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A Parameter-free Method for Sensor Fault Detection and Isolation in Bilinear Systems

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***Abstract*— This paper is concerned with Fault Detection and Isolation (FDI) and more specifically it focuses on a parameter-free residual generation method. The residual signals are obtained by projecting the measured signals onto the kernel of an extended input matrix, which depends on the structure of the system model. The method was not easily applicable in real-world applications due to a high computational complexity. In that paper, fault indicators are constructed differently, using kernels properties, to avoid this complexity problem. A simulated electromechanical actuator example is taken to illustrate the applicability of the method.**

***Index Terms*— Fault detection and isolation, data driven methods, bilinear systems, electromechanical actuator.**

I. INTRODUCTION

Real plants are subject to faults, that can affect the process parameters, actuators or sensors. Online fault detection and isolation (FDI) is an important task for human safety and system dependability. Many approaches have been reported in the literature to achieve this task. Two kinds of FDI methods are distinguished [16]. Model-based and model-free methods. Model-based methods consist in comparing the actual system behavior with the one given by an analytical model, i.e. a set of nonlinear differential equations. A signal called residual is used to evaluate this comparison. In the absence of fault and noise, if the process and the model are exactly matched, the residual is zero, otherwise it is different from zero which characterizes fault occurrence. The main common methods for model-based residual generation are:

- observer-based methods [5], [6] and [7]
- analytical redundancy relation (ARR) -based methods [1]
- parameters estimation methods [4], [8]

Unfortunately, the values of the model parameters are unknown in most practical applications. For such case, model-free FDI methods [2], [13] have been proposed. Some of these methods use signal processing techniques to extract special properties of measured signals, these methods are called signal-based methods (see [16] and [17]). Other data driven methods have been proposed recently for switching systems in [10] and [11], for bilinear systems in [11].

The residual generation method that is proposed in that paper is situated between model-based and model-free methods, since the only information we need is the knowledge of the input-output data and of the structure. The values of the model parameters are not needed.

The advantages of the proposed method are as follows:

- The only needed data are inputs and outputs.
- The generated residuals are structured which allows faults isolation.
- Multiple faults may be considered.

The bilinear system is a particular structure of nonlinear systems, this special class of systems has been widely studied in recent years [3]. Many real-world dynamical systems may be represented by a bilinear model and such model can approximate a large class of nonlinear systems. Consequently, bilinear models study is interesting from both theoretical and practical points of view.

The data-projection method for residual generation was extended for bilinear structure models in a previous publication by the authors [15].

The target of this paper is to enhance the parameter-free residual generation method proposed in [15]. The general principle of the method is kept. However, using kernel properties, fault indicators are computed differently, to avoid the computational complexity of method in [15].

The remainder of this paper is organized as follows. A general description of our residual generation method for bilinear models is provided in section 3.The input/output relation is derived in section 3 while the residual expression is derived in section 4. In section 5, simulation results on an electromechanical actuator are presented to show the effectiveness of our method. The final section gives a conclusion.

II. OVERVIEW OF THE PARAMETER-FREE RESIDUAL GENERATION METHOD

Consider known inputs $u_k \in R^m$ and outputs $y_k \in R^\ell$ affected by colored white noise $w_k \in R^\ell$. These input/output signals are supposed to be collected on a physical plant that can be modeled as a discrete-time bilinear system given by:

$$\begin{cases} x_{k+1} = Ax_k + G(x_k \otimes u_k) + Bu_k \\ y_k = Cx_k + Du_k + f_k + w_k \end{cases} \quad (1)$$

where $\otimes$ represents the Kronecker product, and $f_k \in R^\ell$ is the sensor fault vector. It is supposed that the linear dynamic is stable i.e. $\|A\|_2^i \to 0$.

The aim is to detect and to isolate sensor faults when supposing that the only available information is the system structure (bilinear) and input/output data. The system parameters($A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{\ell\times n}$, $D \in R^{\ell\times m}$, $G \in R^{n\times nm}$) are supposed to be unknown.

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The data-based residual generation method for bilinear systems is detailed in the next section. Let us give the general principle of this method. Under the stability conditions, it is possible to express the vector of measured outputs on a given time-window as a function of the inputs. The following expression is thus obtained.

$$Y \cong HM(u, y \otimes u) \tag{2}$$

where H depends on the system parameters, Y is a matrix of outputs collected on a given time-window, $M(u, y \otimes u)$ is a matrix constructed using inputs and the Kronecker product between inputs and outputs. If the chosen time-window is sufficiently large, we can then project equation (2) on the right kernel Π of $M(u, y \otimes u)$ ($UM(u, y \otimes u)\Pi = 0$) and we can derive the relation:

$$Y\Pi = 0 \tag{3}$$

This relation must be verified in absence of disturbances and faults. Consider Y_{online} and Π_{online}, the Y and Π matrices which are computed using online taken values of inputs and outputs signals. In the no fault situation, the signal $\epsilon = Y_{online}\Pi_{online}$, is not exactly null because of the measurement noise. However, it can be proved that $r = E[\epsilon]$ equals 0, with $E[.]$ the mathematical expectation. When a fault occurs, $r = E[Y_{online}\Pi_{online}]$ becomes different from zero. Thus, r can be considered as a fault indicator (residual) to be used for FDI.

It is clear that no system parameter or state estimation is needed for residual computation since Π depends only on inputs, which makes residual expression (3) independent on model parameters.

III. Input-output expression of bilinear systems

The objective of this section is to show how to derive equation (2) from system (1). A general expression of the output y_k is first obtained. Then it is shown that the influence of the state may be neglected under the stability conditions, which leads to equation (2).

A general output expression y_k can be derived, which is given in the following proposition and proved by induction.

Proposition 1: The general expression of the output y_k in function of the state x_{k-i}, the inputs and system parameters A, B, C, D, G is given by:

$$\forall i \geq 0 : y_k = CA^i x_{k-i} + \widetilde{H}_i \overline{z}_{k,i} + \overline{H}_i \overline{u}_{k,i} + f_k + w_k \tag{4}$$

where $\widetilde{H}_i$, $\overline{z}_{k,i}$, $\overline{H}_i$, $\overline{u}_{k,i}$ are given as follows.

a) $\widetilde{H}_i$ and $\overline{H}_i$ depend only on system matrices.

- $\widetilde{H}_i$ depends on the system matrices C, A and G:

$\widetilde{H}_i = \left[CA^{i-1}G| \cdots | CAG| CG\right] \in R^{\ell \times nmi}$.

- $\overline{H}_i$ depends on the system parameters C, A and B:

$\overline{H}_i = \left[CA^{i-1}B| \cdots | CB| D\right] \in R^{\ell \times m(i+1)}$.

b) $\overline{z}_{k,i}$ and $\overline{u}_{k,i}$ depend on the system states and inputs on a time-window of size i.

$\overline{z}_{k,i} = \left[z_{k-i}^T| \cdots | z_{k-2}^T| z_{k-1}^T\right]^T \in R^{nmi \times 1}$.
with $z_{k-i} = x_{k-i} \otimes u_{k_i}$.
and $\overline{u}_{k,i} = \left[u_{k-i}^T| \cdots | u_{k-1}^T| u_k^T\right]^T \in R^{m(i+1) \times 1}$.

Proof:

Inductive method is used to prove correctness of the general output expression (4), which can be written differently without using matrix representation:

$$\begin{aligned} y_k &= CA^i x_{k-i} + C\sum_{j=0}^{i-1} A^j G.z_{k-j-1} \\ &+C(Du_k + \sum_{j=0}^{i-1} A^j Bu_{k-j-1}) + f_k + w_k \end{aligned} \tag{5}$$

Expression (5) is verified for $i = 0$ and $i = 1$. Assuming that the proposal holds for i, let us prove that it holds for $i + 1$ also:

$$\begin{aligned} y_k &= CA^{i+1} x_{k-i-1} + C\sum_{j=0}^{i} A^j G.z_{k-j-1} \\ &+C(Du_k + \sum_{j=0}^{i} A^j Bu_{k-j-1}) + f_k + w_k \end{aligned} \tag{6}$$

We replace $x_{k-i} = Ax_{k-i-1} + G.z_{k-i-1} + Bu_{k-i-1}$ into (5), and derive the following equation:

$$\begin{aligned} &y_k = CA^{i+1} x_{k-i-1} + \\ &C(A^i G.z_{k-i-1} + \sum_{j=0}^{i-1} A^j G.z_{k-j-1}) \\ &+C(A^i Bu_{k-i-1} + Du_k + \sum_{j=0}^{i-1} A^j Bu_{k-j-1}) \\ &+f_k + w_k \end{aligned} \tag{7}$$

By identifying expressions (6) and (7), it is straightforward to prove that the proposal for $i+1$ (equation (6)) holds. This ends the proof. ■

Because the linear dynamic is supposed to be stable, i.e. A^i tends to zero for i sufficiently large which results to

$$CA^i x_{k-i} \to 0 \tag{8}$$

As a consequence, for i sufficiently large, the state influence may be neglected in expression (4). This leads to the following approximated expression of the output y_k:

$$\begin{aligned} y_k &\cong C\sum_{j=0}^{i-1} A^j G.z_{k-j-1} \\ &+C(Du_k + \sum_{j=0}^{i-1} A^j Bu_{k-j-1}) \\ &+f_k + w_k \end{aligned} \tag{9}$$

The matrix representation of expression (9) is given by:

$$\forall i \geq 0 : y_k = \widetilde{H}_i \overline{z}_{k,i} + \overline{H}_i \overline{u}_{k,i} + f_k + w_k \tag{10}$$

IV. Data-projection residual generation

In this section, a data-based residual ϵ_k is generated for sensor fault detection and isolation.
By right-multiplying (Kronecker product) the measurement equation of system (1) by u_k, and using the following Kronecker product properties:

- $(Q_1 \otimes Q_2)(Q_3 \otimes Q_4)=(Q_1\ Q_3) \otimes (Q_2\ Q_4)$
- $(Q_1 \otimes Q_2)=(Q_1 \otimes I)Q_2$

where Q_1, Q_2, Q_3 and Q_4 are matrices with appropriate dimensions, we can derive the following expression:

$$\begin{aligned} p_k = (C \otimes I_m)z_k + (D \otimes I_m)q_k+ \\ (f_k \otimes u_k) + (w_k \otimes u_k) \end{aligned} \tag{11}$$

where I_m is the identity matrix of dimension $m \times m$ and

$$\begin{cases} z_k = x_k \otimes u_k \\ p_k = y_k \otimes u_k \\ q_k = u_k \otimes u_k \end{cases}.$$

Consider an integer L, which is chosen such that $L > mi + \ell$. The following subsequent vectors and matrices are introduced:

$$s_k = \begin{bmatrix} p_k \\ q_k \end{bmatrix} = \begin{bmatrix} y_k \otimes u_k \\ u_k \otimes u_k \end{bmatrix} \in R^{(\ell+m)m\times 1}$$

$$\overline{s}_{k,i} = \left[s_{k-i}^T | \cdots | s_{k-2}^T | s_{k-1}^T \right]^T \in R^{(\ell+m)mi\times 1}.$$

$$S_k = \left[\overline{s}_{k-L+1,i} \cdots \overline{s}_{k-1,i}\, \overline{s}_{k,i} \right] \in R^{(\ell+m)mi\times L}.$$

$$Z_k = \left[\overline{z}_{k-L+1,i} \cdots \overline{z}_{k-1,i}\, \overline{z}_{k,i} \right] \in R^{nmi\times L}.$$

$$\overline{f}_{k,i} = \left[(f_{k-i} \otimes u_{k-i})^T | \cdots | (f_{k-2} \otimes u_{k-2})^T | (f_{k-1} \otimes u_{k-1})^T \right]^T \in R^{\ell mi\times 1}.$$

$$\overline{F}_k = \left[\overline{f}_{k-L+1,i} \cdots \overline{f}_{k-1,i}\, \overline{f}_{k,i} \right] \in R^{\ell mi\times L}.$$

$$\overline{w}_{k,i} = \left[(w_{k-i} \otimes u_{k-i})^T | \cdots | (w_{k-2} \otimes u_{k-2})^T | (w_{k-1} \otimes u_{k-1})^T \right]^T \overline{w}_{k,i} \in R^{\ell mi\times 1}.$$

$$\overline{W}_k = \left[\overline{w}_{k-L+1,i} \cdots \overline{w}_{k-1,i}\, \overline{w}_{k,i} \right] \in R^{\ell mi\times L}.$$

$$M_i = \begin{bmatrix} C \otimes I_m & 0_{\ell m\times nm} & \cdots & 0_{\ell m\times nm} \\ 0_{\ell m\times nm} & C \otimes I_m & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0_{\ell m\times nm} \\ 0_{\ell m\times nm} & \cdots & 0_{\ell m\times nm} & C \otimes I_m \end{bmatrix} \in R^{\ell mi\times nmi}.$$

$$K_i = \begin{bmatrix} \left[I_{\ell m} | D \otimes I_m \right] & 0_{\ell m\times(\ell m+m^2)} & \cdots & 0_{\ell m\times(\ell m+m^2)} \\ 0_{\ell m\times(\ell m+m^2)} & \left[I_{\ell m} | D \otimes I_m \right] & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0_{\ell m\times(\ell m+m^2)} \\ 0_{\ell m\times(\ell m+m^2)} & \cdots & 0_{\ell m\times(\ell m+m^2)} & \left[I_{\ell m} | D \otimes I_m \right] \end{bmatrix}$$

where $K_i \in R^{\ell mi\times(\ell+m)mi}$.
By putting the terms dependent on the system input and output on the left side of equality (11), the rest of the terms are put on the right side, the following expression is derived at time k:

$$\begin{aligned} \left[I_{\ell m} | D \otimes I_m \right] s_k = \\ (C \otimes I_m)z_k + (f_k \otimes u_k) + (w_k \otimes u_k) \end{aligned} \tag{12}$$

Expression (12) at $k-1$ is given similarly as follows:

$$\begin{aligned} \left[I_{\ell m} | D \otimes I_m \right] s_{k-1} = \\ (C \otimes I_m)z_{k-1} + (f_{k-1} \otimes u_{k-1}) + (w_{k-1} \otimes u_{k-1}) \end{aligned} \tag{13}$$

Equation (13) can be rewritten differently as follows:

$$\begin{aligned} & 0_{\ell m\times(\ell m+m^2)}s_{k-i} + \cdots + 0_{\ell m\times(\ell m+m^2)}s_{k-2}+ \\ & \left[I_{\ell m} | D \otimes I_m \right] s_{k-1} = 0_{\ell m\times nm}z_{k-i} + \cdots + 0_{\ell m\times nm}z_{k-2} \\ & +(C \otimes I_m)z_{k-1} + (f_{k-1} \otimes u_{k-1}) + (w_{k-1} \otimes u_{k-1}) \end{aligned} \tag{14}$$

By writing expression (12) at $k-2$

$$\begin{aligned} \left[I_{\ell m} | D \otimes I_m \right] s_{k-2} = \\ (C \otimes I_m)z_{k-2} + (f_{k-2} \otimes u_{k-2}) + (w_{k-2} \otimes u_{k-2}) \end{aligned} \tag{15}$$

Equation (15) can be rewritten differently as follows:

$$\begin{aligned} & 0_{\ell m\times(\ell m+m^2)}s_{k-i} + \cdots + 0_{\ell m\times(\ell m+m^2)}s_{k-3}+ \\ & \left[I_{\ell m} | D \otimes I_m \right] s_{k-2} + 0_{\ell m\times(\ell m+m^2)}s_{k-1} = \\ & 0_{\ell m\times nm}z_{k-i} + \cdots + 0_{\ell m\times nm}z_{k-3} + (C \otimes I_m)z_{k-2}+ \\ & 0_{\ell m\times nm}z_{k-1} + (f_{k-2} \otimes u_{k-2}) + (w_{k-2} \otimes u_{k-2}) \end{aligned} \tag{16}$$

By following the same procedure till $k-i$, the common matrix representation of all the obtained equations is given by:

$$K_i\overline{s}_k = M_i\overline{z}_k + \overline{f}_k + \overline{w}_k \tag{17}$$

By concatenating equation (17) over columns on a time-window of size L, we can derive the following equation:

$$K_iS_k = M_iZ_k + \overline{F}_k + \overline{W}_k \tag{18}$$

The derived equation (18) and following theorems will be useful in the sequel. We are now ready to express the main results of our paper as Theorem 4.2 and Proposition 2.

Theorem 4.1: If the number of independent rows of the matrix C is equal or greater than the number of independent columns of C, then the matrix M_i is left invertible. In other words, it exists a matrix V_i such that the following relation holds:

$$V_iM_i = I_{i\alpha} \tag{19}$$

where α is the number of independent rows of the matrix C. If the number of independent rows of the matrix C is equal to the number of independent columns of C, then $V_i = M_i^{-1}$.
If the number of independent rows of the matrix C is greater than the number of independent columns of C, then $V_i = (M_i^TM_i)^{-1}M_i^T$.

Theorem 4.2: If $\Gamma \in R^{\cdot\times L}$ is a matrix, where the number of independent rows is equal or less than the number of independent columns, then the right kernel of Γ is given by:

$$\Pi_\Gamma = I_L - \Gamma^T(\Gamma\Gamma^T)^{-1}\Gamma \in R^{L\times L} \tag{20}$$

where Π_Γ is the right projection matrix of Γ, and consequently we have:

$$\Gamma\Pi_\Gamma = 0 \tag{21}$$

Using left invertibility property of M_i, and by left multiplying equation (18) by V_i, matrix Z_k is given by

$$Z_k = V_iK_iS_k - V_i\overline{F}_k - V_i\overline{W}_k \tag{22}$$

Right projecting matrix Z_k on the right kernel matrix $\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]}$ of $\begin{bmatrix} S_k \\ U_k \end{bmatrix}$, the equation (22) becomes:

$$Z_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} = -V_i\overline{F}_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} - V_i\overline{W}_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} \tag{23}$$

where $U_k = \left[\overline{u}_{k-L+1,i}|\cdots|\,\overline{u}_{k-1,i}|\,\overline{u}_{k,i}\right] \in R^{m(i+1)\times L}$.

Proposition 2: The proposed parameter-free residual is defined as follows:

$$\epsilon_k = Y_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} \tag{24}$$

where $Y_k = \left[y_{k-L+1,i}|\cdots|\,y_{k-1,i}|\,y_{k,i}\right] \in R^{\ell\times L}$.

The mathematical expectation of ϵ_k,$r = E[\epsilon_k]$ is used for fault detection.

Proof:

By concatenating equation (10) over columns on a time-window of size L, we can derive the following equation:

$$\forall i \geq 0 : Y_k = \widetilde{H}_iZ_k + \overline{H}_iU_k + F_k + W_k \tag{25}$$

where F_k and W_k are constructed similarly as Y_k.

1) If there is no sensor fault ($f_k = 0$): [we will prove in this case that $E[\epsilon_k] = 0$]

   By concatenating the output in equation (4) on a time-window of size L, the evaluation form of the proposed parameter-free residual is given by:

$$\epsilon_k = Y_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} \cong \widetilde{H}_iZ_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} + W_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} \tag{26}$$

   where W_k and F_k are constructed similarly as Y_k. By replacing (23) into (26), and knowing that $\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]}$ is a right kernel of U_k, we get:

$$\epsilon_k \cong -\widetilde{H}_iV_i\overline{W}_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} + W_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} \tag{27}$$

   The evaluation form (27) of the proposed residual is a linear combination of a centered noise w, which implies that $E[\epsilon_k] = 0$ when there is no sensor fault.

2) If there is a sensor fault: [we will prove in this case that $E[\epsilon_k] \neq 0$]

   From equation (27), the evaluation form of the proposed parameter-free residual is given by:

$$\begin{aligned}\epsilon_k \cong &-\widetilde{H}_iV_i\overline{F}_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} - \widetilde{H}_iV_i\overline{W}_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} + \\ &W_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} + F_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]}\end{aligned} \tag{28}$$

   Following the same procedure as in the no fault case to get the equation (27), the evaluation form of the mathematical expectation of the proposed (28) parameter-free residual becomes:

$$\begin{aligned}E[\epsilon_k] \cong &-E[\widetilde{H}_iV_i\overline{F}_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]}] + \\ &E[F_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]}]\end{aligned} \neq 0 \tag{29}$$

   which shows the sensitivity of the mathematical expectation of the proposed residual to sensor faults.

A. Sensor fault isolability

As shown previously the sensor fault is detectable, in addition to that an important process is to isolate this fault which means the decision on which sensor is in faulty case. To achieve this process we distinguish two cases, when the sensor fault is a constant bias fault at least during the time window of $L + i + 1$ and when it is not.

When the sensor fault is a constant bias fault at least during the time window of $L+i+1$, we have the following expression:

$$-\widetilde{H}_iV_i\overline{F}_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} = 0 \tag{30}$$

As a result the expression (28) becomes:

$$\epsilon_k \cong -\widetilde{H}_iV_i\overline{W}_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} + W_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} + F_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} \tag{31}$$

Following the same procedure as in the no fault case to get the equation (27), the evaluation form of the mathematical expectation of the proposed (31) parameter-free residual becomes:

$$E[\epsilon_k] \cong F_k\Pi_{\left[\begin{smallmatrix} S_k \\ U_k \end{smallmatrix}\right]} \tag{32}$$

which shows that the mathematical expectation of the proposed residual is structured, which means that the first row of $E[\epsilon_k]$ is dedicated for the first sensor, the second row of $E[\epsilon_k]$ is dedicated for the second sensor and so on, in this case only the corresponding mathematical expectation of the residual is not zero when the corresponding sensor is in a faulty case.

If the fault fluctuates instantaneously and $\widetilde{H}_iV_i = 0$, it is the same case as for constant bias sensor faults.

But if the fault fluctuates instantaneously and $\widetilde{H}_iV_i \neq 0$, a more convenient decision algorithm should be developed and this case is not treated in this paper for brevity reasons (for signature table the reader is referred to [20] and [21]).

■

V. EXAMPLE AND SIMULATION

An electromechanical actuator [19] is used to show the effectiveness of the proposed residual generation method for sensor fault detection and isolation.

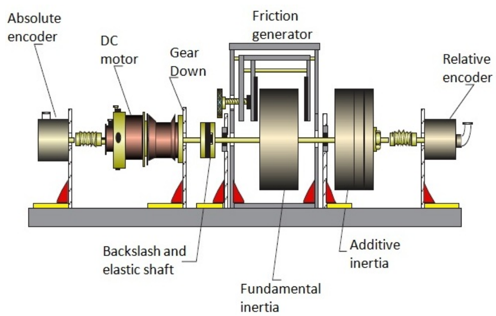

Fig. 1. Electromechanical actuator

This plant may be modeled by a bilinear state-space model:

$$\begin{cases} x_{k+1} = Ax_k + G(x_k \otimes u_k) + Bu_k \\ y_k = Cx_k + Du_k + f_k + w_k \end{cases} \tag{33}$$

with

$$A = \begin{pmatrix} -\frac{R_a}{L_a} & 0 & 0 & 0 \\ 0 & -\frac{F_m}{J_m} & -\frac{k_r}{NJ_m} & 0 \\ 0 & \frac{1}{N} & 0 & -1 \\ 0 & 0 & \frac{k_r}{J_c} & -\frac{F_c}{J_c} \end{pmatrix},$$

$$G = \begin{pmatrix} 0 & 0 & -\frac{k_a}{L_a} & 0 & 0 & 0 & 0 & 0 \\ \frac{k_a}{J_m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 & \frac{1}{L_a} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, D = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

The model parameters are defined in table 1, these parameters are not used for residuals generation but they are used to simulate the model and to generate output data.

Parameter	Description	Value	Unit
T_e	sampling time	0.03	$[sec]$
J_m	motor shaft inertia	$2.4e-4$	$[m^2kg]$
J_c	load shaft inertia	0.0825	$[m^2kg]$
F_m	motor viscous friction	0.0032	$[m^2kg/sec]$
F_c	load viscous friction	0	$[m^2kg/sec]$
k_a	motor torque constant	0.156	$[m^2kg/sec^2]$
k_r	coupling rigidity coefficient	37.7	$[m^2kg/sec^2]$
R_a	motor resistance	1	$[\Omega]$
L_a	motor inductance	0.05	$[H]$
N	gear ratio	20	
i	time-window	16	
L	time-window	339	

Table 1

The 4 states are given in table 2:

State	Description
i_a	armature current
w_m	motor shaft velocity
Δ	angular rotation
w_c	load shaft angular velocity

Table 2

The input vector is plotted in Fig.2:

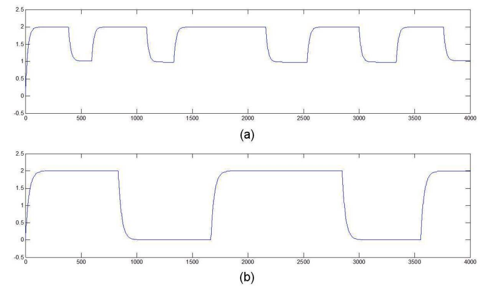

Fig. 2. (a): $u(1,:) = i_e$ is the stator current, (b): $u(2,:) = v_a$ is the armature voltage

The two outputs are plotted in Fig.3:

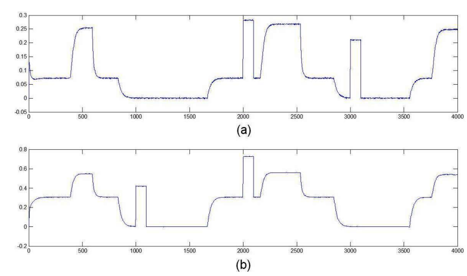

Fig. 3. (a): $y(1,:)$ is the armature current, (b): $y(2,:)$ is the angular velocity

There are two calculated residuals since the number of sensors is 2, the calculated residuals using the proposed method are presented in Fig.4.

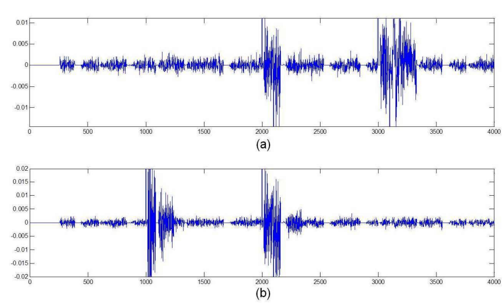

Fig. 4. (a): $\epsilon_k(1)$ is the first residual, (b): $\epsilon_k(2)$ is the second residual

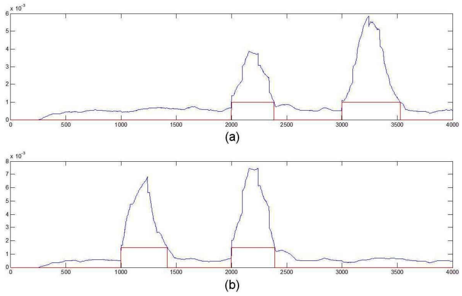

Fig. 5. Blue:(a): $E[\epsilon_k(1)]$, (b): $E[\epsilon_k(2)]$, Red: Thresholding the finite moving average of the proposed residual

In Fig. 5, the blue curve represents the mathematical expectation of the proposed residuals, this mathematical expectation is calculated in a moving time-window of size 339. The red curve represents the result of the decision procedure called Finite Moving Average (FMA) [22] (see the FMA algorithm in Fig. **??**), this is due to the fault sensitivity of the residual mean, which seems to be well dedicated to decide whether there is a fault or not, a good choice of the threshold is needed which can be achieved offline using a healthy database, where the threshold is chosen greater than the maximum value of $E[\epsilon_k]$, for this example the threshold is equal to 0.001 for the first sensor and 0.0015 for the second sensor. Moreover, the FDI can be realized if multiple faults occur simultaneously.

CONCLUSION

A data-projection residual generation method is presented for bilinear systems, where an output matrix is projected on the input right kernel matrix. A new way for input-output matrices construction is proposed to avoid complexity problem of the method presented in paper [15]. The online diagnosis is then easily implemented. Simulation results of an electromechanical actuator show the effectiveness of the proposed method.

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