

Mean-Square Exponential Stabilization of Packet-Based Networked Systems with Time-Varying Transmission Delays, Packet Losses and Input Missing

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Abstract—This paper is concerned with the mean-square exponential stabilization of packet-based networked systems with time-varying transmission delays, packet losses and input missing. Based on a multivariate i.i.d. model of input delays and a Bernoulli model of input missing, the packet-based networked system is formulated as a switching system with multiple subsystems of different input delays. The sufficient fast-switching conditions are directly established for the closed-loop mean-square exponential stability via an appropriate Lyapunov-Krasovskii approach different from average dwell time approach. The resulting controller design method can be obtained by cone complementarity linearization (CCL). Numerical example is also given to substantiate the effectiveness of our results.

Keywords—Networked systems; packet-based; transmission delays; packet losses; input missing

I. INTRODUCTION

Networked control systems (NCSs) have attracted much attention in recent years for its advantages over conventional control systems, such as low installation cost, less power consumption, simple maintenance and flexible configuration [1]-[3]. But transmission delays, packet losses, and input missing are common phenomena in NCSs, which often degrade the system performance and even lead to the closed-loop instability.

Transmission delays and packet losses are time-varying and random in nature, and often taken as input delays of NCSs [2], [3]. Bernoulli process is widely used to model discrete transmission delays and packet losses [4]-[6]. By transforming continuous input delays as discrete interval-distributed variables, Bernoulli process is also an appropriate model to explore the binary interval-distributed property of continuous input delays [7]-[9]. Recently, the multivariate independent and identically distributed process (i.i.d) has been proposed as a more realistic and general model to formulate discrete and continuous random input delays [10]-[13]. Based on i.i.d. models, NCSs with time-varying transmission delays and packet losses were essen-

tially switching systems. NCSs with discrete i.i.d. input delays have attracted much research attention and many results have been reported in the literatures, see [10] and [11] for overall system performance conditions dependent and independent on average dwell time of switching subsystems, respectively. But for continuous i.i.d. input delays, most of results are additionally dependent on average dwell time of switching subsystems to guarantee closed-loop mean-square stability, except [7]-[9] where continuous binary i.i.d. input delays, i.e. continuous Bernoulli input delays, were considered. Unfortunately, the results of [7]-[9] could not be extended to the case of continuous multivariate i.i.d. input delays.

Input missing, resulted from control failure, actuator failure or energy-saving consideration, is a common event in NCSs and has an innegligible effect on system performance [14]-[16]. So it is practically important to take it into account in the modeling, analysis and synthesis of NCSs. Input missing is often modeled as Bernoulli process. But all results in [14]-[16] are derived via the average dwell time approach and are essentially slow switching conditions, which would reduce the performance room in system synthesis.

In this paper, the mean-square exponential stabilization problem is considered for packet-based networked systems with time-varying transmission delays, packet losses and input missing. It is known that packet-based scheme is an effective way to compensate time-varying transmission delays and packet losses [12], [13]. But the existing results are all dependent on average dwell time of subsystems. Here input delays are modeled as a continuous multivariate i.i.d. process, and input missing is modeled as a Bernoulli process. Then the NCS is described as a switching system with multiple subsystems of different input delays. A new Lyapunov-Krasovskii approach is proposed for the packet-based networked systems, which takes the distribution information of input delays into account both in constructing the Lyapunov-Krasovskii functional and bounding its infinitesimal. Based on this approach, the fast-switching conditions are directly derived for the closed-loop mean-square

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exponential stability as well as the resulting controller design method. Numerical examples show the effectiveness and advantage of our results.

Notation: The superscript ‘ T ’ denotes matrix transposition. \mathbb{N} and \mathbb{R}^n denote the sets of positive integers and $n \times n$ real matrices, respectively. The notation $P > 0$ (≥ 0) means that P is real symmetric and positive definite (semi-definite). I and 0 denote an identity matrix and a zero matrix of appropriate dimensions, respectively. For given positive integers n_1 and n_2 , I_{n_1} denotes the $n_1 \times n_1$ identity matrix, and $0_{n_1 \times n_2}$ the $n_1 \times n_2$ zero matrix. $E\{\cdot\}$ denotes the mathematical expectation. The space of functions $\phi: [-\bar{\eta}, 0] \rightarrow \mathbb{R}^n$, which are absolutely continuous functions on $[-\bar{\eta}, 0]$, have a finite $\lim_{\theta \rightarrow 0^-} \phi(\theta)$, and have square integrable first-order derivatives is denoted by Γ with the norm

$$\|\phi\|_{\Gamma} = \max_{\theta \in [-\bar{\eta}, 0]} |\phi(\theta)| + \left[\int_{-\bar{\eta}}^0 |\dot{\phi}(s)|^2 ds \right]^{1/2}$$

Denote $x_i(\theta) = x(t + \theta)$ where $\theta \in [-\bar{\eta}, 0]$.

II. PROBLEM FORMULATION

Packet-based networked system is plotted in Figure 1. Assume that: 1) the sensor is clock-driven while the controller and the zero-order hold (ZOH) are event-driven, and all of them are connected through network; 2) Feedback and control data packets are transmitted in one packet, and there exist time-varying transmission delays and packet losses in the channels from the sensor to the controller and from the controller to the ZOH; 3) The ZOH suffers from input missing; 4) The controller and the ZOH can determine if a data packet is new, only new data packets are accepted, and $u(t) = 0$ before the first updating instant.

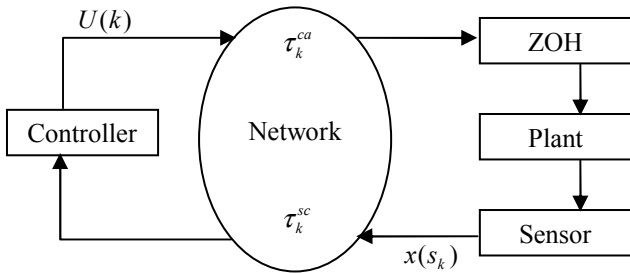


Figure 1. Block diagram of packet-based networked systems

Let t_0 denotes the initial instant of NCS, $k \in \mathbb{N}$ the number of new data packet arriving at the ZOH, t_k the corresponding arriving instant, s_k the associated sensor's sampling instant, and τ_k^{sc} and τ_k^{ca} are the transmission delays from the sensor to the controller and from the controller to the ZOH, respectively. It is seen that $t_k - s_k = \tau_k^{sc} + \tau_k^{ca}$. In the packet-based NCS, when the controller receives a new state feedback $x(s_k)$, it computes a sequence $U(k)$ of controls associated with diffe-

rent input delays, and then sends the sequence in one data packet to the ZOH. The ZOH selects different controls from the received new data packet according to the real input delays of the closed-loop systems, and then performs them.

For $t \in [t_k, t_{k+1})$, let $\tau(t) = t - s_k$ denotes the time-varying input delays induced by transmission delays and packet losses. It is assumed that there exist scalars $\bar{\tau} > 0$ and $\underline{\tau} \geq 0$ such that

$$\underline{\tau} \leq \tau(t) < \bar{\tau}, \quad t \in [t_k, t_{k+1}) \quad (1)$$

To implement the packet-based framework, the distributed interval $[\underline{\tau}, \bar{\tau})$ of input delays interval is divided as

$$[\underline{\tau}, \bar{\tau}) = \sum_{i=1}^m [\bar{\tau}_{i-1}, \bar{\tau}_i) \quad (2)$$

where $\bar{\tau}_0 = \underline{\tau}$ and $\bar{\tau}_m = \bar{\tau}$, and the controller adopts different feedback gains for $\tau(t)$ in different subintervals $[\bar{\tau}_{i-1}, \bar{\tau}_i)$, i.e. $\tau(t) \in [\bar{\tau}_{i-1}, \bar{\tau}_i)$. Let $\tau_i(t)$ denote the input delays $\tau(t) \in [\bar{\tau}_{i-1}, \bar{\tau}_i)$. It is seen that $\tau(t) \in [\underline{\tau}, \bar{\tau})$ are now represented as m discrete variables $\tau_i(t) \in [\bar{\tau}_{i-1}, \bar{\tau}_i)$.

The plant considered in this paper is of the form

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (3)$$

where $x(t) \in \mathbb{R}^{n_x}$ and $u(t) \in \mathbb{R}^{n_u}$ are state vector and control input vector, respectively, and A and B are constant matrices with appropriate dimensions. In packet-based NCS with input missing, the input is represented as

$$u(t) = \alpha(t) \sum_{i=1}^m \beta_i(t) u_i(k), \quad t \in [t_k, t_{k+1}) \quad (4)$$

where $u_i(k)$ is a control signal of the sequence $U(k) = [u_1(k), u_2(k), \dots, u_m(k)]$ and is given as

$$u_i(k) = K_i x(s_k) \quad (5)$$

where $x(s_k)$ is the state feedback, and K_i is the feedback gain associated with $\tau_i(t)$. The random variable $\beta_i(t)$ is defined as follows

$$\beta_i(t) = \begin{cases} 1 & \tau(t) \in [\bar{\tau}_{i-1}, \bar{\tau}_i) \\ 0 & \tau(t) \notin [\bar{\tau}_{i-1}, \bar{\tau}_i) \end{cases} \quad (6a)$$

with the probability $\bar{\beta}_i$ given as

$$\bar{\beta}_i = \Pr\{\tau(t) \in [\bar{\tau}_{i-1}, \bar{\tau}_i)\} = E\{\beta_i(t)\} = \lim_{t \rightarrow \infty} \frac{1}{t} \int_{t_1}^t \beta_i(s) ds \quad (6b)$$

It is seen that $\{\beta(t)\}$ is actually a continuous multivariate i.i.d. process with respect to t , where $\beta(t) = \text{col}\{\beta_1(t), \beta_2(t), \dots, \beta_m(t)\}$. The random variable $\alpha(t) \in \{1, 0\}$ denotes if the input missing occurs, where 1 and 0 correspond to the situations with and without input missing, respectively, and the related probability is computed as

$$\bar{\alpha} = \Pr\{\alpha(t) = 1\} = E\{\alpha(t)\} = \lim_{t \rightarrow \infty} \frac{1}{t} \int_{t_1}^t \alpha(s) ds \quad (7)$$

So $\{\alpha(t)\}$ is a continuous Bernoulli process with respect to t . Then the closed-loop packet-based NCS in (3)-(5) is formulated as

$$\dot{x}(t) = Ax(t) + B\alpha(t) \sum_{i=1}^m \beta_i(t) K_i x(t - \tau_i(t)) \quad (8)$$

It is seen that (8) is essentially a switching system.

The purpose of this paper is to develop the fast-switching conditions of the mean-square exponential stability for the packet-based NCS with time-varying transmission delays, packet losses and input missing basing on (6)-(8). Before giving our main results, we introduce the following definitions.

Definition 1: The closed-loop system (3)-(5) is said to be mean square exponentially stable if for any finite initial condition x_{t_1} , there exist scalars $\varepsilon > 0$ and $\lambda > 0$ such that

$$\mathbb{E}\{\|x(t)\|^2\} \leq \varepsilon e^{-\lambda(t-t_1)} \|x_{t_1}\|_{\Gamma}^2$$

Definition 2: The infinitesimal operator is defined as

$$\mathcal{L}V(t, x_t, \dot{x}_t) = \lim_{\vartheta \rightarrow 0^+} \frac{1}{\vartheta} \mathbb{E}\{V(t + \vartheta, x_{t+\vartheta}, \dot{x}_{t+\vartheta}) - V(t, x_t, \dot{x}_t)\}$$

where $X_t = \{x_t\}$.

III. MAIN RESULTS

To derive the fast-switching conditions of the mean-square exponential stability of (3)-(5), we construct the following Lyapunov-Krasovskii functional via delay-decomposition approach:

$$V(t, x_t, \dot{x}_t) = V_1(t, x_t) + V_2(t, x_t) + V_3(t, \dot{x}_t) \quad (9a)$$

$$V_1(t, x_t, \dot{x}_t) = x^T(t) P x(t) \quad (9b)$$

$$V_2(t, x_t, \dot{x}_t) = \sum_{i=1}^{m_0} \int_{t-\underline{\tau}_i}^{t-\underline{\tau}_i-1} e^{\lambda(s-t)} x^T(s) Q_{0,i} x(s) ds + \sum_{i=1}^m \int_{t-\bar{\tau}_i}^{t-\bar{\tau}_i-1} e^{\lambda(s-t)} x^T(s) Q_{2,i} x(s) ds \quad (9c)$$

$$V_3(t, \dot{x}_t, \dot{x}_t) = \sum_{i=1}^{m_0} (\underline{\tau}_i - \underline{\tau}_{i-1}) \int_{-\underline{\tau}_i}^{-\underline{\tau}_i-1} \int_{t+\theta}^t e^{\lambda(s-t)} \dot{x}^T(s) R_0 \dot{x}(s) ds d\theta + \sum_{i=1}^m (\bar{\tau}_i - \bar{\tau}_{i-1}) \int_{-\bar{\tau}_i}^{-\bar{\tau}_i-1} \int_{t+\theta}^t e^{\lambda(s-t)} \dot{x}^T(s) R_{2,i} \dot{x}(s) ds d\theta \quad (9d)$$

where $[0, \underline{\tau}] = \bigcup_{i=1}^{m_0} [\underline{\tau}_{i-1}, \underline{\tau}_i]$, $P > 0$, $Q_{0,i} > 0$ ($i=1, 2, \dots, m_0$), $R_0 > 0$, $Q_{2,i} > 0$, $R_{2,i} > 0$ ($i=1, 2, \dots, m$). It is seen from (9) that (8) is not simply deemed as a system with multiple delays $\tau_i(t)$ ($i=1, 2, \dots, m$) and $\tau_i(t)$ are still taken as a case of $\tau(t)$ belong to $[\bar{\tau}_{i-1}, \bar{\tau}_i]$, which is more accurate than the method in [11].

Theorem 1: Given (1), (2), (6), (7), $[0, \underline{\tau}] = \bigcup_{i=1}^{m_0} [\underline{\tau}_{i-1}, \underline{\tau}_i]$, $\rho_i = \bar{\beta}_i / \sum_{j=1}^m \bar{\beta}_j$ for $\bar{\beta}_i \neq 0$ and $\rho_i = 0$ for $\bar{\beta}_i = 0$ and K_i ($i=1, 2, \dots, m$), the closed-loop system (3)-(5) is mean-square exponentially stable with a decay rate λ , if there exist matrices $P > 0$, $Q_{0,i} > 0$ ($i=1, 2, \dots, m_0$), $Q_{2,i} > 0$, $R_0 > 0$, $R_{2,i} > 0$

($i=1, 2, \dots, m$) of appropriate dimensions such that the following inequalities hold:

$$\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4 + \Pi \Lambda_2 \Pi^T < 0 \quad (10)$$

for $\sigma_i = 0$ and 1 with $i=1, 2, \dots, m$, where

$$\Omega_1 = \begin{bmatrix} PA + A^T P + \lambda P & 0_{n_x \times m_0 n_x} & \Omega_{11} \\ * & & \\ * & 0_{(m_0+2m)n_x \times (m_0+2m)n_x} & \end{bmatrix}$$

$$\Omega_{11} = \bar{\alpha} P B \begin{bmatrix} \bar{\beta}_1 K_1 & 0 & \bar{\beta}_2 K_2 & 0 & \dots & \bar{\beta}_m K_m & 0 \end{bmatrix}$$

$$\Omega_2 = \text{diag}\{Q_{0,1}, -e^{-\lambda \underline{\tau}_1} Q_{0,1} + e^{-\lambda \underline{\tau}_1} Q_{0,2}, \dots, -e^{-\lambda \underline{\tau}_{m_0-1}} Q_{0,m_0-1} + e^{-\lambda \underline{\tau}_{m_0-1}} Q_{0,m_0}, -e^{-\lambda \underline{\tau}_{m_0}} Q_{0,m_0} + e^{-\lambda \bar{\tau}_0} Q_{2,1}, 0, -e^{-\lambda \bar{\tau}_1} Q_{2,1} + e^{-\lambda \bar{\tau}_1} Q_{2,2}, \dots, 0, -e^{-\lambda \bar{\tau}_{m-1}} Q_{2,m-1} + e^{-\lambda \bar{\tau}_{m-1}} Q_{2,m}, 0, -e^{-\lambda \bar{\tau}_m} Q_{2,m}\}$$

$$\Omega_3 = \Omega_{31} + \begin{bmatrix} 0_{(m_0+2m)n_x \times n_x} & \Omega_{32} \\ 0_{n_x \times (m_0+2m+1)n_x} & \end{bmatrix} + \begin{bmatrix} 0_{(m_0+2m)n_x \times n_x} & \Omega_{32} \\ 0_{n_x \times (m_0+2m+1)n_x} & \end{bmatrix}^T + \begin{bmatrix} 0_{(m_0+2m-1)n_x \times 2n_x} & \Omega_{33} \\ 0_{2n_x \times (m_0+2m+1)n_x} & \end{bmatrix} + \begin{bmatrix} 0_{(m_0+2m-1)n_x \times 2n_x} & \Omega_{33} \\ 0_{2n_x \times (m_0+2m+1)n_x} & \end{bmatrix}^T$$

$$\Omega_{31} = \text{diag}\{-e^{-\lambda \underline{\tau}_1} R_0, -e^{-\lambda \underline{\tau}_1} R_0 - e^{-\lambda \underline{\tau}_2} R_0, \dots, -e^{-\lambda \underline{\tau}_{m_0-1}} R_0 - e^{-\lambda \underline{\tau}_{m_0}} R_0, -e^{-\lambda \underline{\tau}_{m_0}} R_0, -e^{-\lambda \bar{\tau}_1} (\bar{\beta}_1 \rho_1 - \sigma_{2,1} \bar{\beta}_1 \rho_1 + 1) R_{2,1}, -3e^{-\lambda \bar{\tau}_1} \bar{\beta}_1 \rho_1 R_{2,1}, -e^{-\lambda \bar{\tau}_1} (\sigma_{2,1} \bar{\beta}_1 \rho_1 + 1) R_{2,1} - e^{-\lambda \bar{\tau}_2} (\bar{\beta}_2 \rho_2 - \sigma_{2,2} \bar{\beta}_2 \rho_2 + 1) R_{2,2}, -3e^{-\lambda \bar{\tau}_2} \bar{\beta}_2 \rho_2 R_{2,2}, \dots, -e^{-\lambda \bar{\tau}_{m-1}} (\sigma_{2,m-1} \bar{\beta}_{m-1} \rho_{m-1} + 1) R_{2,m-1}, -e^{-\lambda \bar{\tau}_m} (\bar{\beta}_m \rho_m - \sigma_{2,m} \bar{\beta}_m \rho_m + 1) R_{2,m}, -3e^{-\lambda \bar{\tau}_m} \bar{\beta}_m \rho_m R_{2,m}, -e^{-\lambda \bar{\tau}_m} (\sigma_{2,m} \bar{\beta}_m \rho_m + 1) R_{2,m}\}$$

$$\Omega_{32} = \text{diag}\{e^{-\lambda \underline{\tau}_1} R_0, \dots, e^{-\lambda \underline{\tau}_{m_0}} R_0, e^{-\lambda \bar{\tau}_1} (2 - \sigma_{2,1}) \bar{\beta}_1 \rho_1 R_{2,1}, e^{-\lambda \bar{\tau}_1} (1 + \sigma_{2,1}) \bar{\beta}_1 \rho_1 R_{2,1}, \dots, e^{-\lambda \bar{\tau}_m} (2 - \sigma_{2,m}) \bar{\beta}_m \rho_m R_{2,m}, e^{-\lambda \bar{\tau}_m} (1 + \sigma_{2,m}) \bar{\beta}_m \rho_m R_{2,m}\}$$

$$\Omega_{33} = \text{diag}\{0, \dots, 0, \underbrace{e^{-\lambda \bar{\tau}_1} (1 - \bar{\beta}_1 \rho_1) R_{2,1}}_{m_0}, 0, \dots, e^{-\lambda \bar{\tau}_{m-1}} (1 - \bar{\beta}_{m-1} \rho_{m-1}) R_{2,m-1}, 0, e^{-\lambda \bar{\tau}_m} (1 - \bar{\beta}_m \rho_m) R_{2,m}\}$$

$$\Pi = \begin{bmatrix} \sqrt{\bar{\alpha} \bar{\beta}_1} A^T & \sqrt{\bar{\alpha} \bar{\beta}_2} A^T & \dots & \sqrt{\bar{\alpha} \bar{\beta}_m} A^T \\ 0_{m_0 n_x \times n_x} & 0_{m_0 n_x \times n_x} & \dots & 0_{m_0 n_x \times n_x} \\ \begin{bmatrix} \sqrt{\bar{\alpha} \bar{\beta}_1} K_1^T B^T \\ 0 \end{bmatrix} & 0_{2n_x \times n_x} & \dots & 0_{2n_x \times n_x} \\ 0_{2n_x \times n_x} & \begin{bmatrix} \sqrt{\bar{\alpha} \bar{\beta}_2} K_2^T B^T \\ 0 \end{bmatrix} & \dots & 0_{2n_x \times n_x} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{2n_x \times n_x} & 0_{2n_x \times n_x} & \dots & \begin{bmatrix} \sqrt{\bar{\alpha} \bar{\beta}_m} K_m^T B^T \\ 0 \end{bmatrix} \end{bmatrix}$$

$$\begin{aligned}\Lambda_2 &= \text{diag}\{\Lambda_1 \quad \Lambda_1 \quad \cdots \quad \Lambda_1\} \\ \Lambda_1 &= \sum_{i=1}^{m_0} (\underline{\tau}_i - \underline{\tau}_{i-1})^2 R_0 + \sum_{i=1}^m (\bar{\tau}_i - \bar{\tau}_{i-1})^2 R_{2,i} \\ \Omega_4 &= \begin{bmatrix} (1-\bar{\alpha})A^T \Lambda_1 A & 0_{n_x \times (m_0+2m)n_x} \\ * & 0_{(m_0+2m)n_x \times (m_0+2m)n_x} \end{bmatrix}.\end{aligned}$$

Proof: Along the trajectories of the closed-loop system (8), it follows from (9) that

$$\mathcal{L}V(t, x_i, \dot{x}_i) = \mathcal{L}V_1(t, x_i) + \mathcal{L}V_2(t, x_i) + \mathcal{L}V_3(t, \dot{x}_i) \quad (11)$$

where

$$\begin{aligned}\mathcal{L}V_1(t, x_i) &= -\lambda V_1(t, x_i) + x^T(t) \lambda P x(t) \\ &\quad + 2x^T(t) P \{Ax(t) + \bar{\alpha} \sum_{i=1}^m \bar{\beta}_i(k) BK_i x(t - \tau_i(t))\} \\ \mathcal{L}V_2(t, x_i) &= -\lambda V_2(t, x_i) + E\left\{ \sum_{i=1}^{m_0} x^T(t - \underline{\tau}_{i-1}) e^{-\lambda(\underline{\tau}_i - t)} Q_{0,i} x(t - \underline{\tau}_{i-1}) \right. \\ &\quad \left. - x^T(t - \underline{\tau}_i) e^{-\lambda \underline{\tau}_i} Q_{0,i} x(t - \underline{\tau}_i) \mid \mathbf{X}_i \right\} \\ &\quad + E\left\{ \sum_{i=1}^m x^T(t - \bar{\tau}_{i-1}) e^{-\lambda \bar{\tau}_{i-1}} Q_{2,i} x(t - \bar{\tau}_{i-1}) \right. \\ &\quad \left. - x^T(t - \bar{\tau}_i) e^{-\lambda \bar{\tau}_i} Q_{2,i} x(t - \bar{\tau}_i) \mid \mathbf{X}_i \right\} \\ \mathcal{L}V_3(t, \dot{x}_i) &= -\lambda V_3(t, \dot{x}_i) + E\{ \dot{x}^T(t) \Lambda_1 \dot{x}(t) \mid \mathbf{X}_i \} \\ &\quad + E\left\{ \sum_{i=1}^{m_0} -(\underline{\tau}_i - \underline{\tau}_{i-1}) \int_{t-\underline{\tau}_i}^{t-\underline{\tau}_{i-1}} \dot{x}^T(s) e^{\lambda(s-t)} R_0 \dot{x}(s) ds \mid \mathbf{X}_i \right\} \\ &\quad + E\left\{ \sum_{i=1}^m -(\bar{\tau}_i - \bar{\tau}_{i-1}) \int_{t-\bar{\tau}_i}^{t-\bar{\tau}_{i-1}} \dot{x}^T(s) e^{\lambda(s-t)} R_{2,i} \dot{x}(s) ds \mid \mathbf{X}_i \right\}.\end{aligned}$$

With the closed-loop system in (8), it follows that

$$\begin{aligned}& E\{ \dot{x}^T(t) \Lambda_1 \dot{x}(t) \mid \mathbf{X}_i \} \\ &= (1-\bar{\alpha})x^T(t) A^T \Lambda_1 A x(t) \\ &\quad + \sum_{i=1}^m \bar{\alpha} \bar{\beta}_i \left\{ x^T(t) A^T \Lambda_1 A x(t) + 2x^T(t) A^T \Lambda_1 B K_i x(t - \tau_i(t)) \right. \\ &\quad \left. + x^T(t - \tau_i(t)) K_i^T B^T \Lambda_1 B K_i x(t - \tau_i(t)) \right\} \\ &= \xi^T(t) \Omega_4 \xi(t) + \xi^T(t) \Pi \Lambda_2 \Pi^T \xi(t)\end{aligned} \quad (12)$$

where

$$\begin{aligned}\xi(t) &= \text{col}\{x(t), x(t - \underline{\tau}_1), \dots, x(t - \underline{\tau}_{m_0}), \\ &\quad x(t - \tau_1(t)), x(t - \bar{\tau}_1), \dots, x(t - \tau_m(t)), x(t - \bar{\tau}_m)\}.\end{aligned}$$

By Jensen's inequality, it follows that for $i = 1, 2, \dots, m_0$

$$\begin{aligned}& E\left\{ -(\underline{\tau}_i - \underline{\tau}_{i-1}) \int_{t-\underline{\tau}_i}^{t-\underline{\tau}_{i-1}} e^{\lambda(s-t)} \dot{x}^T(s) R_0 \dot{x}(s) ds \mid \mathbf{X}_i \right\} \\ &\leq e^{-\lambda \underline{\tau}_i} \begin{bmatrix} x(t - \underline{\tau}_{i-1}) \\ x(t - \underline{\tau}_i) \end{bmatrix}^T \begin{bmatrix} -R_0 & R_0 \\ R_0 & -R_0 \end{bmatrix} \begin{bmatrix} x(t - \underline{\tau}_{i-1}) \\ x(t - \underline{\tau}_i) \end{bmatrix}.\end{aligned} \quad (13)$$

In this paper the bounding of the other integrals associated with $R_{2,i}$ in (11) takes the distribution of $\tau(t)$ into account. For this purpose, the following decomposition is made:

$$\begin{aligned}& E\{ -(\bar{\tau}_i - \bar{\tau}_{i-1}) \int_{t-\bar{\tau}_i}^{t-\bar{\tau}_{i-1}} \dot{x}^T(s) e^{\lambda(s-t)} R_{2,i} \dot{x}(s) ds \mid \mathbf{X}_i \} \\ &= \rho_i(k) E\{ -(\bar{\tau}_i - \bar{\tau}_{i-1}) \int_{t-\bar{\tau}_i}^{t-\bar{\tau}_{i-1}} \dot{x}^T(s) e^{\lambda(s-t)} R_{2,i} \dot{x}(s) ds \mid \mathbf{X}_i \} \\ &\quad + (1 - \rho_i(k)) E\{ -(\bar{\tau}_i - \bar{\tau}_{i-1}) \int_{t-\bar{\tau}_i}^{t-\bar{\tau}_{i-1}} \dot{x}^T(s) e^{\lambda(s-t)} R_{2,i} \dot{x}(s) ds \mid \mathbf{X}_i \}\end{aligned} \quad (14)$$

for $i = 1, 2, \dots, m$. In (14), the first decomposing term is related to the input delays $\tau_i(t)$, and the second one is related to other input delays $\tau_j(t)$ with $j = i+1, \dots, m$.

As seen in [17], different ways are needed in bounding

$$-\int_{t-\bar{\tau}_i}^{t-\bar{\tau}_{i-1}} \dot{x}^T(s) e^{\lambda(s-t)} R_{2,i} \dot{x}(s) ds$$

with respect to $\tau_i(k, t) \in [\bar{\tau}_{i-1}, \bar{\tau}_i]$ and $\tau_i(k, t) \notin [\bar{\tau}_{i-1}, \bar{\tau}_i]$, respectively. Based on this idea, it follows from Jensen's inequality and the technique similar to that of [17] that

$$\begin{aligned}& \rho_i(k) E\{ -(\bar{\tau}_i - \bar{\tau}_{i-1}) \int_{t-\bar{\tau}_i}^{t-\bar{\tau}_{i-1}} \dot{x}^T(s) e^{\lambda(s-t)} R_{2,i} \dot{x}(s) ds \mid \mathbf{X}_i \} \\ &\leq \zeta_i^T(t) e^{-\lambda \bar{\tau}_i} \rho_i(k) \\ &\quad \times \left\{ \bar{\beta}_i(k) \begin{bmatrix} -(2-\sigma_i)R_{2,i} & (2-\sigma_i)R_{2,i} & 0 \\ (2-\sigma_i)R_{2,i} & -3R_{2,i} & (1+\sigma_i)R_{2,i} \\ 0 & (1+\sigma_i)R_{2,i} & -(1+\sigma_i)R_{2,i} \end{bmatrix} \right. \\ &\quad \left. + (1-\bar{\beta}_i(k)) \begin{bmatrix} -R_{2,i} & 0 & R_{2,i} \\ 0 & 0 & 0 \\ R_{2,i} & 0 & -R_{2,i} \end{bmatrix} \right\} \zeta_i(t) \\ &= \zeta_i^T(t) e^{-\lambda \bar{\tau}_i} \rho_i(k) \begin{bmatrix} \Upsilon_i & (2-\sigma_i)\bar{\beta}_i R_{2,i} & (1-\bar{\beta}_i)R_{2,i} \\ * & -3\bar{\beta}_i R_{2,i} & (1+\sigma_i)\bar{\beta}_i R_{2,i} \\ * & * & -(\sigma_i \bar{\beta}_i + 1)R_{2,i} \end{bmatrix} \zeta_i(t)\end{aligned} \quad (15)$$

and

$$\begin{aligned}& (1 - \rho_i(k)) E\{ -(\bar{\tau}_i - \bar{\tau}_{i-1}) \int_{t-\bar{\tau}_i}^{t-\bar{\tau}_{i-1}} \dot{x}^T(s) e^{\lambda(s-t)} R_{2,i} \dot{x}(s) ds \mid \mathbf{X}_i \} \\ &\leq \zeta_i^T(t) e^{-\lambda \bar{\tau}_i} (1 - \rho_i(k)) \begin{bmatrix} -R_{2,i} & 0 & R_{2,i} \\ 0 & 0 & 0 \\ R_{2,i} & 0 & -R_{2,i} \end{bmatrix} \zeta_i(t).\end{aligned} \quad (16)$$

where $\zeta_i^T(t) = [x^T(t - \bar{\tau}_{i-1}), x^T(t - \tau_i(t)), x^T(t - \bar{\tau}_i)]$, $\Upsilon_i = -(\bar{\beta}_i - \sigma_i \bar{\beta}_i + 1)R_{2,i}$, $\sigma_i = (\tau_i(t) - \bar{\tau}_{i-1}) / (\bar{\tau}_i - \bar{\tau}_{i-1})$. It is seen that (15) further takes the distribution of $\tau_i(t)$ into account. From (15) and (16), we have the following

$$\begin{aligned}& E\{ -(\bar{\tau}_i - \bar{\tau}_{i-1}) \int_{t-\bar{\tau}_i}^{t-\bar{\tau}_{i-1}} \dot{x}^T(s) e^{\lambda(s-t)} R_{2,i} \dot{x}(s) ds \mid \mathbf{X}_i \} \\ &= \zeta_i^T(t) e^{-\lambda \bar{\tau}_i} \\ &\quad \times \left\{ \bar{\beta}_i \rho_i \begin{bmatrix} -(1-\sigma_i)R_{2,i} & (2-\sigma_i)R_{2,i} & -R_{2,i} \\ (2-\sigma_i)R_{2,i} & -3R_{2,i} & (1+\sigma_i)R_{2,i} \\ -R_{2,i} & (1+\sigma_i)R_{2,i} & -\sigma_i R_{2,i} \end{bmatrix} \right.\end{aligned}$$

$$+ \begin{bmatrix} -R_{2,i} & 0 & R_{2,i} \\ 0 & 0 & 0 \\ R_{2,i} & 0 & -R_{2,i} \end{bmatrix} \zeta_i(t). \quad (17)$$

If inequality (10) holds with $\sigma_i = 0$ and 1 for $i = 1, 2, \dots, m$, it follows from (11)-(13), (17) that

$$\mathcal{L}V(t, x_t, \dot{x}_t) + \lambda V(t, x_t, \dot{x}_t) \leq \xi^T(t)(\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4 + \Pi\Lambda_2\Pi^T)\xi(t) < 0 \quad (18)$$

which means that $E\{V(t, x_t, \dot{x}_t)\} \leq e^{-\lambda(t-t_0)} E\{V(t_0, x_{t_0}, \dot{x}_{t_0})\}$. So it can be concluded by Definition 1 that the closed-loop system (3)-(5) is mean-square exponentially stable under the given conditions of Theorem 1. The proof of Theorem 1 is completed.

Theorem 2: Given (1), (2), (6), (7), $[0, \underline{\tau}] = \bigcup_{i=1}^{m_0} [\underline{\tau}_{i-1}, \underline{\tau}_i]$, $\rho_i = \bar{\beta}_i / \sum_{j=i}^m \bar{\beta}_j$ for $\bar{\beta}_i \neq 0$ and $\rho_i = 0$ for $\bar{\beta}_i = 0$ ($i = 1, 2, \dots, m$), the closed-loop system (3)-(5) is mean-square exponentially stabilizable with a decay rate λ , if there exist matrices $X > 0$, $\tilde{Q}_{0,i} > 0$ ($i = 1, 2, \dots, m_0$), $\tilde{Q}_{2,i} > 0$, $\tilde{R}_0 > 0$, $\tilde{R}_{2,i} > 0$, Y_i ($i = 1, 2, \dots, m$) of appropriate dimensions such that the following inequalities hold:

$$\begin{bmatrix} \tilde{\Omega}_1 + \tilde{\Omega}_2 + \tilde{\Omega}_3 & \tilde{\Omega}_4\Phi_1 & \tilde{\Pi}\Phi_2 \\ * & -\tilde{\Lambda}_1 & 0 \\ * & * & -\tilde{\Lambda}_2 \end{bmatrix} < 0 \quad (19)$$

for $\sigma_i = 0$ and 1 with $i = 1, 2, \dots, m$, where

$$\begin{aligned} \tilde{\Omega}_1 &= \begin{bmatrix} AX + XA^T + \lambda X & 0_{n_x \times m_0 n_x} & \tilde{\Omega}_{11} \\ * & & \\ * & 0_{(m_0+2m)n_x \times (m_0+2m)n_x} & \end{bmatrix} \\ \tilde{\Omega}_{11} &= \bar{\alpha}B \begin{bmatrix} \bar{\beta}_1 Y_1 & 0 & \bar{\beta}_2 Y_2 & 0 & \dots & \bar{\beta}_m Y_m & 0 \end{bmatrix} \\ \tilde{\Omega}_2 &= \text{diag}\{\tilde{Q}_{0,1}, -e^{-\lambda \underline{\tau}_1} \tilde{Q}_{0,1} + e^{-\lambda \underline{\tau}_1} \tilde{Q}_{0,2}, \dots, \\ & -e^{-\lambda \underline{\tau}_{m_0-1}} \tilde{Q}_{0,m_0-1} + e^{-\lambda \underline{\tau}_{m_0-1}} \tilde{Q}_{0,m_0}, \\ & -e^{-\lambda \underline{\tau}_{m_0}} \tilde{Q}_{0,m_0} + e^{-\lambda \underline{\tau}_0} \tilde{Q}_{2,1}, 0, -e^{-\lambda \underline{\tau}_1} \tilde{Q}_{2,1} + e^{-\lambda \underline{\tau}_1} \tilde{Q}_{2,2}, \dots, \\ & 0, -e^{-\lambda \underline{\tau}_{m-1}} \tilde{Q}_{2,m-1} + e^{-\lambda \underline{\tau}_{m-1}} \tilde{Q}_{2,m}, 0, -e^{-\lambda \underline{\tau}_m} \tilde{Q}_{2,m}\} \\ \tilde{\Omega}_3 &= \tilde{\Omega}_{31} + \begin{bmatrix} 0_{(m_0+2m)n_x \times n_x} & \tilde{\Omega}_{32} \\ 0_{n_x \times (m_0+2m+1)n_x} & \end{bmatrix} + \begin{bmatrix} 0_{(m_0+2m)n_x \times n_x} & \tilde{\Omega}_{32} \\ 0_{n_x \times (m_0+2m+1)n_x} & \end{bmatrix}^T \\ & + \begin{bmatrix} 0_{(m_0+2m-1)n_x \times 2n_x} & \tilde{\Omega}_{33} \\ 0_{2n_x \times (m_0+2m+1)n_x} & \end{bmatrix} + \begin{bmatrix} 0_{(m_0+2m-1)n_x \times 2n_x} & \tilde{\Omega}_{33} \\ 0_{2n_x \times (m_0+2m+1)n_x} & \end{bmatrix}^T \\ \tilde{\Omega}_{31} &= \text{diag}\{-e^{-\lambda \underline{\tau}_1} \tilde{R}_0, -e^{-\lambda \underline{\tau}_1} \tilde{R}_0 - e^{-\lambda \underline{\tau}_1} \tilde{R}_0, \dots, \\ & -e^{-\lambda \underline{\tau}_{m_0-1}} \tilde{R}_0 - e^{-\lambda \underline{\tau}_{m_0}} \tilde{R}_0, -e^{-\lambda \underline{\tau}_{m_0}} \tilde{R}_0 \\ & -e^{-\lambda \underline{\tau}_1} (\bar{\beta}_1 \rho_1 - \sigma_{2,1} \bar{\beta}_1 \rho_1 + 1) \tilde{R}_{2,1}, -3e^{-\lambda \underline{\tau}_1} \bar{\beta}_1 \rho_1 \tilde{R}_{2,1}, \\ & -e^{-\lambda \underline{\tau}_1} (\sigma_{2,1} \bar{\beta}_1 \rho_1 + 1) \tilde{R}_{2,1} - e^{-\lambda \underline{\tau}_2} (\bar{\beta}_2 \rho_2 - \sigma_{2,2} \bar{\beta}_2 \rho_2 + 1) \tilde{R}_{2,2}, \end{aligned}$$

$$\begin{aligned} & -3e^{-\lambda \underline{\tau}_2} \bar{\beta}_2 \rho_2 \tilde{R}_{2,2}, \dots, -e^{-\lambda \underline{\tau}_{m-1}} (\sigma_{2,m-1} \bar{\beta}_{m-1} \rho_{m-1} + 1) \tilde{R}_{2,m-1} \\ & -e^{-\lambda \underline{\tau}_m} (\bar{\beta}_m \rho_m - \sigma_{2,m} \bar{\beta}_m \rho_m + 1) \tilde{R}_{2,m}, \\ & -3e^{-\lambda \underline{\tau}_m} \bar{\beta}_m \rho_m \tilde{R}_{2,m}, -e^{-\lambda \underline{\tau}_m} (\sigma_{2,m} \bar{\beta}_m \rho_m + 1) \tilde{R}_{2,m}\} \\ \tilde{\Omega}_{32} &= \text{diag}\{e^{-\lambda \underline{\tau}_1} \tilde{R}_0, \dots, e^{-\lambda \underline{\tau}_{m_0}} \tilde{R}_0, \\ & e^{-\lambda \underline{\tau}_1} (2 - \sigma_{2,1}) \bar{\beta}_1 \rho_1 \tilde{R}_{2,1}, e^{-\lambda \underline{\tau}_1} (1 + \sigma_{2,1}) \bar{\beta}_1 \rho_1 \tilde{R}_{2,1}, \dots, \\ & e^{-\lambda \underline{\tau}_m} (2 - \sigma_{2,m}) \bar{\beta}_m \rho_m \tilde{R}_{2,m}, e^{-\lambda \underline{\tau}_m} (1 + \sigma_{2,m}) \bar{\beta}_m \rho_m \tilde{R}_{2,m}\} \\ \tilde{\Omega}_{33} &= \text{diag}\{\underbrace{0, \dots, 0}_{m_0}, e^{-\lambda \underline{\tau}_1} (1 - \bar{\beta}_1 \rho_1) \tilde{R}_{2,1}, 0, \dots, \\ & e^{-\lambda \underline{\tau}_{m-1}} (1 - \bar{\beta}_{m-1} \rho_{m-1}) \tilde{R}_{2,m-1}, 0, e^{-\lambda \underline{\tau}_m} (1 - \bar{\beta}_m \rho_m) \tilde{R}_{2,m}\} \\ \tilde{\Omega}_4 &= \begin{bmatrix} \sqrt{1 - \bar{\alpha}} XA^T, \dots, \sqrt{1 - \bar{\alpha}} XA^T \\ 0_{(m_0+2m)n_x \times (1+m)n_x} \end{bmatrix} \\ \tilde{\Lambda}_2 &= \text{diag}\{\underbrace{\tilde{\Lambda}_1, \tilde{\Lambda}_1, \dots, \tilde{\Lambda}_1}_m\} \\ \tilde{\Lambda}_1 &= \text{diag}\{X\tilde{R}_0^{-1}X, X\tilde{R}_{2,1}^{-1}X, \dots, X\tilde{R}_{2,m}^{-1}X\} \\ \Phi_2 &= \text{diag}\{\underbrace{\Phi_1, \Phi_1, \dots, \Phi_1}_m\} \\ \Phi_1 &= \text{diag}\{\sqrt{\sum_{i=1}^{m_0} (\underline{\tau}_i - \underline{\tau}_{i-1})^2}, \bar{\tau}_1 - \bar{\tau}_0, \dots, \bar{\tau}_m - \bar{\tau}_{m-1}\} \\ \tilde{\Pi} &= [\tilde{\Pi}_1, \tilde{\Pi}_2, \dots, \tilde{\Pi}_m] \\ \tilde{\Pi}_i &= \begin{bmatrix} \sqrt{\bar{\alpha}\bar{\beta}_i} XA^T & \sqrt{\bar{\alpha}\bar{\beta}_i} XA^T \\ 0_{(m_0+2i-2)n_x \times n_x} & \dots & 0_{(m_0+2i-2)n_x \times n_x} \\ \sqrt{\bar{\alpha}\bar{\beta}_i} Y_i^T B^T & & \sqrt{\bar{\alpha}\bar{\beta}_i} Y_i^T B^T \\ 0_{(2m-2i+1)n_x \times n_x} & & 0_{(2m-2i+1)n_x \times n_x} \end{bmatrix}_{1+m} \end{aligned}$$

Furthermore, the controller gains are given by $K_i = Y_i X^{-1}$.

Proof: Applying the well known Schur Lemma and congruence transformation to (10) yields Theorem 2.

Inequality (19) is nonlinear for the existing of $X\tilde{R}_{2,i}^{-1}X$, $i = 1, 2, \dots, m$, and can not be directly solved by LMI toolbox. The feasible problem of nonlinear inequality in Theorem 2 can be transformed by CCL algorithm into a nonlinear convex optimization problem subject to LMI constraints, which can be directly solved by LMI toolbox.

IV. NUMERICAL EXAMPLE

Consider the linear system in [12] with

$$A = \begin{bmatrix} -1 & 0 & -0.5 \\ 1 & -0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Let $m_0 = 1$, $m = 6$, $\underline{\tau} = 0$, $\bar{\tau}_i - \bar{\tau}_{i-1} = (\bar{\tau} - \underline{\tau})/m$, $\bar{\beta}_i = 1/m$ with $i = 1, 2, \dots, m$. It is obtained by Theorem 2 that:

Case 1: Given $\bar{\alpha} = 1$, $\lambda = 0$, the maximum value of $\bar{\tau}$ is 2.64 with the following feedback gains

$$\begin{aligned}
K_1 &= [-0.0015, 0.0002, -0.5285] \\
K_2 &= [-0.0005, -0.0001, -0.5309] \\
K_3 &= [-0.0002, -0.0002, -0.5287] \\
K_4 &= [0.0000, -0.0003, -0.5235] \\
K_5 &= [0.0002, -0.0003, -0.5172] \\
K_6 &= [0.0008, -0.0005, -0.5132].
\end{aligned}$$

Case 2: Given $\bar{\alpha} = 0.8$, $\lambda = 0$, the maximum value of $\bar{\tau}$ is 1.70 with the following feedback gains

$$\begin{aligned}
K_1 &= [-0.0045, -0.0031, -0.9652] \\
K_2 &= [0.0068, -0.0068, -0.9844] \\
K_3 &= [0.0103, -0.0078, -0.9693] \\
K_4 &= [0.0122, -0.0082, -0.9269] \\
K_5 &= [0.0140, -0.0085, -0.8719] \\
K_6 &= [0.0167, -0.0092, -0.8386].
\end{aligned}$$

Case 3: Given $\bar{\alpha} = 0.8$, $\bar{\tau} = 1.0$, the maximum value of λ is 0.44 with the following feedback gains

$$\begin{aligned}
K_1 &= [0.0033, -0.0021, -1.4844] \\
K_2 &= [0.0095, -0.0024, -1.5318] \\
K_3 &= [0.0111, -0.0024, -1.5150] \\
K_4 &= [0.0116, -0.0023, -1.4373] \\
K_5 &= [0.0117, -0.0022, -1.3199] \\
K_6 &= [0.0126, -0.0021, -1.2355].
\end{aligned}$$

V. CONCLUSION

This paper gives the fast-switching conditions for packet-based NCSs with time-varying transmission delays, packet losses and input missing, which are independent on average dwell time of subsystems. With a multivariate i.i.d. model of input delays and a Bernoulli model of input missing, the NCS is formulated as a switching system with multiple subsystems of different input delays. Appropriate Lyapunov-Krasovskii functional is constructed by a delay decomposition approach. By using the bounding techniques based on Jensen's inequality as in [17], the distribution of input delays is taken into account when bounding the integral terms of the infinitesimal of the functional. The resulting controller design method can be solved by CCL algorithm. Numerical example shows the effectiveness of our results.

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