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Trajectory tracking control of a car-like mobile robot in presence of sliding

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Abstract- This paper investigates the trajectory tracking control problem of a nonholonomic car-like mobile robot in the presence of sliding effects. Sliding (slipping and skidding) effects are treated as disturbances and introduced into the kinematic model of the car-like using the singular perturbation approach. In order to compensate for the effects of tire slipping and skidding, a robust second order sliding mode controller is developed based on the super twisting algorithm. It is theoretically proven that for car-like vehicle subjected to sliding, the lateral-longitudinal deviations and the orientation errors can be stabilized near the origin. Simulations results show the effectiveness and the robustness of the proposed controller with respect to the sliding effects.

Index Terms- Car-like Mobile Robot, Second order sliding mode, Tracking control, Sliding effects.

I. Introduction

The control problem of nonholonomic systems with parameter uncertainties has been extensively studied in past decades. Regarding the trajectory tracking problem of the Wheeled Mobile Robots WMR, many works address violation of the nonholonomic constraints. A robust control law against decoupled skidding and slipping effects has been proposed for solving a velocity tracking problem for a unicycle-type robot [1]. For a dynamic model of the unicycle-type robot including sliding effects, simulation results were reported in [2], showing the robustness of a second order sliding tracking control with respect to the non ideal constraints and to the nonlinearities. In [3], a path tracking control of an autonomous robot was considered under slipping and unknown dynamics. A robust adaptive neural network controller was proposed for WMR with the aid of backsteeping techniques and the learning ability. Based on the backsteeping methods, a robust adaptive tracking controller was designed for farm agriculture vehicles in presence of sliding [4]. The sliding effects are introduced as time-varying parameters to the ideal kinematic model. Both simulation and experimental results confirm the high longitudinal-lateral tracking controller accuracy. In [5], it was derived kinematic models of WMR that explicitly relate the perturbations to the vehicle skidding and slipping that are geometrically well defined. This description allows the analyse of these perturbations from a control perspective. The reference [6] copes with the control of WMR not satisfying the ideal kinematic constraints by using slow manifolds methods, but the parameters characterizing the sliding effects are assumed to be exactly known. In [7], a time varying stabilizing control law based on the linear quadratic theory was proposed and the necessity for the trajectory to satisfy the dynamics of the skidding effects is pointed out. This feedback control law ensures the local asymptotic convergence of the error dynamics of the unicycle but only under some conditions on the reference trajectory (accelerations should be sufficiently small). In [8], a singular perturbation formulation was derived which leads to robust linearizing feedback laws ensuring trajectory tracking in presence of sufficiently small sliding effects. In [9], a discrete-time sliding mode control was proposed for trajectory tracking of the kinematic model of a unicycle type mobile robot in the presence of skidding effects. The sliding effect is modeled taking into account its most important component which is skidding for no straight trajectories. In [10], a robust adaptive controller based on the tunable dynamic oscillator was developed for a skid steer mobile robot in the presence of disturbances violating nonholonomic nonslipping constraint. Simulation results were performed to solve both the tracking and regulation problems.

In this paper, the robust trajectory tracking problem for a kinematic model of a car-like WMR in the presence of sliding effects is solved by means of a higher order sliding mode control. Using the singular perturbation approach [8], the kinematic tracking model of the car-like WMR is derived. Then, a second order sliding mode controller of the super twisting algorithm is used. The proposed control law is based on two nonlinear sliding manifolds ensuring the asymptotic tracking of the output variables in spite of the transgression of the nonholonomic constraints during the motion.

This paper is organized as follows, in Section 2 a kinematic tracking model considering sliding effects is derived and the problem statement is presented. The second order sliding mode control law is presented in Section 3, while Section 4 shows simulation results.

II. Model of the car-like and problem statement

The Robucar in a single drive mode is an example of car-like WMR. For model simplicity, we assume that each axle of the physical wheels (front and rear) is represented by a virtual wheel located in the middle axle. Then, the schematic representation of the car-like WMR can be given by Figure 1. It is fully described by a four-dimensional vector of generalized coordinates $q = (x_P, y_P, \theta, \phi)^t$ where x_P, y_P are the coordinates of point P, θ is the orientation angle of the vehicle w.r.t to a fixed frame and ϕ represents the front steering angle relative to the car body. L denotes the WMR's wheelbase.

Note that mobile robot of type (δ_m, δ_s) with a degree of mobility $\delta_m = 1$ and a degree of steerability $\delta_s = 1$ present four independent constraints (the non skidding constraint being the same for both driving wheels) of the form:

$$A^t(q)\dot{q} = 0 \tag{1}$$

with

$$\begin{aligned} A &= \begin{pmatrix} s(\theta) & s(\theta+\phi) & c(\theta) & c(\theta+\phi) \\ -c(\theta) & -c(\theta+\phi) & s(\theta) & s(\theta+\phi) \\ 0 & -s(\theta) & 0 & s(\theta) \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\ c(\theta) &= \cos(\theta), s(\theta) = \sin(\theta). \end{aligned}$$

which correspond to the ideal hypothesis of a " pure rolling and non slipping " condition.

Defining a full matix $S(q)$ such that $A^t(q)S(q) = 0$, a vector $v = (v_1, v_2)^t$ exists satisfying

$$\dot{q} = S(q)v \tag{2}$$

with

$$S = \begin{pmatrix} c(\theta) & 0 \\ s(\theta) & 0 \\ \tan(\phi) & 0 \\ 0 & 1 \end{pmatrix},$$

where v_1, v_2 represent respectively the linear driven (steering) velocity of the car-like WMR

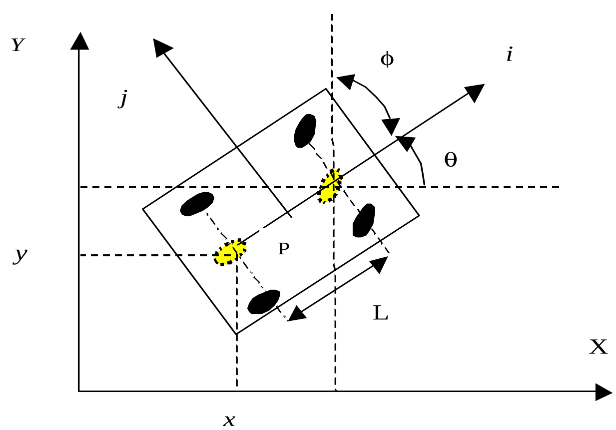

Figure 1. The car-like WMR.

In practise, it is well known that the nonholonomic constraints are never strictly satisfied during the motion of robot, due to various factors such as sliding, deformability or flexibility of the wheels. Hence, the kinematic model given by (2) is no longer valid. So, the interaction forces and slipping effects have to be modelled. Based on the singular perturbation formalism [8], the kinematic model of the car-like WMR with sliding can be expressed as:

$$\dot{q} = S(q)v + A_1\epsilon\mu_1 + A_2\epsilon\mu_2 + A_3\epsilon\mu_3 + A_4\epsilon\mu_4,$$

where A_1, A_2, A_3, A_4 are the colomns of the matrix A, μ is a four dimension vector reflecting the violation of the constraints and ϵ is a positive scalar, which is the inverse of the largest stiffness. $\epsilon\mu_1$ and $\epsilon\mu_2$ corresponds to skidding effects while $\epsilon\mu_3$ et $\epsilon\mu_4$ represents slipping effects.

Let v_x and v_y be the longitudinal and the lateral velocities of point P (see Figure 1), respectively, and v_θ the vehicle angular velocity:

$$\begin{cases} v_x = v_1 + s(\phi)\varepsilon\mu_2 + \varepsilon\mu_3 + c(\phi)\varepsilon\mu_4, \\ v_y = \varepsilon\mu_1 + c(\phi)\varepsilon\mu_2 - s(\phi)\varepsilon\mu_4, \\ v_\theta = \tan(\phi)v_1 - c(\phi)\varepsilon\mu_2 + s(\phi)\varepsilon\mu_4, \end{cases} \tag{3}$$

Under the hypothesis of a quite small front steering angle, the kinematic model of the car-like WMR is given in the base frame by time evolution of point P of coordinates:

$$\begin{pmatrix} \dot{x}_P \\ \dot{y}_P \\ L\dot{\theta} \end{pmatrix} = M(\theta) \begin{pmatrix} v_x \\ v_y \\ v_\theta \end{pmatrix}$$

with

$$M(\theta) = \begin{pmatrix} c(\theta) & s(\theta) & 0 \\ s(\theta) & -c(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The objective is to make the car-like robot follow a given reference trajectory that satisfies the kinematic equations of a virtual WMR:

$$\begin{pmatrix} \dot{x}_{Pr} \\ \dot{y}_{Pr} \\ L\dot{\theta}_r \end{pmatrix} = M(\theta_r) \begin{pmatrix} v_{xr} \\ v_{yr} \\ v_{\theta r} \end{pmatrix}, \tag{4}$$

where (x_{Pr}, y_{Pr}) is the position of a virtual reference vehicle at the point (P_r) in the base frame, θ_r is the orientation. v_{xr}, v_{yr} are the longitudinal lateral reference velocities at the point (P_r) and $v_{\theta r}$ is the angular velocity. The reference velocities, as well as their first and second time derivatives, are assumed to be bounded for all t.

Denote by $e = \begin{pmatrix} \widetilde{x} & \widetilde{y} & L\widetilde{\theta} \end{pmatrix}^t$ the position tracking error vector expressed in the frame linked to the virtual vehicle, i.e.:

$$\begin{pmatrix} \widetilde{x}(t) \\ \widetilde{y}(t) \\ L\widetilde{\theta}(t) \end{pmatrix} = M^t(\theta_r) \begin{pmatrix} x_P(t) - x_{Pr}(t) \\ y_P(t) - y_{Pr}(t) \\ L\theta(t) - L\theta_r(t) \end{pmatrix} \tag{5}$$

By time derivation of (4), the full kinematic tracking model for the car-like can be easily developed (see these for detail):

$$\begin{cases} \dot{\widetilde{x}} = \frac{v_{\theta r}}{L}\widetilde{y} + c(\widetilde{\theta})\widetilde{v}_x + s(\widetilde{\theta})\widetilde{v}_y + p_{\widetilde{x}}(t, \widetilde{\theta})\widetilde{\theta}, \\ \dot{\widetilde{y}} = -\frac{v_{\theta r}}{L}\widetilde{x} + s(\widetilde{\theta})\widetilde{v}_x - c(\widetilde{\theta})\widetilde{v}_y + p_{\widetilde{y}}(t, \widetilde{\theta})\widetilde{\theta}, \\ L\dot{\widetilde{\theta}} = \widetilde{v}_\theta. \end{cases} \tag{6}$$

where

$$p_{\widetilde{x}}(t,\widetilde{\theta}) = \frac{(\cos(\widetilde{\theta})-1)}{\widetilde{\theta}}v_{xr} + \frac{\sin(\widetilde{\theta})}{\widetilde{\theta}}v_{yr},$$
$$p_{\widetilde{y}}(t,\widetilde{\theta}) = \frac{\sin(\widetilde{\theta})}{\widetilde{\theta}}v_{xr} + \frac{(1-\cos(\widetilde{\theta}))}{\widetilde{\theta}}v_{yr}.$$

Note that $p_{\widetilde{x}}(t,\widetilde{\theta})$ and $p_{\widetilde{y}}(t,\widetilde{\theta})$ are bounded functions. Sliding effects are treated as kinematic perturbations. As we can see, the model rely on the measurement of the parameter perturbations. Hence, its accurate measurement is difficult to obtain. To introduce the perturbations which can be present during the motion of the robot (Robucar), we have negligated the sliding of the rear wheels ($\mu_2 = \mu_4 = 0$) and take into account only the sliding of the front wheels ($\epsilon\mu_1 \neq 0$, $\epsilon\mu_3 \neq 0$). Then, the system (3) becomes:

$$\begin{cases} v_x = v_1 + \varepsilon\mu_3 \\ v_y = \varepsilon\mu_1 \\ v_\theta = \tan(\phi)v_1 \end{cases} \tag{7}$$

In this situation, two solution are proposed here. The first one, by adding a term which represent the perturbations on the longitudinal velocity control v_x (longitudinal sliding or slipping), and the second one, by modifying the reference trajectory that the vehicle must follow (lateral sliding or skidding). We introduce the longitudinal (lateral) perturbation in the expression of the velocity control (the reference trajectory) respectively, as follow:

$$\begin{cases} v_x = v_1 + \varepsilon\mu_3 \\ v_{yr} = -c(\widetilde{\theta}).\varepsilon\mu_1 \end{cases} \tag{8}$$

In the case when no sliding occurs, it is obvious that $v_x = v_1$ and $v_y = 0$.

The problem addressed in this paper is to find a state feedback controller which can guarantee the asymptotic stabilization of system (6) about the origin, i.e:

$$\lim_{t\to\infty}\widetilde{x}(t) = 0, \lim_{t\to\infty}\widetilde{y}(t) = 0, \lim_{t\to\infty}\widetilde{\theta}(t) = 0.$$

For this, a robust nonlinear control law based on second order sliding mode is derived in this paper. It will be assumed that only x, y, θ are available for measurement. it is shown that both the position and the angular tracking errors of the robot are asymptotically stabilized in an arbitrarily small neighborhood of the origin.

III. Second order sliding mode controller

Sliding mode control approach exhibits relative simplicity of design and some robustness properties with respect to matching perturbations [11]. To overcome the well know chattering phenomenon while guaranteeing better convergence acuracy, second order sliding mode controllers have been proposed [12]. They are characterized by a discontinuous control acting on the second time derivatives of the sliding constraint s (instead of the first time derivative in classical sliding mode), whose vanishing defines the sliding manifold. The main principle of the second order sliding mode is to obtain a finite time convergence onto the second order sliding set $\{s = \dot{s} = 0\}$ (see, [14][13]).

Different kind of algorithms (twisting, drift , super twisting, sub-optimal...) able to ensure the finite time convergence have been proposed [13]. Here, the super twisting algorithm that only requires the knowlegde of the sliding surface will be adopted. In what follow, it is shown that both the position and the angular tracking errors of the robot are asymptotically stabilized in an arbitrarily small neighborhood of the origin.

A. Design and attractivity of the sliding manifold

Let us define the sliding constraint $s = [s_1, s_2]^T$ as:

$$s_1 = \lambda_1\widetilde{x} + p_{\widetilde{x}}(t,\widetilde{\theta})\widetilde{\theta}, \tag{9}$$
$$s_2 = p_{\widetilde{y}}(t,\widetilde{\theta})\widetilde{\theta} + \lambda_2\widetilde{y}. \tag{10}$$

where λ_1, λ_2 are positive parameters. Note that the system has relative degree one with respect to both s_1 and s_2 and that the second time derivatives $s_1$and s_2 of are given by:

$$\ddot{s}_1 = \psi_1(\widetilde{x},\widetilde{y},\widetilde{\theta},t) + \dot{v}_1$$
$$\ddot{s}_2 = \psi_2(\widetilde{x},\widetilde{y},\widetilde{\theta},t) + \dot{v}_2$$

For sake of place, the expression of $\psi_1(\widetilde{x},\widetilde{y},\widetilde{\theta},t)$ and $\psi_2(\widetilde{x},\widetilde{y},\widetilde{\theta},t)$ are not reported here. The task is to generate a second order sliding mode on the second order sliding set given by the equalities: $s = \dot{s} = 0$. For this, assume that the reference velocities (v_{1r}, v_{2r}) and their first and second time derivatives are bounded and that the functions $\psi_i(\widetilde{x},\widetilde{y},\widetilde{\theta},t)$ are bounded such that $\left|\psi_i(\widetilde{x},\widetilde{y},\widetilde{\theta},t)\right| \leq K_i,\ i = 1,2$ where K_i are positive constants. Then, it is known that one can apply the super twisting algorithm defined by the following control law [13]:

$$\begin{aligned} v_i &= -\lambda_{mi}\,|s_i|^{\frac{1}{2}}\, sign(s_i) + v_{1i}, \\ \dot{v}_{1i} &= -W_i sign(s_i), \qquad i = 1,2 \end{aligned} \tag{11}$$

where W_i and λ_{mi} are positive constants that satisfy the following conditions:

$$\begin{cases} W_i > K_i \\ \lambda_{mi}^2 \geq 4K_i\frac{W_i+K_i}{W_i-K_i} \end{cases}$$

It can be shown that the control laws (11) generate a second order sliding mode on the second order sliding set $\{s = \dot{s} = 0\}$ (see [14], [13]). In particular this implies that:

$$\lambda_1\widetilde{x} = -p_{\widetilde{x}}(t,\widetilde{\theta})\widetilde{\theta} \tag{12}$$
$$\lambda_2\widetilde{y} = -p_{\widetilde{y}}(t,\widetilde{\theta})\widetilde{\theta} \tag{13}$$

B. Asymptotic stability of the sliding motion

In order to show that, once in sliding mode, the posture errors of the robot are vanishing asymptotically, let us introduce the following candidate Lyapunov function:

$$V = \frac{1}{2}\left(\widetilde{x}^2 + \widetilde{y}^2\right).$$

The time derivative of V along the trajectories of the system is given by:

$$\dot{V} = \widetilde{x}\left(\frac{v_{\theta r}}{L}\widetilde{y} + c(\widetilde{\theta})\widetilde{v}_x + s(\widetilde{\theta})\widetilde{v}_y + p_{\widetilde{x}}(t,\widetilde{\theta})\widetilde{\theta}\right) + \widetilde{y}\left(-\frac{v_{\theta r}}{L}\widetilde{x} + s(\widetilde{\theta})\widetilde{v}_x - c(\widetilde{\theta})\widetilde{v}_y + p_{\widetilde{y}}(t,\widetilde{\theta})\widetilde{\theta}\right). \quad (14)$$

Replacing the expressions (12) and (13) in (14), one gets:

$$\dot{V} = -\lambda_1\widetilde{x}^2 - \lambda_2\widetilde{y}^2 + \widetilde{x}\left[c(\widetilde{\theta})\widetilde{v}_x + s(\widetilde{\theta})\widetilde{v}_y\right] + \widetilde{y}\left[s(\widetilde{\theta})\widetilde{v}_x - c(\widetilde{\theta})\widetilde{v}_y\right]$$

From the equations (12) and (13), we have:

$$\widetilde{y} = k(t,\widetilde{\theta})\widetilde{x}$$

with

$$k(t,\widetilde{\theta}) = \frac{\lambda_1 p_{\widetilde{y}}(t,\widetilde{\theta})}{\lambda_2 p_{\widetilde{x}}(t,\widetilde{\theta})}$$

Then one gets:

$$\dot{V} = -\lambda_1\widetilde{x}^2 - \lambda_2\widetilde{y}^2 + \widetilde{x}\widetilde{v}_x\left[c(\widetilde{\theta}) + k(t,\widetilde{\theta})s(\widetilde{\theta})\right] + \widetilde{x}\widetilde{v}_y\left[s(\widetilde{\theta}) - k(t,\widetilde{\theta})c(\widetilde{\theta})\right].$$

As $p_{\widetilde{x}}(t,\widetilde{\theta})$ and $p_{\widetilde{y}}(t,\widetilde{\theta})$ are bounded for all $\widetilde{\theta}$, one can write:

$$\left|(c(\widetilde{\theta}) + k(t,\widetilde{\theta})s(\widetilde{\theta}))\right| \leq \Gamma_1,$$
$$\left|(s(\widetilde{\theta}) - k(t,\widetilde{\theta})c(\widetilde{\theta}))\right| \leq \Gamma_2.$$

and suppose that

$$\|\widetilde{v}_x\| \leq \Gamma_3 + \Gamma_4\|X\|,$$
$$\|\widetilde{v}_y\| \leq \Gamma_5 + \Gamma_6\|X\|.$$

where Γ_1, Γ_2 Γ_3, Γ_4, Γ_5, Γ_6 are positive constants and $X = \begin{pmatrix} \widetilde{x} & \widetilde{y} \end{pmatrix}^t$.

Let us define $\lambda^\star = \min_{i=1,2}(\lambda_i)$, then one can write:

$$\dot{V} \leq -\lambda^\star\|X\|^2 + \|X\|^2\left(\frac{\Gamma_1\Gamma_3 + \Gamma_2\Gamma_5}{\|X\|} + \Gamma_1\Gamma_4 + \Gamma_2\Gamma_6\right)$$

Taking $\|X\| \geq \frac{\varepsilon}{2}$, this implies that

$$\dot{V} \leq -\lambda^*\|X\|^2 + \|X\|^2\left(\frac{2(\Gamma_1\Gamma_3 + \Gamma_2\Gamma_5)}{\varepsilon} + \Gamma_1\Gamma_4 + \Gamma_2\Gamma_6\right).$$

Define the ball $\mathcal{B}_{\varepsilon/2} = \left\{X : \|X\| \leq \frac{\epsilon}{2}\right\}$. It results that outside the ball $\mathcal{B}_{\varepsilon/2}$, one has

$$\dot{V} \leq -\widetilde{\lambda}\|X\|^2 = -2\widetilde{\lambda}V$$

with

$$\widetilde{\lambda} = \lambda^\star - \left(\frac{2(\Gamma_1\Gamma_3 + \Gamma_2\Gamma_5)}{\varepsilon} + \Gamma_1\Gamma_4 + \Gamma_2\Gamma_6\right).$$

Thus $\dot{V}$ will be negative definite if the the parameter $\lambda^\star$ is chosen as:

$$\lambda^\star > \left(\frac{2(\Gamma_1\Gamma_3 + \Gamma_2\Gamma_5)}{\varepsilon} + \Gamma_1\Gamma_4 + \Gamma_2\Gamma_6\right)$$

Hence, with this choice of $\lambda^\star$ the solution of the system is given by

$$\|X(t)\| \leq \|X(0)\|\exp\left(-\widetilde{\lambda}t\right)$$

and there exists a finite time t_1 such that $\forall\, t > t_1 : X(t) \in \mathcal{B}_\varepsilon$. Thus, $\widetilde{x}$, $\widetilde{y}$ are stabilized in an arbitrarily small neighborhood of the origin. From the equations (12) and (13), it can be seen that the orientation angle error $\widetilde{\theta}$ is also stabilized in an arbitrarily small neighborhood of the origin.

IV. Simulation results

In this section, simulation results on the kinematic model (6) using the proposed controller are presented. The sampling time is chosen to be $0.01s$ with the physical paramter $L = 1.2m$. The car-like WMR must follow a reference trajectory of a circular path in a time interval $T = 40s$. Simulation was performed testing the developed control both without and with sliding. The design of parameters of sliding surfaces and control gain are: $\lambda_1 = 1.5, \lambda_2 = 0.4, \lambda_{mi} = 1, W_i = 10^{-5}(i = 1,2)$. Simulated tracking responses of the car-like WMR given in the base frame are reported relative to the two classes of simulations. Simulation results of the trajectory tracking of the car-like considering the ideal case (without sliding) are given in Figure 2 with initial condition: $\widetilde{x}(0) = 0.2m, \widetilde{y} = 0.1m, \widetilde{\theta}(0) = -0.1rd, v_{xr}(t) = 1m.s^{-1}, v_{\theta r}(t) = 0.03rd.s^{-1}$. The position tracking errors are given in the first line and the second line provides the orientation and steering angles errors, while the last line gives the input time error response (driven velocity) and the trajectories in the phase plane (x, y). We remark that all the errors converge to the origin after a time equal to $10s$. Despite the initial errors of the car-like, this one joins the path made by the virtual vehicle and the control activity is accpetable and exhibits no chattering.

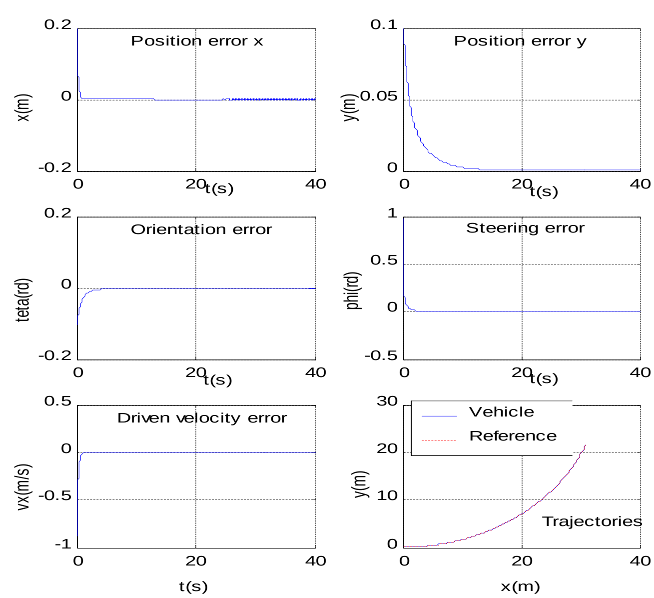

Figure 2. Tracking of the car-like without sliding.

As noted in the second section, two types of pertubations were introduced and applied to the car-like WMR at time $5s$. The first one is reported in Figure 3, representing the vehicle slipping and the second one the vehicle skidding while curving, see Figure 4. Simulation results for the trajectory tracking of the car-like considering only the slipping and both slipping and skidding are shown respectively in Figures $5, 6$ with null initial posture condition and $v_{xr}(t) = 0.6m.s^{-1}, v_{\theta r}(t) = 0.06rd.s^{-1}$. The Figures $5, 6$ shown that at the introduction of pertubations, the control inputs (driven velocity and steering angle) changes the values for converging to the desired reference one after the annulation of perturbations at time $t = 10s$. It can be seen that, the tracking of the car-like diverge quietly from the reference trajectory at time $t = 5s$ and its remains close to the reference trajectory after $t = 10s$. From these results, the robustness with respect to the sliding effects of the proposed sliding mode controller is confirmed.

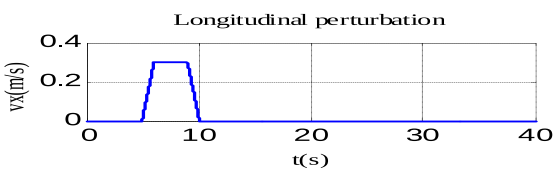

Figure 3. Perturbation: case of slipping.

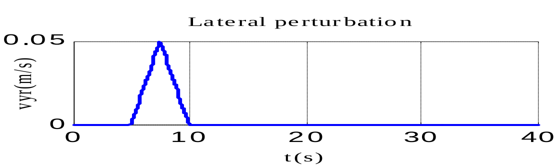

Figure 4. Perturbation: case of skidding.

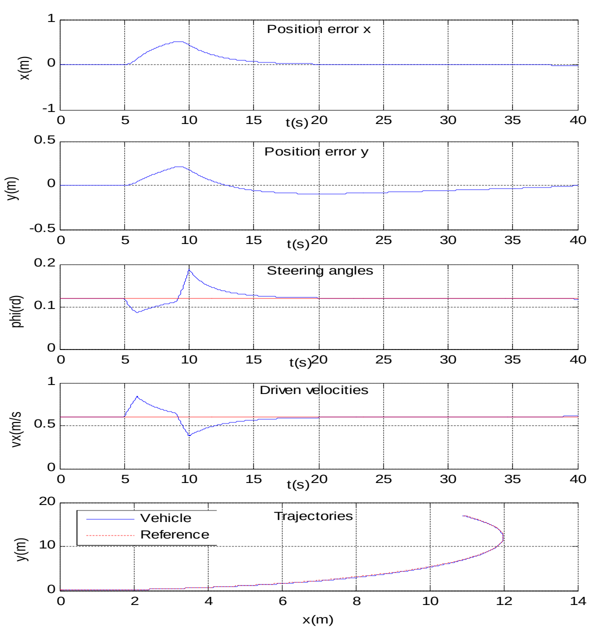

Figure 5. Tracking of the car-like with slipping.

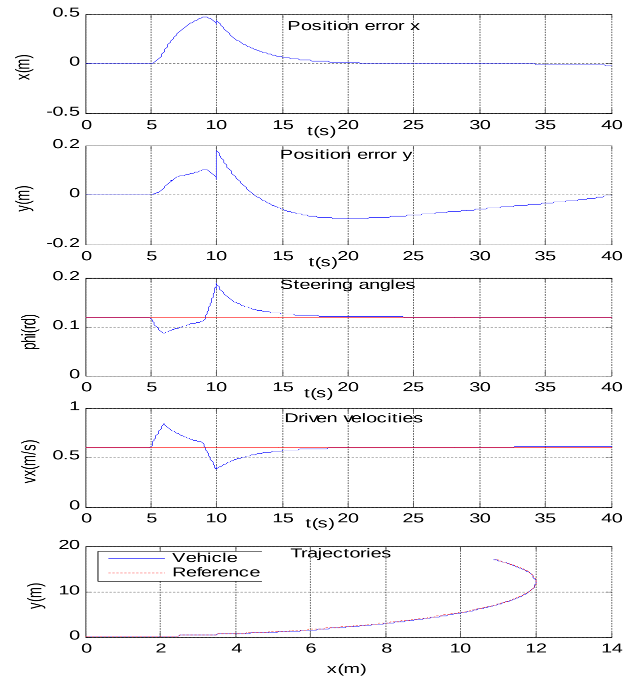

Figure 6. Tracking of the car-like with sliding.

V. Conclusion

In this paper, we suggest higher order sliding mode control approach for solving the trajectory tracking problem for the car-like WMR considering sliding effects. Based on the tracking kinematic model of the car-like WMR, a second order sliding mode controller has been developed using the super twisting algorithm. The asymptotic convergence of the tracking errors has been proven by means of the Lyapunov function method. However, due to the nonlinearities, those errors are only stabilized in an arbitrarily small neighborhood of the origin. Simulation results are reported, showing the robustness of the proposed control law against nonlinearities and sliding effects. As a future work, it will be interesting to test the proposed tracking controller in experimentation and show how this approach works in real problems.

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