gamma\|w\| + \mu(t))\sum_{i=2}^{N}\|w_i\|\|P_i(t)\| \\ &\leq -\zeta\|w\|^2 + 2(\gamma\|w\| + \mu(t))\|w\|(\sum_{i=2}^{N}\|P_i(t)\|^2)^{\frac{1}{2}} \\ &= \|w\|((2\gamma\beta - \zeta)\|w\| + 2\beta\mu(t)). \end{aligned} \tag{22}$$

Thus, when

$$\|w\| \geq \frac{2\beta\mu(t)}{\zeta - 2\gamma\beta - \delta}, \tag{23}$$

we have

$$\dot{V} \leq -\delta\|w\|^2. \tag{24}$$

Applying the result in [22] completes the proof.

IV. Adaptive pinning controller

In this section, we will derive globally consensus criteria via direct adaptive pinning control method. Without loss of generality, we still assume that the first l agents are selected as pinned agents with the adaptive controllers:

$$\begin{cases} u_i = -d_i(x_i - s), & 1 \leq i \leq l, \\ \dot{d}_i = h_i e_i^T P_i(t) e_i, & \\ u_i = 0, & l+1 \leq i \leq N, \end{cases} \tag{25}$$

where constant $h_i > 0$ and positive definite matrix $P_i(t) \in R^{n\times n}$. Applying Newton-Leibniz formula, then the error N-

MAS can be rewritten as

$$
\begin{cases}
\dot{e}_i = D\bar{f}(s)e_i + c\sum_{j\in\mathcal{N}_i} a_{ij}\Gamma e_j(t-\tau(t)) \\
\qquad + \int_0^1 (Df_i(s+\tau e_i) - D\bar{f}(s))e_i d\tau \\
\qquad -\frac{1}{N}\sum_{k=1}^N \int_0^1 Df_k(s+\tau e_k)e_k d\tau \\
\qquad +f_i(s) - \bar{f}(s) - d_i e_i, \qquad 1 \le i \le l, \\
\dot{d}_i = h_i e_i^T P_i(t) e_i, \\
\dot{e}_i = D\bar{f}(s)e_i + c\sum_{j\in\mathcal{N}_i} a_{ij}\Gamma e_j(t-\tau(t)) \\
\qquad + \int_0^1 (Df_i(s+\tau e_i) - D\bar{f}(s))e_i d\tau \\
\qquad -\frac{1}{N}\sum_{k=1}^N \int_0^1 Df_k(s+\tau e_k)e_k d\tau \\
\qquad +f_i(s) - \bar{f}(s), \qquad l+1 \le i \le N.
\end{cases}
\tag{26}
$$

Repeating a similar procedure to the previous subsection, the controlled consensus problem of system (1) is equivalent to the stability problem of the following $N-1$ of n-dimensional systems.

$$
\begin{cases}
\dot{w}_i = D\bar{f}(s(t))w_i - d_i w_i + c\lambda_i\Gamma w_i(t-\tau(t)) \\
\qquad +(\Phi_i^T \otimes I_n)I(t)(\Phi\otimes I_n)w \\
\qquad +(\Phi_i^T \otimes I_n)F(t), \qquad 2 \le i \le l, \\
\dot{d}_i = h_i w_i^T P_i(t) w_i, \\
\dot{w}_i = D\bar{f}(s)w_i + c\lambda_i\Gamma w_i(t-\tau(t)) \\
\qquad +(\Phi_i^T \otimes I_n)I(t)(\Phi\otimes I_n)w \\
\qquad +(\Phi_i^T \otimes I_n)F(t), \qquad l+1 \le i \le N,
\end{cases}
\tag{27}
$$

where w_i, w, Φ, Φ_i, $I(t)$ and $F(t)$ are the same as the previous subsection.

Theorem 2 Suppose there exist positive definite matrices $P_i(t) \in \mathcal{PC}^1_{n\times n}$, Q_i and constants $\bar{\zeta} > 0$, $\gamma \ge 0$, $a > 0$ and $b > 0$ such that

$$
a\|x\|^2 \le x_i^T P_i(t)x_i + \int_{t-\tau(t)}^t x_i^T(\alpha)Q_i x_i(\alpha)d\alpha + \frac{(d_i-d)^2}{h_i} \le b\|x\|^2, \forall t \in R^+,\ x\in R^n, i = 2,3,\cdots,N,
\tag{28}
$$

$$
\dot{P}_i(t) + P_i(t)D\bar{f}(s) + (D\bar{f}(s))^T P_i(t) + Q_i - 2dP_i(t) + c^2\lambda_i^2 P_i(t)\Gamma Q_i^{-1}\Gamma^T P_i(t) + \bar{\zeta} I \le 0,\ i = 1,2,\cdots,N,
\tag{29}
$$

(11) and $\bar{\zeta} > 2\gamma\beta$ are satisfied, then the system (6) converges to the set (14) for any time-varying delay $\tau(t) > 0$, where $\mu(t)$ and β are the same as in (12) and (13) respectively, $\bar{\delta} > 0$ is any constant satisfying $\bar{\delta} < \bar{\zeta} - 2\gamma\beta$, and then the NMAS (1) achieves bounded consensus for any fixed time delay $\tau(t) > 0$, $0 \le \dot{\tau}(t) \le 1$.

Proof. Construct the following Lyapunov-Krasovskii functional as

$$
V = \sum_{i=2}^N V_i + \sum_{i=2}^l \frac{(d_i-d)^2}{h_i},
\tag{30}
$$

where

$$
\begin{cases}
V_i = w_i^T P_i(t) w_i + \int_{t-\tau(t)}^t w_i^T(\alpha)Q_i w_i(\alpha)d\alpha \\
\qquad + \frac{(d_i-d)^2}{h_i}, \qquad 2 \le i \le l, \\
V_i = w_i^T P_i(t) w_i + \int_{t-\tau(t)}^t w_i^T(\alpha)Q_i w_i(\alpha)d\alpha, \\
\qquad\qquad l+1 \le i \le N,
\end{cases}
\tag{31}
$$

where d is a positive constant to be determined.

Differentiating (31) along the trajectory of (27) gives

$$
\begin{aligned}
\dot{V}_i &= w_i^T(\dot{P}_i(t) + P_i(t)D\bar{f}(s) + (D\bar{f}(s))^T P_i(t) + Q_i \\
&- 2dP_i(t))w_i + 2w_i^T P_i(t)(\Phi_i^T\otimes I_n)I(t)(\Phi_i\otimes I_n)w \\
&+ 2w_i^T P_i(t)(\Phi_i^T\otimes I_n)F(t) + 2w_i^T(c\lambda_i P_i(t)\Gamma)w_i(t-\tau(t)) \\
&- w_i^T(t-\tau(t))Q_i w_i(t-\tau(t)).
\end{aligned}
\tag{32}
$$

The remaining part of the proof is similar to that of Theorem 1, so is therefore omitted here. This completes the proof.

V. Examples

To demonstrate the theoretical results obtained above, we construct a NMAS consisting of 12 agents described as follows

$$
\dot{x}_i(t) = f_i(x_i(t)) + c\sum_{j\in\mathcal{N}_i} a_{ij}\Gamma x_j(t-\tau(t)),
\tag{33}
$$

where $f_i(x_i(t)) = B_i x_i(t) + g(x_i(t))$, $B_i(i = 1,2,\cdots,6)$ and $B_i(i = 7,8,\cdots,12)$ are chosen as follows:

$$
\begin{pmatrix}
-10+0.1\times(i-1) & 10-0.1\times(i-1) & 0 \\
1 & -1 & 1 \\
0 & -15-0.1\times(i-1) & 0
\end{pmatrix},
$$

$$
\begin{pmatrix}
-10-0.1\times(i-6) & 10+0.1\times(i-6) & 0 \\
1 & -1 & 1 \\
0 & -15+0.1\times(i-6) & 0
\end{pmatrix},
$$

and

$$
g(x_i(t)) = (-9.5sin(\frac{\pi x_{i1}(t)}{3.2} + \pi)\ 0\ 0)^T, \quad i = 1,2,\cdots,12.
$$

The communication coupling matrix $C = (C_1^T C_2^T \cdots C_{12}^T)$, $C_1 = (-8\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 0)$, $C_2 = (1\ -8\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 0)$, $C_3 = (1\ 1\ -7\ 1\ 0\ 0\ 0\ 1\ 0\ 1\ 1\ 1)$, $C_4 = (0\ 1\ 1\ -6\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 1)$, $C_5 = (1\ 1\ 0\ 0\ -6\ 0\ 1\ 1\ 1\ 1\ 0\ 0)$, $C_6 = (1\ 0\ 0\ 1\ 0\ -5\ 1\ 0\ 1\ 1\ 0)$, $C_7 = (1\ 1\ 0\ 1\ 1\ 1\ -7\ 1\ 0\ 1\ 0)$, $C_8 = (0\ 0\ 1\ 0\ 1\ 0\ 1\ -6\ 0\ 1\ 1\ 1)$, $C_9 = (1\ 1\ 0\ 0\ 1\ 1\ 0\ 0\ -7\ 1\ 1\ 1)$, $C_{10} = (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ -10\ 1)$, $C_{11} = (1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ -7\ 1)$, $C_{12} = (0\ 0\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 0\ 1\ -5)$. $\Gamma = diag\{2,2,2\}$, respectively, where the matrix A is produced by means of the Scale-Free network program.

Design the following controllers

$$
\begin{cases}
u_{i_k} = -d_{i_k}(x_{i_k}(t) - s(t)), & i_k = 1,2 \text{ and } 10, \\
u_{i_k} = 0, & \text{else},
\end{cases}
$$

with $d_1 = 0.5$, $d_2 = 0.5$, $d_{10} = 0.5$ and

$$
\begin{cases}
u_{i_k} = -d_{i_k}(t)(x_{i_k}(t) - s(t)), & i_k = 1,2 \text{ and } 10, \\
\dot{d}_{i_k}(t) = h_{i_k} e_{i_k}^T P_{i_k}(t) e_{i_k}, \\
u_{i_k} = 0, & \text{else},
\end{cases}
$$

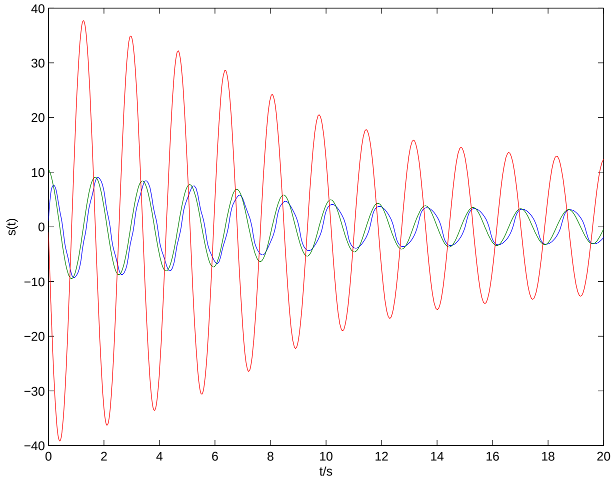

Fig.1. Desired agent dynamics under pinning control.

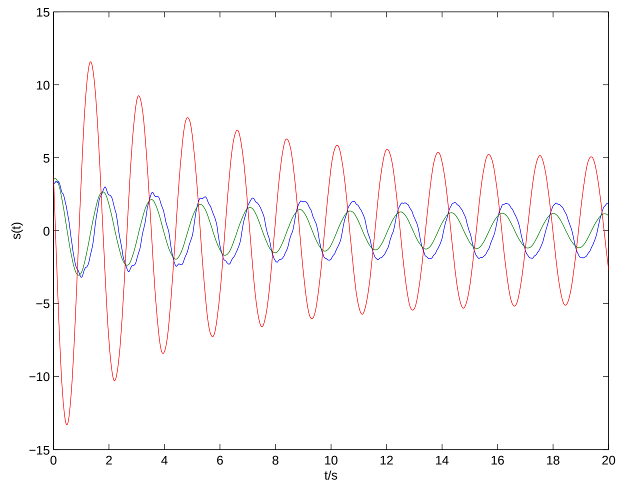

Fig.2. Desired agent dynamics under adaptive pinning control.

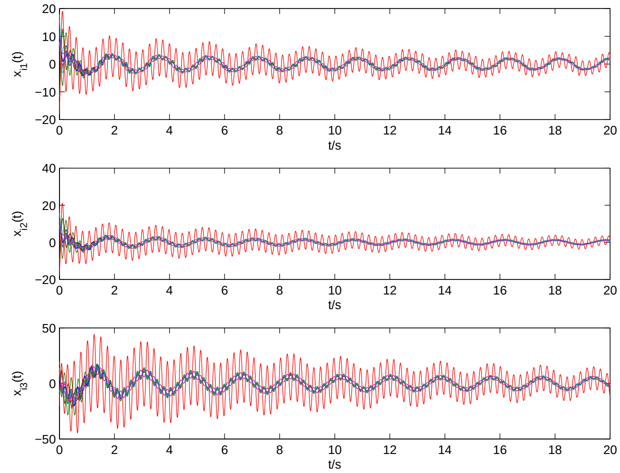

Fig.3. All agent dynamics under pinning control.

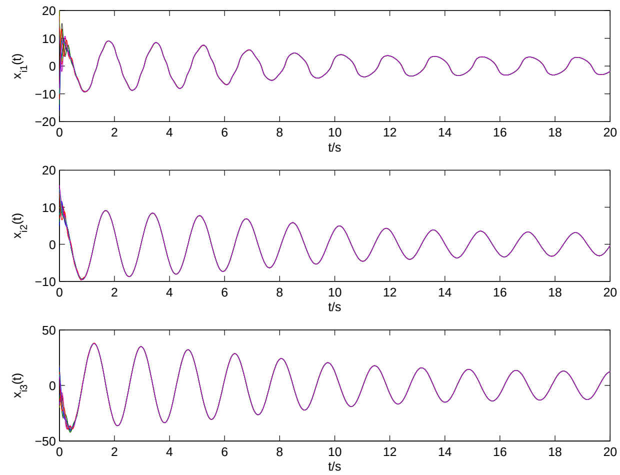

Fig.4. All agent dynamics under adaptive pinning control.

with $h_1 = 0.1$, $h_2 = 0.2$, $h_{10} = 0.3$, $s(t)$ can then be evaluated by simulation.

Given the initial values of 12 agents as $(10\ 5\ \ -10)^T$, $(12\ 6\ \ -12)^T$, $(14\ 7\ \ -14)^T$, $(16\ 8\ \ -16)^T$, $(18\ 9\ \ -18)^T$, $(20\ 10\ -20)^T$, $(-18\ 11\ 18)^T$, $(-16\ 12\ 16)^T$, $(-14\ 13\ 14)^T$, $(-12\ 14\ 12)^T$, $(-10\ 15\ 10)^T$, $(-8\ 16\ 8)^T$ respectively and $P_{i_k}(t) = I_3$, $d_1(0) = 1$, $d_2(0) = 1$, $d_{10}(0) = 1$ and $\tau(t) = \frac{\pi}{2} + arctan(t)$. The conditions of Theorem 1 and Theorem 2 are satisfied readily. Bounded consensus of the NMAS is achieved for any time varying delay satisfying $0 < \tau \leq \frac{\pi}{2} + arctan(t)$. Simulation results are depicted in Fig.1 to Fig.8 for $\tau(t) = \frac{\pi}{2} + arctan(t)$ and $c = 1$.

VI. Conclusion

In this paper, we've investigated the controlled consensus problems of NMAS with different agent dynamics. The derived criteria are verified via theoretical analysis and numerical simulation. The consensus for the NMAS is achieved based on pinning control and adaptive pinning control methods. Many related results for the case of identical agent dynamics have been viewed as the special cases of the proposed results. However, it should be noted that the conditions are still restrictive and the time-varying delay is chosen as fixed case. Further investigations will focus on relaxing these limitations and more generalized cases.

References

[1] Sandro Zampieri. Trends in networked control systems. *Proceedings of 17th World Congress of IFAC*, Seoul, Korea, July 6-11, 2008, pp. 2886 – 2894.

[2] J.P. Desai, J.P. Ostrowski and V. Kumar. Modeling and control of formation of nonholonomic mobile robust. *IEEE Int. Trans. on Robotics and Automation*, 2001, 17(6): 905 – 908.

[3] M. Porfiri, D.G. Roberson and D.J. Stilwell. Tracking and formation control of multiple autonomous agents: a two-level consensus approach. *Automatica*, 2007, 43: 1318 – 1328.

[4] J. Cortés. Global formation-shape stabilization of relative sensing networks. *2009 American Control Systems*, Hyatt Regency Riverfront, St. Louis, USA, June 10-12, 2009, pp. 1460 – 1465.

[5] F. Xiao and L. Wang. Asynchronous consensus in continuous-time multi-agent systems with switching topology and time-varying delays. *IEEE Transactions on Automatic Control*, 2008, 53(8): 1804 – 1816.

[6] T. Li and J.F. Zhang. Decentralized tracking-type games for multi-agent systems with coupled ARX models: asymptotic Nash equilibria. *Automatica*, 2008, 44: 713 – 725.

[7] E.S. Kazerooni and K. Khorasani. Optimal consensus algorithms for cooperative team of agents subject to partial information. *Automatica*, 2008, 44: 2766 – 2777.

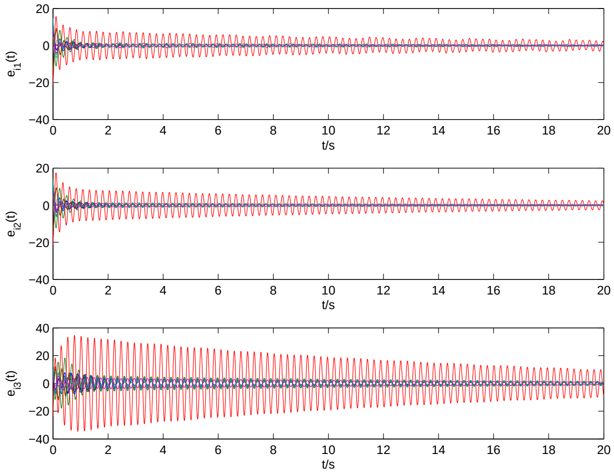

Fig.5. All agent error dynamics under pinning control.

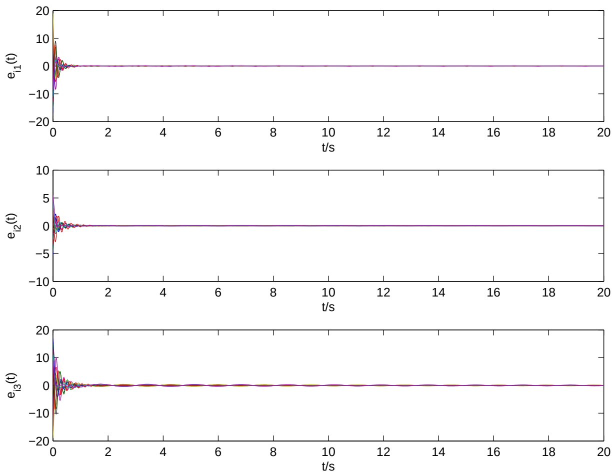

Fig.6. All agent error dynamics under adaptive pinning control.

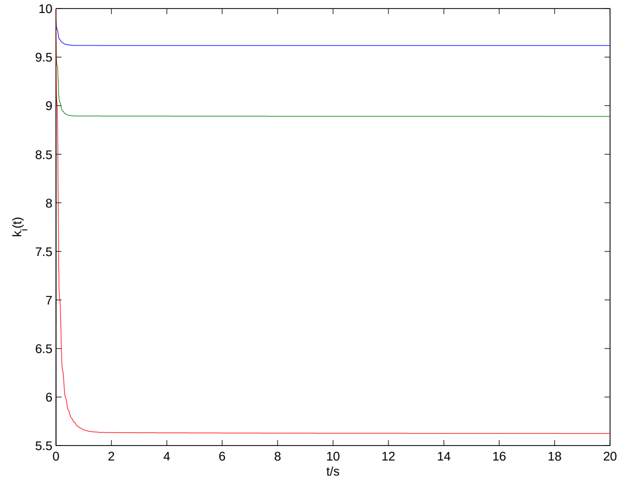

Fig.7. Adaptive gain curves.

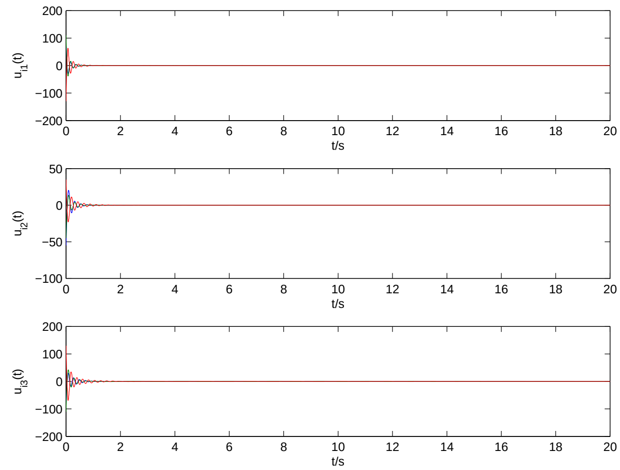

Fig.8. Adaptive pinning controllers curves.

[8] Z.K. Li, Z.S. Duan, G.R. Chen and L. Huang. Consensus of multiagent systems and synchronization of complex networks: a unified viewpoint. *IEEE Transactions on Circuits and Systems-I*, 2010, 57(1): 213 – 224.

[9] R. Olfati-Saber and R.M. Murray. Consensus problmes in networks with switching topology and time-delays. *IEEE Transactions on Automatic Control*, 2004, 49(9): 1520 – 1533.

[10] M. Arack. Passivity as a design tool for group coordination. *IEEE Transactions on Automatic Control*, 2007, 52(8): 1380 – 1390.

[11] F. Chen, Z.Q. Chen, L.Y. Xiang, Z.X. Liu and Z.Z. Yuan. Reaching a consensus via pinning control. *Automatica*, 2009, 45: 1215 – 1220.

[12] Y.P. Tian and C.L Liu. Robust consensus of multi-agent systems with diverse input delays and asymmetric interconnection perturbations. *Automatica*, 2009, 45: 1347 – 1353.

[13] P. Lin, Y.M. Jia and L. Li. Distributed robust H_∞ Consensus control in directed networks of agents with time-delay. *Systems and Control Letters*, 2008, 57(8): 643 – 653.

[14] W. Ren, R.W. Beard. Formation feedback control for multiple spacecraft via virtual structures. *IEE Proc. Control Theory and Applications*, 2004, 151(3): 357 – 368.

[15] H. Yamaguchi, T. Arai and G. Beni. A distributed control scheme for multiple robotic vehicles to make group formations. *Robitics and Autonomous Systems*, 2001, 36: 125-147.

[16] Z.P. Wu, Z.H. Guan, T. Li. Group motion control of multi-agent systems based on complex network, *Proceedings of the 26th Chinese Control Conference*, Hunan, China, July 26-31, 2007, pp: 63 – 67.

[17] J.Q. Lu, D.W.C. Ho and J. Kurths. Consensus over directed static networks with arbitrary finite communication delays. *Physical Review E*, 2009, 80(6): 066121.

[18] J. Cortes. Distributed algorithms for reaching consensus on general functions. *Automatica*, 2008, 44(3): 726 – 737.

[19] J. Zhao, D. J. Hill and T. Liu. Synchronization of Dynamical Networks With Nonidentical Nodes: Criteria and Control. *IEEE Tran.Circuits and Syst.I*, 2011, 58(3): 584 – 594.

[20] P.A. Bliman and G.F. Trecate. Average consensus problems in networks of agents with delayed communications. *Automatica*, 2008, 44: 1985 – 1995.

[21] Z.S Duan and G.R. Chen. Global Robust Stability and Synchronization of Networks With Lorenz-Type Nodes. *IEEE Transactions on Circuits and Systems-II: Express Briefs*, 2009, 56(8): 679 – 683.

[22] D. J.Hill and J. Zhao. Global synchronization of complex dynamical networks with non-identical nodes. *Proceedings of the 47th IEEE Conference on Decision and Control*, Cancun, Mexico, Dec. 9-11, 2008, 817 – 822.

[23] J.C. Zhao, J.A. Lu and Q.J. Zhang. Pinning a complex dynamical network to a homogenous trajectory. *IEEE Transactions on Circuits and Systems-II: Express Briefs*, 2009, 56(6): 514 – 518.

[24] X. Li, X.F. Wang and G.R. Chen. Pinning a complex dynamical network to its equilibrium. *IEEE Tran.Circuits and Syst.I*,2004, 51: 2074 – 2087.

[25] W. Wu, W.J. Zhou and T.P. Chen. Cluster Synchronization of Linearly Coupled Complex Networks Under Pinning Control. *IEEE Transactions on Circuits and Systems-I*, 2009, 56(4): 829 – 839.

[26] Y.Y. Wu, W. Wei, G.Y. Li and J. Xiang. Pinning Control of Uncertain Complex Networks to a Homogeneous Orbit. *IEEE Transactions on Circuits and Systems-II: Express Briefs*, 2009, 56(3): 235 – 239.