UKACC International Conference on Control 2012
Cardiff, UK, 3-5 September 2012

Free Flight Concept Formulation Exploiting Neighbouring Optimal Control Concepts

S.N.Hasan
Department of Automatic Control and Systems Engineering
University of Sheffield
Mappin Street, Sheffield S1 3JD
Email: saam.najat@sheffield.ac.uk

J.A.Rossiter
Department of Automatic Control and Systems Engineering
University of Sheffield
Mappin Street, Sheffield S1 3JD
Email: j.a.rossiter@sheffield.ac.uk

***Abstract*—The goal of this paper is to develop a new approach for the Free Flight concept based on neighbouring Optimal Control (NOC) that can deliver an aircraft to its designated position safely and reliably when faced with conflicts, and to have some form of recovery measure when the pilot is faced with uncertainty that are due to wind and other uncertainties in the FF environment. The benefits of the algorithm are that it reduces fuel consumption, and it achieves optimal safe inter-aircraft separation while maintaining near-optimality under disturbances.**

***Index Terms*—NOC, Air Traffic Control, Fuel Consumption, Conflict Detection, Separation Constraints.**

I. Introduction

Recent advancements in navigation aids, communication technologies, and computing power makes it possible for new concepts of Air Traffic Control (ATC) to be implemented, namely the Free Flight (FF), which is an environment where pilots have the liberty to select their routes in real time, and also define their own cruising altitudes and speeds for better fuel efficiency [2]. The workload of ATC would be greatly reduced, since pilots would be responsible for immediate threat avoidance when encountered. ATC then would be responsible for higher level tasks, such as monitoring traffic flow, airspace control, sectorisation, and scheduling runways. In the current centralised environment, ATC has to deal with potentially thousands of aircraft spread over a vast geographic area in order to ensure resolution of all possible conflicts [2]. Furthermore, as the number of aircraft increases, the complexity of conflict resolution grows and quickly becomes computationally intractable. Similarly, the possibility that a single failure could disable a significant portion of the centralized system creates a highly undesirable risk.

One of the most important tasks for ATC is conflict detection and resolution (CDR). Much of the current research for automating ATC focuses on this problem. Solutions for Conflict Resolution (CR) that are based on fixed rule sets that dictate actions based on situational geometry has been widely studied in both centralized and distributed algorithms. Very few of these techniques incorporate free flight rules in the algorithm, and that FF research has mainly been focused on the concept rather than defining automation tools for this concept. The goal of this paper is to exploit an approach based on neighbouring Optimal Control (NOC) to achieve a near-optimal FF air traffic control when faced with conflicts, and to have some form of recovery measure when the pilot is faced with uncertainty that are due to wind and other uncertainties in the FF environment. In addition, reduction in fuel consumption is desirable, and to achieve near-optimal safe inter-aircraft separation while maintaining near-optimality under disturbances.

The rest of the paper is organized as follows. Section II presents the Free Flight concept and its associated rules and conditions and some previous work that was carried out will be described. Section III outlines a multi aircraft model used, its associated constraints and a subsection also outlines fuel efficient resolution maneuvers. Subsequent sections will outline a derivation for Trajectory redirection using NOC and it also shows that feasible trajectories exist when faced with uncertainties that the pilot may experience. Finally a simulation example with a worst case scenario will be shown. A summary (Pseudo Code) of the algorithm is also shown on Section V.

II. The Free Flight Concept

The following definition was defined in the Report of the Radio Technical Commission for Aeronautics (RTCA) Board of Directors' Select Committee on Free Flight [2]. "Free Flight" was defined as :

"A safe and efficient flight operating capability under instrument flight rules (IFR) in which the operators have the freedom to select their path and speed in real time. Air traffic restrictions are only imposed to ensure separation, to preclude exceeding airport capacity, to prevent unauthorized flight through special use airspace, and to ensure safety of flight. Restrictions are limited in extent and duration to correct the identified problem. Any activity which removes restrictions represents a move towards free flight."

Sometimes the words "Free Flight" are used for concepts, which include direct routing but no airborne separation. ATC will be overseeing traffic flow management (TFM) and intervening only when there is a loss of separation.

A. Conditions and Rules of Free Flight

The FF definition used by RTCA roughly defines the concept. To define the context, a more detailed definition and rules are required. The following questions need to be clarified after applying the RTCA definition:

1) What is the role & responsibility of ATC?
2) How is an alert zone and protected zone defined?

Based on the questions above, the following choices and assumptions were made:

1) NO ATC: An inflexible proposition of FF is chosen with no ATC on the ground. The idea behind this concept is to explore the limits of the FF concept.

2) ALL AIRCRAFT FULLY EQUIPPED: All aircraft are assumed to be set up with Automatic Dependent Surveillance-Broadcast (ADS-B) transmitter & receiver and CDR advisory modules. The transmitter sends the aircraft's position and intent information needed by the CD module to all other aircraft. The ADS-B receiver collects all the information of the traffic within a certain range in the FF sector. The scenarios of mixed equipage is not considered here even though it is required for the transition to FF.

3) UPPER AIRSPACE ONLY: This work is limited to the uppermost airspace only, in order to concentrate on general conceptual problems. In other flight phases, a transition to controlled flight is foreseen and required. Where and how that transition should be implemented is not addressed in this work, since more benefit is attained when contemplating direct routing of aircraft(s) through FF in the uppermost airspace, so the feasibility for this airspace is a desirable result in itself.

4) PROTECTED ZONE: A conflict is defined in this work as an intrusion of a protected zone. The protected zone is defined using current ATC standards to be able to relate to existing traffic densities, even though studies have shown that the protected zone can be smaller. The protected zone must not be entered by any other aircraft. The task of the CD module is to predict an intrusion of the protected zone. This protected zone was chosen to reflect current ATC separation standards, which is 5 nautical mile radius and a height of 2000 feet ($-1000ft \leqslant altitude \leqslant +1000ft$). This means the ratio diameter to height is about 30 to 1.

The following section outlines the aircraft model & objective function formulation with its associated constraints.

III. Multi Aircraft Model & Constraints Formulation

Consider I aircraft flying within an area of interest, their continuous-time dynamics for level flight cruise is outlined in [9]. An aircraft in level flight can be modelled as a four state Point Mass Model (PMM). The states considered in this model are the following

- horizontal position (X) and (Y)
- the true airspeed (V)
- flight path angle (ψ)

The control inputs considered for the model are the engine thrust (T), and the bank angle (ϕ). Assuming a PMM, and that the applied forces acting on the aircraft will lead to the following equations of motion [9]:

$$\begin{bmatrix} \dot{X}_i \\ \dot{Y}_i \\ \dot{V}_i \\ \dot{\psi}_i \end{bmatrix} = \begin{bmatrix} V_i \cos(\psi_i) \\ V_i \sin(\psi_i) \\ \frac{-C_{D_i} S_i \rho}{2} \frac{V_i^2}{m_i} + \frac{1}{m_i} T_i \\ \frac{C_{L_i} S_i \rho V_i}{2m_i} \sin(\phi_i) \end{bmatrix} = f(t, x, u) \tag{1}$$

for $i \in \Upsilon \triangleq \{1, ..., I\}$ and S is the surface area of the wings, ρ is the air density and C_D, C_L, are aerodynamic lift and drag coefficients. Since we are considering commercial aircraft, it is assume that the aircraft operates on a near trimmed flight conditions [9]. Eurocontrol's Base of Aircraft Database (BADA) [9] manual defines this as Total Energy Model (TEM) [9]. The TEM combines the rate of work done by all the forces that is affecting the aircraft to the rate of increase in its total energy (Potential and Kinetic).The aircraft motion has four states (X, Y, V, ψ) and two inputs (T_i, ϕ_i). For a specific aircraft, the values of such bounds can be derived from the BADA database [9]. There are many models available in the literature when it involves predicting the current and future positions of an aircraft. For example in [8], an interpolating algorithm was developed to fit a vector field to the observed velocities at aircraft positions and at given sample times. Worst case techniques were used in [2] to estimate aircraft's position. Probabilistic modeling was covered and outlined in [1,3]. In this paper we are using a four state point mass model to estimate the aircraft's position.

A. Velocity and acceleration constraints

Aerodynamics impose physical constraints on the minimum and maximum speeds an aircraft can fly at each altitude. In addition, passenger comfort and other factors impose constraints on the acceleration and the turning rate. These constraints can be expressed in the following form:

$$V_{max} \geqslant V_i(t) \geqslant V_{min} \tag{2}$$

$$|\dot{V}_i(t)| \leqslant \delta V \tag{3}$$

$$|\dot{\psi}_i(t)| \leqslant \delta \psi \tag{4}$$

B. Maneuvering constraints

Considering enroute traffic, the speed range of an aircraft is narrow. Passenger comfort is also taken into consideration which requires smooth trajectories, so in this case additional constraints are required for the maneuvers. The approach was proposed in [2], and was also used in [8] where constraints are imposed on speed changes that are bounded on a set around the current aircraft speed. The possible changes is the convex set of possible speed commands and can be described by a combination of quadratic and linear constraints as follows:

$$\|V_{0i} + \dot{V}_i\| \leq V_{max} \tag{5}$$

$$(V_{0i} + \dot{V}_i)^T \frac{V_{0i}}{\|V_{0i}\|} \geq V_{min} \tag{6}$$

Values of V_{max} and V_{min} can be obtained from the BADA database [15] but generally for most commercial aircraft cruising at high altitudes it can be approximated by:

$$\frac{V_{max} - V_{min}}{V_{max}} \leq 0.1 \tag{7}$$

C. Conflict Avoidance Constraints & Overall Cost Function

Conflict avoidance constraints can be mathematically formalised in numerous ways. Many authors express collision avoidance constraints in terms of a given minimum miss distance, and it appears to be the most attractive option from a geometrical point of view, In this work, a time-based separation criterion is considered. All aircraft should remain separated at all times, by at least a minimum distance, which is set to $d_s = 5nm$ for cruising altitudes. This constraint can be described as follows:

$$\left\| [X_i, Y_i]^T - [X_j, Y_j]^T \right\|_2 \geqslant d_s \tag{8}$$

for all times $t > 0$ and for all aircraft pairs $i \neq j$, where $(i, j) \in \Upsilon$. The constraint described by (8) is not convex and, in order to be able to handle it computationally, it requires tightening. One approach would be using the norm inequality $\|\bullet\|_2 \geq \|\bullet\|_\infty$ where $\|\bullet\|$ is the euclidean norm, then the so-called "*big-M*" technique (Refer to the formulation in [1]) can be used. This will ensure that at least one of the inequality constraints is active, and consequently that the two aircraft are separated by the required distance along at least one of the axes. The problem with this approach is that a model mismatch may be prevalent and there is a possibility that aircraft will encounter a situation where a solution satisfying both the dynamics and the separation constraints may no longer exist.

So, in this case an alternative separation constraint which was adopted in [8] is considered. For notational convenience, let's define $P_i = [X_i, Y_i]^T$, $P_j = [X_j, Y_j]^T$ which denotes the aircraft position and $P_{ij} = P_i - P_j$ is the relative position. $V_{ij} = V_i - V_j$ is the relative speed and $\dot{V}_{ij} = \dot{V}_i - \dot{V}_j$ is the relative velocity $P_{i,0}$ and $V_{i,0}$ are the initial position and initial speed of aircraft i respectively. the constraint can be described as:

$$P_{0ij}^T(V_{0ij} + \dot{V}_{ij}) + \|V_{0ij} + \dot{V}_{ij}\| \sqrt{\|P_{0ij}\|^2 - d_s^2} \geq 0 \tag{9}$$

Non-convexity is also prevalent in the above formulation, so a new slack variable is introduced φ_{ij} where $\|V_{0ij}\|^2 \geq \varphi_{ij}$ and $\varphi_{ij} \geq 0$ which leads to

$$P_{0ij}^T(V_{0ij} + \dot{V}_{ij}) + \varphi_{ij} \sqrt{\|P_{0ij}\|^2 - d_s^2} \geq 0 \tag{10}$$

It can be seen that the above constraints and cost function form a non-convex quadratic program of the form

$$\begin{array}{ll} \min & h^T P_0 h + 2\xi_0^T h + r_0 \\ s.t & h^T P_i h + 2\xi_i^T h + r_i \leq 0 \end{array} \tag{11}$$

Although the above quadratic program can be very difficult to solve in general, it has received a fair share of research attention. Approximate solutions can be obtained based on convex optimization. We also have $H = [\dot{V}_1, \dot{V}_2, ..., \dot{V}_n, \varphi_{12}, ..., \varphi_{(n-1),n}]^T$. If the optimal solution H to the semi-definite relaxation has unit rank, then $\overline{h}$ is the solution. If not, then a form of randomization procedure must be applied. If we consider a gaussian distribution with mean $\overline{h}$ and $cov(H - \overline{h}\,\overline{h}^T)$ then samples can be chosen in accordance to the distribution. As outlined in [8] a linearisation procedure is required to pick the crossing pattern for each aircraft pair by computing the following

$$C_p = sgn(P_{0ij} \times (V_{0ij} + \dot{V}_{ij})) = \frac{P_{0ij} \times (V_{0ij} + \dot{V}_{ij})}{|P_{0ij} \times (V_{0ij} + \dot{V}_{ij})|} \tag{12}$$

If $C_p = 1$ the crossing pattern is then chosen to be counter-clockwise and clockwise if $C_p = -1$. We will assume that the crossing pattern is clockwise in the very unlikely case when $C = 0$. Subsequently, the corresponding convex optimization problem can be solved using barrier methods.

D. Fuel Efficient Resolution Maneuvers

In this section, a formulation for fuel resolution maneuvers outlined in [10] is considered. The strategic encounter is concerned with minimizing the economics of a CR. Economics defined in this context by considering direct operating cost (DOC), which comprises of fuel and flight time costs involved in maneuvering to avoid a conflict. Only trajectory solutions that geometrically pass around the Protected Airspace Zone of an intruder aircraft is considered. In the FF context, it is assumed that the heading maneuver will consist of a series of standard vectoring heading, and speed changes. in such a way that straight line motion materialises in between heading, and speed. The DOC penalty function used incorporates both fuel and time elements. Included in the DOC are

1) The additional fuel required due to the increased drag and flight path distance traveled during a maneuver [10].
2) The additional operating costs due to the additional time required to execute the maneuver and return back to course [10].

$$DOC = C_{Fuel}\Delta W_{Fuel} + C_{Time}\Delta T \tag{13}$$

where C_{Fuel} is the cost of fuel, ΔW_{Fuel} is the additional fuel used in the maneuver, C_{Time} is the time dependent aircraft operating cost and ΔT is the additional time used in the maneuver. This DOC is investigated by [10], where $C_{Fuel} = \pounds 0.10/lb$ and $C_{Time} = \pounds 15.22/min$. Another issue that needs to be considered is the additional DOC incurred by neighbouring traffic that may be affected by CR maneuvers; however, it's not considered here. The fuel burn equation is:

$$\frac{dW}{dT} = T_R C_j \tag{14}$$

where W is the weight, C_j is the specific fuel consumption, and T_R is the required thrust. The required thrust can be described as:

$$T_R = D + sin(\psi_{ss}) \tag{15}$$

where D is the drag and ψ_{ss} is the steady state flight path angle. The fuel consumption model varies linearly with airspeed. The lift coefficient C_L, is described in terms of the aircraft weight W and speed V as follows:

$$C_L = \frac{2W cos(\psi_{ss})}{S_w V^2 \rho} \tag{16}$$

where ρ is air density, and S_w is the reference wing area. The drag is described as:

$$D = \frac{C_D V^2 \rho W S_w}{S_w V^2 \rho} \tag{17}$$

C_D is the drag coefficient which is approximated by:

$$C_D = C_{D0} + K_{CL}^2 \tag{18}$$

C_{D0} is the zero lift drag coefficient, $K = \frac{1}{ARe\pi}$ is the induced drag factor, e is Oswald's efficiency factor, and AR is the wing aspect ratio. Furthermore, the thrust specific fuel consumption of a turbojet or turbofan engine increases nearly linearly with Mach number [10]. For this reason, the fuel consumption can be modeled as:

$$C_j = \sigma V \tag{19}$$

where σ is the coefficient. Next, these endurance equations can be combined to obtain the fuel burn:

$$\frac{dW}{dT} = -\left[0.5\rho V^2 S_w (C_{D0} + K\left(\frac{2W cos(\psi_{ss})}{\rho V^2 S_w}\right)^2 + W sin(\psi ss)\right]\sigma V \tag{20}$$

Equation (20) is integrated to calculate the fuel burn, given the geometry the previously defined heading maneuvers or speed control maneuvers. For the acceleration case, the equation is integrated numerically [10].

IV. Trajectory redirection using neighboring optimal control (NOC)

Solving a nonlinear optimal control problem with a long horizon is generally computationally expensive, especially when FF concepts are considered, since there is limited computation power onboard an aircraft. If a nominal optimal solution is known, it is advantageous to approximate the optimal control solution when the parameters of the optimal control problem are slightly perturbed. The Neighbouring optimal control (NOC) method provides a first order approximation to the optimal solution corresponding to an initial state perturbed from the nominal value. When a disturbance significantly affect the forces on an aircraft, and modifies the flight path, redirecting the trajectory may recover the aircraft. Disturbances considered here may correspond to the wind or cross track errors, so in this case the nominal states and controls can be used to redirect the trajectory [11]. An off-line optimal trajectory is always assumed to be found from a feasible trajectory database. When a failure occurs, NOC is used to determine its neighbouring feasible trajectory on-line. If such a neighboring feasible trajectory exists, the aircraft may then be recovered from that failure in real time [11]. The formulation outlined in [11,4] is outlined in this section, and they are applicable to the formulation of FF concepts.

A. *Off-line trajectory optimization for the Shortest Path Evaluation*

A shortest path problem can be used for the off-line optimal trajectory search. To find the shortest path, several algorithms can be used. In the free flight concept, Dijkstra's algorithm [5] is a reasonable choice since it guarantees to find an optimal path by repeatedly selecting a vertex with the minimum shortest-path estimate. The weight is defined by the mean value of the angle of attack of the aircraft. The reader is reffered to [5] for further information.

B. *On-line trajectory optimization using NOC*

Based on the off-line optimal trajectory, a small perturbation is imposed on each state variable and new control can be obtained by NOC. The aircraft can be recovered from the disturbances. The NOC approach was outlined by Bryson and Ho (1975) [4], Consider the following nonlinear system

$$\dot{x}(t) = f(t, x, u), x(t_0) = x_0 \tag{21}$$

where $x(t)$ are the state variables and $u(t)$ control inputs and the cost function is defined as

$$\min \qquad J(u) = \phi(t_f, x_f, u_f) \tag{22}$$

s.t. Dynamical, Separation constraints, Velocity and acceleration constraints.

"f" denotes the final time. An assumption is made that the optimal solution $(x^*(t), u^*(t))$ can be determined, in which the optimal control is $u^*(t) = g(t, x^*(t), \lambda(t))$ under the following constraints

$$x_{min} \leq x^*(t) \leq x_{max}\ ,\ u_{min} \leq u^*(t) \leq u_{max} \tag{23}$$

when small perturbations $\delta x(t_0)$ and small variations $\delta\lambda$ imposed on x_0 and λ, then the neighbouring feasible trajectory $z(t)$ and new optimal control $\nu(t)$ can be found by $\delta x(t_0, \delta\lambda)$ and $\dot{x}(t)$. In order to ensure that a neighbouring trajectory exists the following assumptions are made:

- **Assumption 1.** For the nonlinear system (22), $f(t, x, u)$ is continuous.
- **Assumption 2.** The open loop optimal solution $x^*(t)$ and optimal control $u^*(t)$ is well defined for system (22).

For a neighbouring feasible initial condition $z(t_0)$ which will be equal to x_0, there will be a neighbouring feasible trajectory $z(t)$ and neighbouring optimal control $\nu(t)$ and satisfies

$$\sup\|z(t) - x^*(t)\| \leq K \tag{24}$$

Where K is an upperbound. And when system parameters varies fromλ_1 to λ_2 and $\lambda_2 = \lambda_1 + \delta\lambda$ then the neighbouring feasible state trajectory $z(t)$ and neighbouring optimal control $\nu(t)$ is:

$$\nu(t) = u^*(t) + G(t)(Z(t) - X*(t)) \qquad or \tag{25}$$
$$\nu(t) = u^*(t) + G(t)\Delta X(t), \tag{26}$$

Where $Z(t) = [z(t), \lambda_2]^T$, $X^*(t) = [x^*(t), \lambda_1]$, and $\Delta X(t)$ is a deviation vector of the new state vector $Z(t)$ from the open loop nominal state trajectory $X^*(t)$, $G(t)$ is the feedback gain matrix which is given by

$$G(t) = \delta U(t)\delta\overline{X}^{-1}(t) \tag{27}$$

Where $\delta U(t)$ are the control errors caused by the perturbation on the state vector $X(t)$ and $\delta\overline{X}(t)$ is a perturbation matrix in the initial states x_{t_0} and initial system parameter λ_1. Since the control law can be put in the form of $u = g(t, x, \lambda)$ then system (22) becomes an unforced system $\dot{x} = \overline{f}(t, x, \lambda), x(t_0) = x_0$, under the above conditions the controller can satisfy the constraints in. In order to ensure that that initial value problem does exist then theorem 3.5 outlined in [6] proves that the conditions are satisfied then in this case perturbations can be imposed.

In the neighbourhood of x^*, the state vector z can be defined as $z = x^* + \delta x$. It was mentioned above that the parameters changed from λ_1 to λ_2 and $\lambda_2 = \lambda_1 + \delta\lambda$. Carrying out a Taylor series expansion for ν^* is the following

$$\nu^* = g(t, z, \lambda_2) = g(t, x^* + \delta x, \lambda_1 + \delta\lambda) \tag{28}$$
$$= g(t, x^*, \lambda_1) + \frac{\partial g}{\partial x}\partial x + \frac{\partial g}{\partial \lambda}\partial\lambda + H.O.T \tag{29}$$
$$= u^* + \frac{\partial g}{\partial x}(z - x^*) + \frac{\partial g}{\partial \lambda}(\lambda_2 - \lambda_1) + H.O.T \tag{30}$$
$$= u^* + \begin{bmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial \lambda} \end{bmatrix} \left(\begin{bmatrix} z \\ \lambda_2 \end{bmatrix} - \begin{bmatrix} x^* \\ \lambda_1 \end{bmatrix} \right) + H.O.T \tag{31}$$

H.O.T denotes higher order terms. If the perturbation δx is small enough or z is in the neighbourhood of x^* then the higher order terms of δx and δy must be small as well. Therefore the higher order terms can be neglected at this point. The new controller now is $\nu = u^* + G(Z - X^*)$, where $Z \in \mathbb{R}^{m\times(n+p)}$ and $X^* \in \mathbb{R}^{n+p}$. If closed loop system was to be considered, the perturbations δx and $\delta\lambda$ will be the measured deviations, in this case the gain matrix is defined by:

$$G = \begin{bmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial \lambda} \end{bmatrix} = \frac{\partial g}{\partial X} \in \mathbb{R}^{m\times(n+p)} \tag{32}$$

The gain matrix G can be calculated numerically at every time $t \in [t_0, t_f]$. The reason for this being is that no partial derivatives will be evaluated. Now from the $u = g(t, x, \lambda)$ and $X = [x^T, \lambda^T]^T$, the differential of u is $du = \frac{\partial g}{\partial X}dX + \frac{\partial g}{\partial t}dt$. For a known perturbation, the perturbation vectors are $\delta X = X_{perturbed} - X^*$ and $\delta u = u_{perturbed} - u^*$. If a linear approximation is used and also noting that $\delta t = 0$ for all $t \in [t_0, t_f]$, δu can be obtained by

$$\delta u = \frac{\partial g}{\partial X}\delta X + \frac{\partial g}{\partial t}\delta t = G\delta X \tag{33}$$

Next an $(n + p) \times (n + p)$ square matrix is constructed for $\delta\overline{X}$, and another for δU which is an $m \times (n + p)$:

$$\delta\overline{X} = [\delta X^1 ....\delta X^{n+p}] \in \mathbb{R}^{(n+p)\times(n+p)} \tag{34}$$
$$\delta U = [\delta u^1 ....\delta u^{n+p}] \in \mathbb{R}^{m\times(n+p)} \tag{35}$$

Where δX^i, $i = 1, 2, ..., n + p$ and δu^i, $i = 1, 2, ..., n + p$ are the ith perturbation of δX and δu then equation () can be expressed by the following:

$$\delta U = G\delta\overline{X} \tag{36}$$

If the perturbations are chosen to be linearly independent and at the same time small enough, then $\delta\overline{X}$ can be guaranteed to be invertible at every instance of time.

V. NOC Application to the Aircraft trajectory retargeting under conflict

It was shown above that a neighbouring feasible trajectory exists when applying NOC to an aircraft [11,4]. Since we have four states (X, Y, V, ψ) and two control variables (T, ϕ). If C_L and C_D are the parameters that vary then a neighbouring solution exists. As mentioned above that a nominal optimal solution exists using Dijikstra's algorithm [5] then a new controller is calculated Ψ^* then the perturbed controller is calculated as $\Psi_{perturbed} = \Psi^* + G(Z - X^*)$. Since it is required to enforce the aircraft to meet a final time, then the state X is chosen as an independent variable. So five perturbations are used which are V, ψ, Y, C_L, C_D. Define $\delta\overline{X} = [V^1 - V^* - \psi^1 - \psi^* x^1 - x^* C^1_{L1} - C^*_{L1} C^1_{D1} - C^*_{D1}]^T ....[V^5 - V^*, \psi^5 - \psi^* x^5 - x^* C^5_{L1} - C^*_{L1} C^5_{D1} - C^*_{D1}]^T$ and $\delta\alpha = [\Psi^1 - \Psi^*, ..., \Psi^5 - \Psi^*]^T$ and $n = 1, ..., 5$ which is the perturbation instances for each state and parameter. Then the gain matrix can be evaluated as $G = \delta\Psi\delta\overline{X}^{-1}$. The algorithm can be summarised below:

Algorithm Free Flight Concept Exploiting NOC

Require: $P_{i0}(t)$ and $P_i(t)$ for all $i \in 1, ..., I$

Require: T_i and ϕ_i for the nominal shortest path

1: **While** $\exists i \; P_{0ij}^T(V_{0ij} + \dot{V}_{ij}) + \|V_{0ij} + \dot{V}_{ij}\| \sqrt{\|P_{0ij}\|^2 - d_s^2} \geq 0$ **do**

2: Solve the Optimisation problem (11)

3: Compute the matrix Gain $\delta\overline{X}$ and δU from (34),(35)

4: Compute the matrix Gain $G(t)$ from (36)

5: Measure new aircraft position $p_i(t)$

6: **End While**

VI. Simulation

A conflict situation with five aircraft are initially located on a circle of 400 km radius and are heading to mid air collision. Without any corrective action, all aircraft would collide at the center. Although unrealistic, this scenario allowed us to test the effectiveness of our method. All conflict constraints are enforced, and fuel consumption maneuvers are computed.

In order to differentiate between the nominal solution and the perturbed on,the results are shown as a box-and-whisker diagram in Figure 1 & 2. Qualitatively, both solutions are very similar, except for the fuel consumption of the aircraft which is outlined in Fig. 2, as the average fuel consumption in the perturbed case is lower than the nominal case which is an excellent demonstration of the ability of NOC to achieve near-optimal results without resorting the need to recompute the optimal solution. On the other hand, in both the nominal and perturbed case one conflict occurred which was not detected but resolved by the algorithm. However, there is a clear disadvantage that introducing any additional constraint in this formulation may impose restrictions when implementation issues are considered since computational times becomes higher, and intractability is also apparent sometimes in the simplest of optimisation problems. While the fuel savings is not very significant, the important point is that a near-optimal conflict resolution that maintains separation has been performed without increasing fuel use.

VII. Conclusion

A Free Flight formulation was presented to deal with conflict detection and resolution. A point mass model for aircraft dynamics was used that allowed a quadratic program formulation for the problem. A new approach based on neighbouring Optimal Control (NOC) that can deliver an aircraft to its designated position safely when faced with conflicts and uncertainties was outlined. A simulation example was provided that illustrated the effectiveness of our approach. The benefits of the algorithm are that it reduces fuel consumption, and it achieves optimal safe inter-aircraft separation while maintaining near-optimality under disturbances. Ongoing research focuses on combining this algorithm with complexity metrics that defines the level of disorder are being explored.

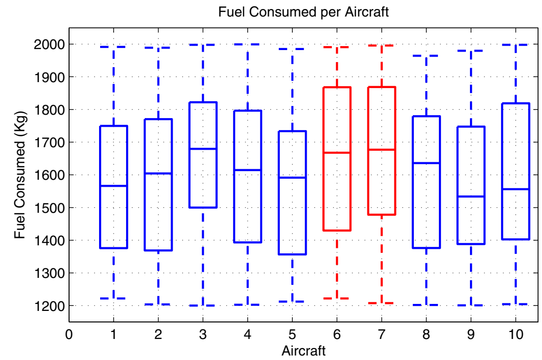

Fig. 1. Fuel Consumption. Blue: No conflict and Red: Conflicting Aircraft

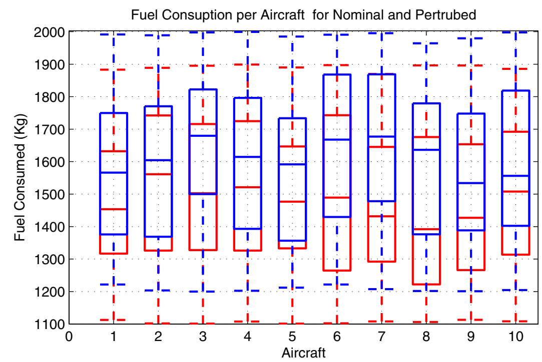

Fig. 2. Fuel Consumption. Blue: Nominal (Dijkstra Algorithm) and Red: Perturbed (NOC)

References

[1] Prandini, M., Hu, J., Lygeros, J., Sastry, S., 2000. A probabilistic approach to aircraft conflict detection. IEEE Transactions on Intelligent Transportation Systems

[2] Eby, M., Kelly, W., 1999. Free Flight separation assurance using distributed algorithms. Proceedings of the IEEE Aerospace Conference, vol. 5. Snowmass, CO, pp. 429441.

[3] Erzberger, H., Paielli, R., 1997. Conflict probability estimation for Free Flight. AIAA Journal of Guidance, Control, and Dynamics 20 (3).

[4] Bryson, A. E., Jr., & Ho, Y. C. (1975). Applied optimal control: Optimization, estimation, and control. New York: Hemisphere.

[5] Dijkstra, E. W. (1959). A note on two problems in connexion with graphs. Numerische Mathematik, 1, 269-271.

[6] Khalil, H. K. (2002). Nonlinear systems (3rd ed.) (p. 97). Upper Saddle River, NJ: Prentice Hall

[7] W. Glover and J. Lygeros, A stochastic hybrid model for air traffic control simulationin Hybrid Systems: Computation and Control.Springer Verlag, 2004, no. 2993, pp. 372386.

[8] E. Frazzoli, Z. Mao, J. Oh, and E. Feron, Aircraft conflict resolution via semi-definite programming, AIAA Journal of Guidance, Control, and Dynamics, vol. 24, no. 1, pp.79 86, 2001.

[9] Eurocontrol Experimental Centre, User Manual for the Base of Aircraft Data (BADA), 2004. [Online]. Available: http://www.eurocontrol.fr/projects/bada/

[10] Krozel, J. and Peters, M. (1997). Strategic Conflict Detection and Resolution for Free Flight, Proceedings of the Conference on Decision and Control, San Diego, USA.

[11] Z. Jiang and R. Ordnez, On-Line Robust Trajectory Generation on Approach And Landing for Reusable Launch Vehicles, Automatica, vol. 45, no. 7, Jul. 2009