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Stochastic Neural Network Control of Rigid Robot Manipulator with Passive Last Joint

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***Abstract*—Stochastic adaptive control of a manipulator with a passive joint which has neither an actuator nor a holding brake is investigated. Aiming at shaping the controlled manipulators dynamics to be of minimized motion tracking errors and joint accelerations, we employ the linear quadratic regulation (LQR) optimization technique to obtain an optimal reference model. Adaptive neural network (NN) control has been developed to ensure the reference model can be matched in finite time, in the presence of various uncertainties and stochastic noise. In addition, due to the stochastic noise, we transform the system equation to the Ito stochastic differential equation (SDE) form and then use the Ito formula to deal with the stochastic terms of the systems. Simulation studies show the effectiveness of the planned trajectory and the feedback control laws.**

Key Words – Stochastic NN control, optimization, LQR, model reference control

I. Introduction

Underactuated robots have received considerable research attention in the last two decades ([1]-[7]). In contrast to conventional robot for which each joint has one actuator and its degree of freedom equals the number of actuators, an underactuated robot has passive joints equipped with no actuators. The underactuation structure make possible for the robots to reduce the weight, energy consumption, and cost of manipulators, which can be applied to the tasks involving an impact, e.g., hitting or hammering an object, will be useful since the impact causes no damage to the joint actuators. It can also contribute to fault tolerance of fully-actuated manipulators in case some of the joint actuators fail.

Though the passive joints are not actuated but they can be controlled by using the dynamic coupling with the active joints, i.e., these passive joints can be indirectly driven by other active joints. The zero torque at the passive joints results in a second-order nonholonomic constraint. This method allows the control of more joints than actuators. In robotics, nonholonomic constraints formulated as nonintegrable differential equations containing time-derivatives of generalized coordinates (velocity, acceleration etc.) are mainly studied. Such constraints include the following: 1) Kinematic constraints which geometrically restrict the direction of mobility; 2) Dynamic constraints due to dynamic balance at passive degrees of freedom where no force or torque is applied. Wheeled vehicles [1], rolling contact between objects [2], trailers [3], [4], and manipulators with nonholonomic gears [5] are mechanical systems which have constraints of the former type. Constraints on space robots [6], [7] belong to the latter. These systems commonly have fewer control inputs than the number of generalized coordinates. Therefore, it is necessary to combine the limited number of inputs skillfully in order to control all the coordinates. So how to efficiently control this kind of nonholonomic systems becomes an interesting research area.

In this paper we consider a n-joints under-actuated system with passive last joint and use the LQR optimization approach to derive a reference model for the first $n-1$ joints subsystems, which guarantees motion tracking and achieves the minimized moving accelerations. High order neural networks (HONNs) have been employed to design the adaptive control in order to make the controlled dynamics to match the reference model dynamics in finite time. Instead of leaving the unactuated joint dynamics uncontrolled, a reference trajectory for the last joint is designed to indirectly affect the movements such that the desired motion can be achieved. HONNs also have been employed to construct a reference trajectory generator of the last joint.

II. Stochastic finite-time attractiveness

Consider the following stochastic nonlinear system with the Ito SDE form

$$dx = f(x)dt + g(x)dw \tag{1}$$

where $x \in \mathbf{R}^n$ is the state, w is an independent r-dimensional standard Wiener process defined on the complete probability space $(\Omega, \mathscr{F}, \mathbf{P})$, the Borel measurable functions $f : \mathbf{R}^n \to \mathbf{R}^n$ and $g : \mathbf{R}^n \to \mathbf{R}^{n\times r}$ are locally Lipschitz continuous with $f(0) = 0$ and $g(0) = 0$. Without loss of generality, we use 0 and x_0 to denote the initial time and the initial state of the system. The solution of system (1) with the initial state x_0 is denoted by $x(t;x_0)$.

The following two definitions come from [8], which will be used to express our system stability.

Definition 1: For system (1), define $T(x_0,w) = \inf\{T \geq 0 : x(t;x_0) = 0, \forall t \geq T\}$, which is called the stochastic settling time function.

Definition 2: The equilibrium $x = 0$ of system (1) is globally stochastically finite-time attractive, if for $x_0 \in \mathbf{R}^n$, the following conditions hold.

(i) Stochastic settling time function $T_0(x_0,w)$ exists with probability one.

(ii) Provided that $T_0(x_0,w)$ exists, then $E[T_0(x_0,w)] < \infty$.

For a given $V(x) \in \mathbf{C}^2$, the infinitesimal generator $\mathscr{L}$ with regard to (1) is defined by

$$\mathscr{L}V(x) = \frac{\partial V}{\partial x} f(x) + \frac{1}{2} Tr\left\{ g^T(x) \frac{\partial^2 V}{\partial x^2} g(x) \right\}. \quad (2)$$

We now state the stochastic Lyapunov lemma for stochastic finite-time attractiveness, which is a combination of Corollary 1 in [8], Theorem 3.1 in [9] and Revision of Corollary 1 in [10], and the proof is omitted here for simplicity.

Lemma 1: Assume that system (1) admits a unique solution. If there exists a $\mathbf{C}^2$ function $V: \mathbf{R}^n \to \mathbf{R}_+$ and class $\mathbf{K}_\infty$ function α_1, α_2, positive numbers $c > 0$ and $0 < \gamma < 1$, such that for $\forall x \in \mathbf{R}^n$ and $t \geq 0$,

$$\begin{array}{l} \alpha_1(\|x\|) \leq V(x) \leq \alpha_2(\|x\|), \\ \mathscr{L}V(x) \leq -c(V(x))^\gamma, \end{array}$$

then the equilibrium $x = 0$ of system (1) is stochastically finite-time attractive, and $E[T_0(x_0,w)] \leq \frac{(V(x_0))^{1-\gamma}}{c(1-\gamma)}$, which implies $T_0(x_0,w) < +\infty$ a.s.

III. Dynamics of Under-actuated Robot Manipulator

Partition of generalized coordinate vector q as $q = [q_a^T,\ q_b^T]^T$ with $q_a = [q_1, q_2, \ldots, q_{n-1}]^T$ and $q_b = q_n$ such that

$$q_a = I_0 q, \quad I_0 = [I_{[n-1,n-1]}, 0_{[n-1,1]}] \in R^{(n-1)\times n} \quad (3)$$

and $\tau_a = [\tau_1, \tau_2, \ldots, \tau_{n-1}]^T$. The dynamics model of robot manipulator with passive last joint is described as follows:

$$\begin{bmatrix} M_a & M_{ab} \\ M_{ba} & M_b \end{bmatrix} \begin{bmatrix} \ddot{q}_a \\ \ddot{q}_b \end{bmatrix} + \begin{bmatrix} C_a & C_{ab} \\ C_{ba} & C_b \end{bmatrix} \begin{bmatrix} \dot{q}_a \\ \dot{q}_b \end{bmatrix} + \begin{bmatrix} g_a \\ g_b \end{bmatrix} + \begin{bmatrix} \dot{w}_a \\ \dot{w}_b \end{bmatrix} = \begin{bmatrix} \tau_a \\ 0 \end{bmatrix} \quad (4)$$

Define

$$M = \begin{bmatrix} M_a & M_{ab} \\ M_{ba} & M_b \end{bmatrix}, g = \begin{bmatrix} g_a \\ g_b \end{bmatrix},$$
$$C = \begin{bmatrix} C_a & C_{ab} \\ C_{ba} & C_b \end{bmatrix}, \dot{w} = \begin{bmatrix} \dot{w}_a \\ \dot{w}_b \end{bmatrix}$$

Then (4) can be written in a compact form as

$$M\ddot{q} + C\dot{q} + g + \dot{w} = I_0^T \tau_a \quad (5)$$

where w is an n-dimensional independent standard Wiener process. The following property are well known for the Lagrange-Euler formulation of robotic dynamics:

Property 1: The matrix M is symmetric and positive definite.

Therefore, the blocks M_a and M_b are also invertable and the inverse of matrix M exist and is

$$M^{-1} = \begin{bmatrix} S_b^{-1} & -M_a^{-1} M_{ab} S_a^{-1} \\ M_b^{-1} M_{ba} S_b^{-1} & S_a^{-1} \end{bmatrix} \quad (6)$$

where S_a and S_b are Schur complements of M_a and M_b, respectively, defined as $S_a = M_b - M_{ba} M_a^{-1} M_{ab}, S_b = M_a - M_{ab} M_b^{-1} M_{ba}$. Multiplying $I_0 M^{-1}$ on both sides of (5) gives us

$$\ddot{q}_a + I_0 M^{-1} C\dot{q} + I_0 M^{-1} g + I_0 M^{-1} \dot{w} = I_0 M^{-1} I_0^T \tau_a = S_b^{-1} \tau_a \quad (7)$$

Then, multiplying S_b on both sides of the above equation, we have

$$S_b \ddot{q}_a + S_b I_0 M^{-1} C\dot{q} + S_b I_0 M^{-1} g + S_b I_0 M^{-1} \dot{w}_a = \tau_a \quad (8)$$

Define $\mathscr{M} \triangleq S_b \in \mathbf{R}^{(n-1)\times(n-1)}$, $\mathscr{C} \triangleq S_b I_0 M^{-1} C = [\mathscr{C}_a, \mathscr{C}_b] \in \mathbf{R}^{(n-1)\times n}$ with $\mathscr{C}_a \in \mathbf{R}^{(n-1)\times(n-1)}$, $\mathscr{C}_b \in \mathbf{R}^{(n-1)\times 1}$, and $\mathscr{G} = S_b I_0 M^{-1} g, \mathscr{S} = S_b I_0 M^{-1}$, then, we have q_a-subsystems as follows

$$\Sigma_{q_a}: \mathscr{M}\ddot{q}_a + \mathscr{C}_a \dot{q}_a + \mathscr{C}_b \dot{q}_b = \tau_a - \mathscr{G} - \mathscr{S}\dot{w}_a \quad (9)$$

At the same time, we obtain the q_b-subsystem as follows:

$$\Sigma_{q_b}: M_b \ddot{q}_b + C_b \dot{q}_b + g_b + \dot{w}_b + M_{ba} \ddot{q}_a + C_{ba} \dot{q}_a = 0 \quad (10)$$

Remark 1: It should be mentioned that due to the unknown system parameters in the above dynamics formulation, the dynamics matrices $\mathscr{M}, \mathscr{S}$ are actually unknown for control design.However, we still can estimate their regions. So we assume there exist the positive constants $\underline{m}$ and $\bar{m}$ such that $\underline{m} \leq |\mathscr{M}^{-1}| \leq \bar{m}$. But it should be mentioned that these bounds are only used in the stability analysis, and their exact values need not to be known in our controller design.

IV. Control of subsystem Σ_a

A. Subsystem dynamics and optimal reference model

For convenience, defining $\bar{q}_a = [q_a^T,\ \dot{q}_a^T]^T$, $\bar{q}_b = [q_b,\ \dot{q}_b]^T$, and $\bar{q} = [q_a^T,\ q_b,\ \dot{q}_a^T,\ \dot{q}_b]^T$, we rewrite (9) as

$$\dot{\bar{q}}_a = A_a \bar{q}_a + A_b \bar{q}_b + B\mathscr{M}^{-1}(\tau_a - \mathscr{G}) - B\mathscr{M}^{-1}\mathscr{S}\dot{w}_a \quad (11)$$

where

$$\begin{array}{rcl} A_a & = & \begin{bmatrix} 0_{[n-1,n-1]} & I_{[n-1,n-1]} \\ 0_{[n-1,n-1]} & -\mathscr{M}^{-1}\mathscr{C}_a \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ A_{a1} & A_{a2} \end{bmatrix} \\ A_b & = & \begin{bmatrix} 0_{[n-1,1]} & 0_{[n-1,1]} \\ 0_{[n-1,1]} & -\mathscr{M}^{-1}\mathscr{C}_b \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ A_{b1} & A_{b2} \end{bmatrix} \\ B & = & \begin{bmatrix} 0_{[n-1,n-1]}, I_{[n-1,n-1]} \end{bmatrix}^T \end{array} \quad (12)$$

For clarity, here and hereafter, the argument $\bar{q}$ of $A_a, A_b, \mathscr{M}, \mathscr{S}$ and $\mathscr{G}$ is omitted.

The control objective is to control the subsystem dynamics (11) to follow a given reference model

$$\dot{\bar{q}}_m = A_m \bar{q}_m + BM_d^{-1} r_m \quad (13)$$

where $\bar{q}_m \in R^{2(n-1)\times 2(n-1)}$ is the desired response of the system,

$$\begin{array}{rcl} A_m & = & \begin{bmatrix} 0_{[n-1,n-1]} & I_{[n-1,n-1]} \\ A_{m1}(\bar{q}_m) & A_{m2}(\bar{q}_m) \end{bmatrix} \\ & = & \begin{bmatrix} 0_{[n-1,n-1]} & I_{[n-1,n-1]} \\ -M_d^{-1} K_d & -M_d^{-1} C_d \end{bmatrix} \in \mathbf{R}^{2(n-1)\times 2(n-1)} \quad (14) \\ r_m & = & -F_\eta(q_d, \dot{q}_d), \quad \bar{q}_m = [q_m^T, \dot{q}_m^T]^T \end{array}$$

In order to choose the optimal values of the reference model parameters, we introduce the following performance index:

$$P_I = \int_{t_0}^{t_f} (e_m^T Q e_m + \ddot{q}_m^T M_d \ddot{q}_m) dt. \tag{15}$$

where $Q = \begin{bmatrix} q_1 & & 0 \\ & \ddots & \\ 0 & & q_{n-1} \end{bmatrix}$, which minimizes both the motion tracking error $e_m = q_m - q_d$ and the joints' angular accelerations. In order to apply the LQR optimization technique[11], we rewrite the reference model (13) as

$$\dot{\bar{q}}_m = A_d \bar{q}_m + Bu \tag{16}$$

with

$$\begin{aligned} A_d &= \begin{bmatrix} 0_{[n-1,n-1]} & I_{[n-1,n-1]} \\ 0_{[n-1,n-1]} & 0_{[n-1,n-1]} \end{bmatrix}, \\ u &= -M_d^{-1}[K_d,\ C_d]\bar{q}_m - M_d^{-1} F_\eta(q_d, \dot{q}_d) \end{aligned} \tag{17}$$

Noting that $u = \ddot{q}_m$ and introducing $\bar{Q}$ defined as

$$\bar{Q} = \begin{bmatrix} Q & 0_{[n-1,n-1]} \\ 0_{[n-1,n-1]} & 0_{[n-1,n-1]} \end{bmatrix} \tag{18}$$

we can then rewrite the performance index (15) as

$$P_{\bar{I}} = \int_{t_0}^{t_f} \left((\bar{q}_m - \bar{q}_d)^T \bar{Q} (\bar{q}_m - \bar{q}_d) + u^T M_d u \right) dt, \tag{19}$$

where $\bar{q}_d = [q_d^T,\ \dot{q}_d^T]^T$. If we regard u as the control input to system (16), then the minimization of (19) subject to dynamics constraint (16) becomes a typical LQR control design problem, where the solution of u that minimizes (19) is

$$u = -M_d^{-1} B^T P \bar{q}_m - M_d^{-1} B^T s \tag{20}$$

where P is the solution of the following differential equation

$$-\dot{P} = P A_d + A_d^T P - P B M_d^{-1} B^T P + \bar{Q}, \quad P(t_f) = 0_{[2(n-1),2(n-1)]}$$

and s is the solution of the following differential equation

$$-\dot{s} = (A_d - B M_d^{-1} B^T P)^T s + \bar{Q}\bar{q}_d, \quad s(t_f) = 0_{[2(n-1)]} \tag{21}$$

Comparing equations (17) and (20), we can see that the matrices K_d and C_d can be calculated in the following manner:

$$[K_d, C_d] = B^T P, \quad F_\eta = B^T s \tag{22}$$

B. NN control and model matching

According to $\mathscr{M}$'s nonsingularity and from the state feedback control for linear systems, we conclude that there exist $K(\bar{q}) \in \mathbf{R}^{(n-1)\times 2(n-1)}, L(\bar{q}) \in \mathbf{R}^{(n-1)\times 2}, T(\bar{q}) \in \mathbf{R}^{(n-1)\times(n-1)}, G(\bar{q}) \in \mathbf{R}^{(n-1)\times 1}$ such that for the control law chosen by

$$\tau_a = G(\bar{q}) + K(\bar{q})\bar{q}_a + L(\bar{q})\bar{q}_b + T(\bar{q}) r_m, \tag{23}$$

the closed-loop system is the same as the reference model (13). By substituting the control law (23) into the system equation (11), the closed-loop system is given by

$$\begin{aligned} d\bar{q}_a = &\{[A_a + B\mathscr{M}^{-1}K(\bar{q})]\bar{q}_a + [A_b + B\mathscr{M}^{-1}L(\bar{q})]\bar{q}_b \\ &+ B\mathscr{M}^{-1}T(\bar{q})r_m + B\mathscr{M}^{-1}[G(\bar{q}) - \mathscr{G}]\}dt \\ &- B\mathscr{M}^{-1}\mathscr{S} dw_a \end{aligned} \tag{24}$$

Comparing it to match the reference model (13), we obtain

$$\begin{aligned} &A_a + B\mathscr{M}^{-1}K(\bar{q}) = A_m, & &\mathscr{M}^{-1}T(\bar{q}) = M_d^{-1} \\ &A_b + B\mathscr{M}^{-1}L(\bar{q}) = 0_{[2(n-1),2]}, & &G(\bar{q}) - \mathscr{G} = 0_{[n-1]} \end{aligned} \tag{25}$$

Then we have

$$K(\bar{q}) = \mathscr{M}([A_{m1}\ A_{m2}] - [A_{a1}\ \ A_{a2}]),\ G(\bar{q}) = \mathscr{G}, \tag{26}$$

$$L(\bar{q}) = -\mathscr{M}[A_{b1}\ A_{b2}],\ T(\bar{q}) = \mathscr{M}M_d^{-1} \tag{27}$$

Unfortunately, according to Remark 1, the dynamic matrices $A_a, A_b, \mathscr{M}, \mathscr{G}$ are not available during practical implementation, and then the exact values of the desired gains $K(\bar{q}), L(\bar{q}), T(\bar{q})$ and $G(\bar{q})$ are also unknown.

We can employ the HONNs [12] to approximate the controller gains as follows

$$K(\bar{q}) = K^*(\bar{q}) + \varepsilon_K,\ L(\bar{q}) = L^*(\bar{q}) + \varepsilon_L \tag{28}$$

$$T(\bar{q}) = T^*(\bar{q}) + \varepsilon_T,\ G(\bar{q}) = G^*(\bar{q}) + \varepsilon_G \tag{29}$$

with

$$K^*(\bar{q}) = [W_K^{\langle T \rangle}\langle\cdot\rangle S_K(\bar{q})], T^*(\bar{q}) = [W_T^{\langle T \rangle}\langle\cdot\rangle S_T(\bar{q})] \tag{30}$$

$$L^*(\bar{q}) = [W_L^{\langle T \rangle}\langle\cdot\rangle S_L(\bar{q})], G^*(\bar{q}) = [W_G^{\langle T \rangle}\langle\cdot\rangle S_G(\bar{q})] \tag{31}$$

where $\langle\rangle$ expresses matrix block-wise operator, defined in [12]. $W_{Ki,j}, W_{Li,k}, W_{Ti,s}, W_{Gi} \in \mathbf{R}^{l\times 1}$ are the NN ideal weights for $K_{i,j}(\bar{q}), L_{i,k}(\bar{q}), T_{i,s}(\bar{q}), G_i(\bar{q})$, respectively $(i = 1, \cdots, n-1; j = 1, \cdots, 2(n-1); k = 1, 2; s = 1, \cdots, n-1)$, l is the number of the neurons. $S_K(\bar{q}), S_L(\bar{q}), S_T(\bar{q}), S_G(\bar{q})$ are the outputs of the bounded basis functions, and $\varepsilon_K, \varepsilon_L, \varepsilon_T, \varepsilon_G$ are the NN approximation errors. For a fixed number of nodes, we know that $\|\varepsilon_K\|, \|\varepsilon_L\|, \|\varepsilon_T\|, \|\varepsilon_G\|$ are bounded, W_K, W_L, W_T, W_G are unknown constant parameters.

Consider the following NN based control law

$$\begin{aligned} \tau_a &= \hat{K}(\bar{q})\bar{q}_a + \hat{L}(\bar{q})\bar{q}_b + \hat{T}(\bar{q})r_m + \hat{G}(\bar{q}) + \tau_r \\ &= [\hat{W}_K^{\langle T \rangle}\langle\cdot\rangle S_K(\bar{q})]\bar{q}_a + [\hat{W}_L^{\langle T \rangle}\langle\cdot\rangle S_L(\bar{q})]\bar{q}_b \\ &\quad + [W_T^{\langle T \rangle}\langle\cdot\rangle S_T(\bar{q})]r_m + [W_G^{\langle T \rangle}\langle\cdot\rangle S_G(\bar{q})] + \tau_r \end{aligned} \tag{32}$$

where τ_r is a robust control term for closed-loop stability which will be defined later to compensate for the approximation errors of the NNs and to suppress the disturbaces.

Define

$$\begin{aligned} &e = \bar{q}_a - \bar{q}_{am},\quad \tilde{W}_K = \hat{W}_K - W_K, \\ &\tilde{W}_L = \hat{W}_L - W_L,\ \tilde{W}_T = \hat{W}_T - W_T,\ \tilde{W}_G = \hat{W}_G - W_G \end{aligned} \tag{33}$$

Substituting the control law (32) into the subsystem dynamics (11), using (33), applying the NN approximations (28)-(29), and recalling (25), we obtain the following error equation

$$\begin{aligned} de = &\Big\{ A_m e + B\mathscr{M}^{-1}(\tau_r - \varepsilon_K\bar{q}_a - \varepsilon_L\bar{q}_b - \varepsilon_T r_m - \varepsilon_G) \\ &+ B\mathscr{M}^{-1}[\tilde{W}_K^{\langle T \rangle}\langle\cdot\rangle S_K(\bar{q})]\bar{q}_a + B\mathscr{M}^{-1}[\tilde{W}_L^{\langle T \rangle}\langle\cdot\rangle S_L(\bar{q})]\bar{q}_b \\ &+ B\mathscr{M}^{-1}[\tilde{W}_T^{\langle T \rangle}\langle\cdot\rangle S_T(\bar{q})]r_m + B\mathscr{M}^{-1}[\tilde{W}_G^{\langle T \rangle}\langle\cdot\rangle S_G(\bar{q})] \Big\} dt \\ &- B\mathscr{M}^{-1}\mathscr{S} dw_a \end{aligned} \tag{34}$$

For stable A_m of the reference model, let P_m be the symmetric positive definite solution of the Lyapunov equation

$$P_m A_m + A_m^T P_m = -Q_m \tag{35}$$

where Q_m is symmetric positive definite.

The following theorem states the stability of the adaptive NN control.

Theorem 1: For the system (11), consider the NN based control laws (32). If the updating laws of the weights of the adaptive NNs are given by

$$\begin{aligned}(\dot{\hat{W}}_{Ki}^{\langle T\rangle})^T &= -\Gamma_{Ki}\langle\cdot\rangle S_{Ki}(\bar{q})\bar{q}_a(e^TP_me)e^TP_m(B)_i\\ (\dot{\hat{W}}_{Ti}^{\langle T\rangle})^T &= -\Gamma_{Ti}\langle\cdot\rangle S_{Ti}(\bar{q})r_m(e^TP_me)e^TP_m(B)_i\\ (\dot{\hat{W}}_{Li}^{\langle T\rangle})^T &= -\Gamma_{Li}\langle\cdot\rangle S_{Li}(\bar{q})\bar{q}_b(e^TP_me)e^TP_m(B)_i\\ \dot{\hat{W}}_{Gi} &= -\Gamma_{Gi}S_{Gi}(\bar{q})\bar{q}_b(e^TP_me)e^TP_m(B)_i\end{aligned} \tag{36}$$

and

$$\begin{aligned}\tau_r &= -k_r(e^TP_me)\,\mathrm{sgn}(B^TP_me)-k_2e-k_3(\|B\|\|P_m\|\|e\|)^2\\ k_r &= k_1+k_{r1}+k_{r2}\end{aligned} \tag{37}$$

where $(B)_i$ stands for the $i-$th column of B, k_1,k_2 are the positive constants, $k_3\geq 6\bar{m}\mathscr{S}^2, k_{r1}\geq\|\varepsilon_K\bar{q}_a+\varepsilon_L\bar{q}_b+\varepsilon_Tr_m+\varepsilon_G\|, k_{r2}\geq\|[\tilde{W}_K^{\langle T\rangle}\langle\cdot\rangle S_K(\bar{q})]\|\|\bar{q}_a\|+\|[\tilde{W}_L^{\langle T\rangle}\langle\cdot\rangle S_L(\bar{q})]\|\|\bar{q}_b\|+\|[\tilde{W}_T^{\langle T\rangle}\langle\cdot\rangle S_T(\bar{q})]\|\|r_m\|+\|[\tilde{W}_G^{\langle T\rangle}\langle\cdot\rangle S_G(\bar{q})]\|$, $\Gamma_{Ki}\in\mathbf{R}^{(2(n-1)\cdot l)\times(2(n-1)\cdot l)}$, $\Gamma_{Ti}\in\mathbf{R}^{((n-1)\cdot l)\times((n-1)\cdot l)}$, $\Gamma_{Li}\in\mathbf{R}^{(2l)\times(2l)}$, $\Gamma_{Gi}\in\mathbf{R}^{l\times l}$ are the symmetric positive definite matrices, then the adaptive NN controller ensures that the closed-loop system are stochastically finite-time attractive, and for each bounded initial condition, and the parameter estimates $\hat{W}_K,\hat{W}_L,\hat{W}_T,\hat{W}_G$ satisfy

$$\begin{aligned}&\mathbf{P}\{\lim_{t\to\infty}\|\hat{W}_K\|,\lim_{t\to\infty}\|\hat{W}_L\|,\lim_{t\to\infty}\|\hat{W}_T\|\text{and}\\ &\lim_{t\to\infty}\|\hat{W}_G\|\text{ exist and are finite}\}=1.\end{aligned} \tag{38}$$

Remark 2: In τ_r, we introduced a positive constant k_2, which is only used in the practical controller design to improve the controller's smoothness and doesn't affect our stability proof.

Proof: Choose the following Lyapunov function

$$V_1=U_1+U_2,\ U_1=\frac{1}{2\bar{m}}(e^TP_me)^2,$$

$$\begin{aligned}U_2 = &\sum_{i=1}^{n-1}\tilde{W}_{Ki}^{\langle T\rangle}\Gamma_{Ki}^{-1}(\tilde{W}_{Ki}^{\langle T\rangle})^T+\sum_{i=1}^{n-1}\tilde{W}_{Li}^{\langle T\rangle}\Gamma_{Ki}^{-1}(\tilde{W}_{Li}^{\langle T\rangle})^T\\ &+\sum_{i=1}^{n-1}\tilde{W}_{Ti}^{\langle T\rangle}\Gamma_{Ti}^{-1}(\tilde{W}_{Ti}^{\langle T\rangle})^T+\sum_{i=1}^{n-1}\tilde{W}_{Gi}^T\Gamma_{Gi}^{-1}\tilde{W}_{Gi}.\end{aligned} \tag{39}$$

Bearing in mind $\underline{m}\leq|\mathscr{M}^{-1}|\leq\bar{m}$ and applying the Ito formula to V_1 yield

$$\begin{aligned}\mathscr{L}V_1 \leq\ & 2(e^TP_me)e^TP_mB(\tau_r-\varepsilon_K\bar{q}_a-\varepsilon_L\bar{q}_b-\varepsilon_Tr_m\\ &-\varepsilon_G)+6\bar{m}\mathscr{S}(\|B\|\|P_m\|\|e\|)^2\\ &+2\sum_{i=1}^{n-1}[\tilde{W}_{Ki}^{\langle T\rangle}\langle\cdot\rangle S_{Ki}(\bar{q})]\bar{q}_a(e^TP_me)e^TP_m(B)_i\\ &+2\sum_{i=1}^{n-1}[\tilde{W}_{Ti}^{\langle T\rangle}\langle\cdot\rangle S_{Ti}(\bar{q})]r_m(e^TP_me)e^TP_m(B)_i\\ &+2\sum_{i=1}^{n-1}[\tilde{W}_{Li}^{\langle T\rangle}\langle\cdot\rangle S_{Li}(\bar{q})]\bar{q}_b(e^TP_me)e^TP_m(B)_i\\ &+2\sum_{i=1}^{n-1}\tilde{W}_{Gi}^TS_{Gi}(\bar{q})(e^TP_me)e^TP_m(B)_i\\ &+2\sum_{i=1}^{n-1}\tilde{W}_{Ki}^{\langle T\rangle}\Gamma_{Ki}^{-1}(\dot{\tilde{W}}_{Ki}^{\langle T\rangle})^T+2\sum_{i=1}^{n-1}\tilde{W}_{Ti}^{\langle T\rangle}\Gamma_{Ti}^{-1}(\dot{\tilde{W}}_{Ti}^{\langle T\rangle})^T\\ &+2\sum_{i=1}^{n-1}\tilde{W}_{Li}^{\langle T\rangle}\Gamma_{Li}^{-1}(\dot{\tilde{W}}_{Li}^{\langle T\rangle})^T+2\sum_{i=1}^{n-1}\tilde{W}_{Gi}^T\Gamma_{Ti}^{-1}\dot{\tilde{W}}_{Gi}\end{aligned} \tag{40}$$

Substituting the adaptive laws (36) to (40), and further substituting τ_r from (37), leads to

$$\begin{aligned}\mathscr{L}V_1 &= -2(k_1+k_{r2})(e^TP_me)e^TP_mB\,\mathrm{sgn}(B^TP_me)\\ &\quad -2k_2(e^TP_me)e^TP_mBe\\ &\leq -k_0\|e\|^3<0,\|e\|\neq 0,\end{aligned} \tag{41}$$

with $k_0=2k_1\|P_m\|^2\|B\|>0$. According to Theorem 1 [14], expression (41) means both U_1 and U_2 are bounded in probability and consequently $\|e\|,\|\tilde{W}_K\|,\|\tilde{W}_T\|,\|\tilde{W}_L\|,\|\tilde{W}_G\|$ are bounded in probability, i.e., $\mathbf{P}\{\lim_{t\to\infty}\|\hat{W}_K\|$, $\lim_{t\to\infty}\|\hat{W}_L\|$, $\lim_{t\to\infty}\|\hat{W}_T\|$, and $\lim_{t\to\infty}\|\hat{W}_G\|$ exist and are finite$\}=1$.

On the other hand, according to (40), (41) and (37), we have

$$\begin{aligned}\mathscr{L}U_1 &\leq -\frac{1}{\bar{m}}(e^TP_me)(e^TQ_me)-k_0\|e\|^3\leq -k_0\|e\|^3\\ &= = -k_0\left(\frac{1}{\|P_m\|^2}(\|e\|\|P_m\|\|e\|)^2\right)^{\frac{3}{4}}\\ &\leq -k_0\left(\frac{1}{\|P_m\|^2}\right)^{\frac{3}{4}}\left((e^TP_me)^2\right)^{\frac{3}{4}}=-c(U_1)^{\frac{3}{4}}\end{aligned} \tag{42}$$

where $c=k_0\left(\frac{2\bar{m}}{\|P_m\|^2}\right)^{\frac{3}{4}}$. Thus, by Lemma 1, the closed-loop system (34) achieves the finite-time attractiveness, i.e., the matching error e will reach the origin in finite time with probability one. This completes the proof. ■

V. Reference Trajectory Generator for q_b Subsystem

For the motion of the passive joint, however, to our best knowledge, there has been very little study to discuss the automatic control of its movement until now. In this work, we attempt to set up a framework to design the reference trajectory for manipulating the last joint to track the desired trajectory. As above discussed, after finite time, q_a will exactly track q_{ad}, such that the dynamics (10) becomes as follows:

$$\begin{aligned}\ddot{q}_b &= -M_b^{-1}C_b\dot{q}_b-M_b^{-1}g_b-M_b^{-1}\dot{w}_b-M_b^{-1}M_{ba}\ddot{q}_{ad}\\ &\quad -M_b^{-1}C_{ba}\dot{q}_{ad}\end{aligned} \tag{43}$$

Let $\varphi = [\varphi_1, \varphi_2]^T = [q_b, \dot{q}_b]^T$, $\phi = [\phi_1^T, \phi_2^T]^T = [q_{ad}^T, \dot{q}_{ad}^T]^T$ and $v = \ddot{q}_{ad}$. Then, equation (43) can be rewritten as

$$\dot{\varphi}_1 = \varphi_2, \dot{\varphi}_2 = f(\varphi, \phi, v) - M_b^{-1}\dot{w}_b \tag{44}$$

with $f(\varphi, \phi, v) = -M_b^{-1}M_{ba}v - M_b^{-1}(C_b\varphi_2 + g_b + C_{ba}\phi_2)$ and $\dot{\phi}_1 = \phi_2, \dot{\phi}_2 = v$.

Consider the desired forward position and forward velocity of the manipulator as q_{bd} and $\dot{q}_{bd}$, respectively. Then, our design objective is to construct a v (subsequently ϕ_1 and ϕ_2) such that φ_1 and φ_2 of system (44) follow φ_{1m} and φ_{2m} generated from the following reference model

$$\dot{\varphi}_{1m} = \varphi_{2m}, \dot{\varphi}_{2m} = f_m(q_{bd}, \dot{q}_{bd}, \varphi_m) \tag{45}$$

where $\varphi_m = [\varphi_{1m}, \varphi_{2m}]^T$ and $f_m(q_{bd}, \dot{q}_{bd}, \varphi_m) = -k_1(\varphi_{1m} - q_{bd}) - k_2(\varphi_{2m} - \dot{q}_{bd}) + \ddot{q}_{bd}$. It can be easily checked that the reference model (45) ensures that $\varphi_{1m} \to q_{bd}$ and $\varphi_{2m} \to \dot{q}_{bd}$. According to implicit function theorem based neural network design [13], there must exist a function $f_v : v^* = f_v(q_{bd}, \dot{q}_{bd}, \varphi, \phi)$ such that $f(\varphi, \phi, v^*) = f_m(q_{bd}, \dot{q}_{bd}, \varphi)$, i.e., there exists the ideal HONNs weight vectors such that

$$v^* = [W_v^{*\langle T \rangle}\langle\cdot\rangle S_v(z)] + \varepsilon_v, \quad z = [q_{bd}, \dot{q}_{bd}, \varphi^T, \phi^T]^T \tag{46}$$

where $\varepsilon_v \in \mathbf{R}^{(n-1)\times 1}$ is the neural network approximation error vector. Let us employ HONNs to approximate v^* as follows:

$$\hat{v} = \hat{W}_v^{\langle T \rangle}\langle\cdot\rangle S_v(z) \tag{47}$$

with $\hat{W}_v^{\langle T \rangle} = [\hat{W}_{v1}^T, \hat{W}_{v2}^T, \cdots, \hat{W}_{v(n-1)}^T]$, where $\hat{W}_{vi}^T \in \mathbf{R}^{l\times 1}(i = 1, 2, \cdots, n-1)$ are the neural network weight vectors. Substituting $\hat{v}$ into (44) and using $f(\phi, \varphi, v^*) = f_m(q_{bd}, \dot{q}_{bd}, \varphi)$, we have

$$\begin{aligned} \dot{\varphi}_1 &= \varphi_2 & (48)\\ \dot{\varphi}_2 &= f_m(q_{bd}, \dot{q}_{bd}, \varphi) - M_b^{-1}(\dot{w}_b + M_{ba}([\tilde{W}_v^{\langle T \rangle}\langle\cdot\rangle S_v(z)] - \varepsilon_v)) \end{aligned}$$

where $\tilde{W} = \hat{W} - W^*$. Define $\tilde{\varphi}_1 = \varphi_1 - \varphi_{1m}$ and $\tilde{\varphi}_2 = \varphi_2 - \varphi_{2m}$ such that $\tilde{\varphi} = \hat{\varphi} - \varphi$. Then, the comparison between (45) and (48) yields

$$\begin{aligned} \dot{\tilde{\varphi}}_1 &= \tilde{\varphi}_2 & (49)\\ \dot{\tilde{\varphi}}_2 &= -k_1\tilde{\varphi}_1 - k_2\tilde{\varphi}_2 - M_b^{-1}(\dot{w}_b + M_{ba}([\tilde{W}_v^{\langle T \rangle}\langle\cdot\rangle S_v(z)] - \varepsilon_v)) \end{aligned}$$

Theorem 2: Consider the following weight adaptation law for HONN employed in (47)

$$\dot{\hat{W}}_{vi} = \Gamma_{vi}S_{vi}(z)\tilde{\varphi}^T P_W[0 \quad 1]^T - \sigma\Gamma_{vi}\hat{W}_{vi} \tag{50}$$

where $\Gamma_{vi} \in \mathbf{R}^{l\times l}$ and σ are suitably chosen as a symmetric positive definite matrix and a positive scalar, respectively. Then, the tracking errors $\tilde{\varphi}_1$ and $\tilde{\varphi}_2$ in (49) will be eventually bounded into a small neighborhood around zero.

Proof: Let us rewrite the error dynamics (49) as the form of Ito SDE

$$\begin{aligned} d\tilde{\varphi} &= \left[A_W\tilde{\varphi} - [0\ 1]^T M_b^{-1}\sum_{i=1}^{n-1} M_{bai}(\tilde{W}_{vi}^T S_{vi}(z) - \varepsilon_{vi})\right]dt \\ &\quad -[0\ 1]^T M_b^{-1}dw_b \end{aligned} \tag{51}$$

where M_{bai} represents the i-th element of vector M_{ba}, $A_W = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix}$ satisfies the Lyapunov equation $A_W^T P_W + P_W A_W = -Q_W$, i.e., for any symmetric positive definite matrix Q_W, there exists a symmetric positive definite P_W satisfying the above equation.

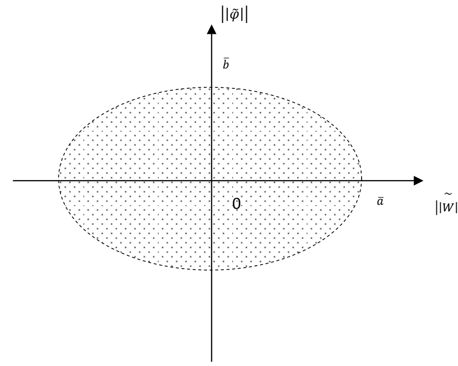

Fig. 1. Bounding set of $\|\tilde{W}_v^{\langle T \rangle}\|$ and $\|\tilde{\varphi}\|$.

Considering the following Lyapunov function

$$V_2(t) = \tilde{\varphi}^T P_W\tilde{\varphi} + M_b^{-1}\sum_{i=1}^{n-1} M_{bai}\tilde{W}_{vi}^T\Gamma_{vi}^{-1}\tilde{W}_{vi}^T \tag{52}$$

and the closed-loop dynamics (51) with the update law (50), we obtain

$$\begin{aligned} \mathscr{L}V_2(t) \leq &-\lambda_{Q_W}\|\tilde{\varphi}\|^2 - 2\sigma M_b^{-1}|M_{ba}|\|\tilde{W}_v^{\langle T \rangle}\|^2 + \varepsilon^2\|\tilde{\varphi}\|^2 \\ &+\varepsilon^2\|\tilde{W}_v^{\langle T \rangle}\|^2 + \frac{1}{\varepsilon^2}\varepsilon_0^2 M_b^{-1}|M_{ba}|^2\|P_W[0\ 1]^T\|^2 \\ &+\frac{1}{\varepsilon^2}\sigma^2 M_b^{-1}|M_{ba}|^2\|W_v^{*\langle T \rangle}\|^2 + Tr(P_W)(M_b^{-1})^2 \end{aligned}$$

where $|\varepsilon_v| \leq \varepsilon_0$, λ_{Q_W} is the minimum eigenvalue of Q_W, ε is any given positive constant and we can choose it sufficiently small. Furthermore, we can choose the suitable Q_W and σ making $\lambda_{Q_W} \geq \varepsilon^2, 2\sigma M_b^{-1}|M_{ba}| \geq \varepsilon^2$, and it follows that $\dot{V}_2(t) \leq 0$ in the complementary set of a set S_b defined as

$$S_b \triangleq \left\{(\tilde{\varphi}, \tilde{W}) \left| \frac{\|\tilde{W}_v^{\langle T \rangle}\|^2}{\bar{a}^2} + \frac{\|\tilde{\varphi}\|^2}{\bar{b}^2} - 1 \leq 0 \right.\right\}$$

with $\bar{a} = \frac{\bar{c}}{\sqrt{\lambda_{Q_W} - \varepsilon^2}}, \bar{b} = \frac{\bar{c}}{\sqrt{2\sigma M_b^{-1}|M_{ba}| - \varepsilon^2}}$, $\bar{c} = \sqrt{\frac{1}{\varepsilon^2}M_b^{-1}|M_{ba}|^2(\varepsilon_0^2\|P_W[0\ 1]^T\|^2 + \sigma^2\|W_v^{*\langle T \rangle}\|^2) + Tr(P_W)(M_b^{-1})^2}$. Obviously, the set S_b defined above is compact. Hence, by Theorem 1 in [14], it follows that all the solutions of (51) are bounded in probability. The set S_b is shown in Fig. 1 and consists of the closed region bounded by the closed oval arc defined by $\frac{\|\tilde{W}_v^{\langle T \rangle}\|^2}{\bar{a}^2} + \frac{\|\tilde{\varphi}\|^2}{\bar{b}^2} = 1$. Thus, the proof is completed. ■

VI. SIMULATION STUDIES

In this section, the developed trajectory generator and controller will be applied to the cart-pendulum system as shown in Fig. 2 [15]. Let $q_1 = x$ and $q_2 = \theta$, then the dynamics can be described as a fully control subsystem of q_1:

$$\Sigma_a : \ddot{q}_1 = \frac{4ml\dot{q}_2^2\sin q_2 - 3mg\sin q_2\cos q_2 + 4F}{4(M+m) - 3m\cos^2 q_2} \tag{53}$$

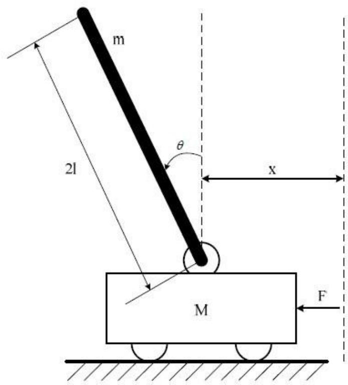

Fig. 2. The cart-pendulum system

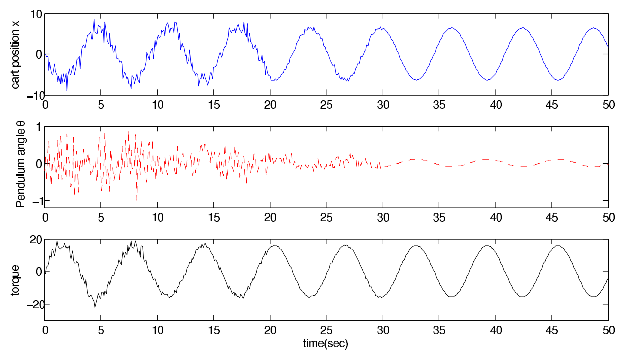

Fig. 3. The simulation results

and an uncontrolled subsystem of q_2

$$\Sigma_b : \ddot{q}_2 = \frac{3(M+m)g\sin q_2 - 3m\dot{q}_2^2 \sin q_2 \cos q_2}{4(M+m)l - 3ml\cos^2 q_2} + \frac{3mg\sin q_2 \cos q_2}{4(M+m) - 3m\cos^2 q_2} - \frac{3}{4}\cos q_2 \ddot{q}_1 \quad (54)$$

where the mass of cart is 2.4kg; the mass of pendulum is 0.23kg; the length of the pendulum (2l) is 0.36m, w_1, w_2 are indepent standard Weiner progresses. The control objective is to make q_2 to track $\frac{\pi}{30}\sin(t)$.

The simulation results are shown in Fig. 3. As clearly shown by the simulation results, in the presence of unknown system parameters and external disturbances, the proposed adaptive NN controller is able to guarantee the pendulum's exact tracking of the given trajectory.

VII. Conclusion

In this paper, adaptive NN control has been designed on the stochastic under-actuated systems for dynamic balance and motion tracking of desired trajectories. The dynamics of the the actuated subsystem has been shaped to follow a reference model, which is derived by using the LQR optimization technique to minimize both the motion tracking error and the transient acceleration. The unactuated subsystem is discussed by suitably generating a reference trajectory. Simulation results have demonstrated the efficiency of the proposed method.

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