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Non-fragile observer design for nonlinear switched time delay systems using delta operator

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Abstract—**This paper considers the non-fragile observer design method for nonlinear switched time delay systems using the delta operator. Based on multiple Lyapunov function method and delta operator theory, an asymptotic stability criterion for delta operator switched system with time delay and Lipschitz nonlinearity is presented. By using the key technical lemma, a new sampling period and delay dependent design approach to the non-fragile observer is addressed. The proposed non-fragile observer can guarantee the estimated state error dynamics of delta operator time delay switched system can be asymptotically convergent for observer gain perturbations. The solution to the observer is formulated in the form of a set of linear matrix inequalities. A numerical example is employed to verify the proposed method.**

Keywords-delta operator; non-fragile observer; Lipschitz nonlinear; switched systems; time delay

I. Introduction

Switched systems have attracted the interest of several scientists in the last several years. Switched systems are a class of hybrid systems consisting of subsystems and a switching law, which define a specific subsystem being activated during a certain interval of time. Switched systems exist widely in engineering and social systems, such as mechanical systems, automotive industry, aircraft and air traffic control and many other fields[1-3]. A lot of research in this direction have appeared recently[4,5]. Many important progress and remarkable achievements have been made on issues about stability and stabilization for the system. Based on common quadratic Lyapunov functions (CQLFs), a series of methods and conditions have been given for analyzing the stability of switched systems for arbitrary switching law[6, 7]. As effective tools, multiple Lyapunov function (MLF), switched Lyapunov function(SLF) and the average dwell-time approaches have been proposed to analyze the stability of switched systems, and many valuable results have been obtained for switched systems[8-10].

It is recognized that most discrete-time signals and systems are the results of sampling continuous-time signals and systems. When sampling is fast, all resulting signals and systems tend to become ill conditioned and thus difficult to deal with using the conventional algorithms. The delta operator-based algorithms are numerically better behaved under finite precision implementations for fast sampling[11]. A class of uncertain systems in delta domain has been studied and several results about robust stability for the system have been developed[12,13]. The problem of system instability in fast sampling can be solved by using delta operator model [13].

Recently, delta operator approach is used to investigate robust control for a class of uncertain switched systems, and the stabilization conditions of the delta operator switched systems are formulated in terms of a set of linear matrix inequalities (LMIs) [14]. The developed method is valid for the system. However, in actual operation, the states of the systems are not all measurable. It is necessary to design state observers for systems of this type. Several design procedures have been proposed to design state observers for switched systems[15, 16]. It is considered that the state observer gain variations could not be avoided in several applications, a kind of non-fragile observer is proposed and the design method is proved to be effective[17]. However, to date and to the best of our knowledge, the problem of the non-fragile observer design for time delay delta operator switched nonlinear systems has not been investigated, which motivated us for this study.

In this paper, we deal with the problem of the non-fragile observer design for time delay switched nonlinear systems using a delta operator, where the observer gain perturbations are assumed to be time-varying and unknown, but are norm-bounded. The aim is to design the non-fragile observer such that the estimated state error dynamics can be asymptotically convergent for observer gain perturbations. The desired non-fragile observer can be constructed by solving a set of LMIs. The remainder of the paper is organized as follows. In Section II, problem formulation and some necessary lemmas are given. In Section III, based on the MLF approach and delta operator theory, the stability analysis for a delta operator time delay switched nonlinear system is considered, and the result is dependent of time delay and will be employed to develop a non-fragile observer. A numerical example is given to illustrate the feasibility and effectiveness of the developed technique in Section IV, and concluding remarks are given in Section V.

II. Problem formulation and Preliminary

Consider the following delta operator switched nonlinear system with time delay:

$$\delta x_k = A_{\sigma(k)} x_k + A_{d\sigma(k)} x_{k-n} + B_{\sigma(k)} u_k + f_{\sigma(k)}(x_k, k) \quad (1)$$

$$y_k = C_{\sigma(k)} x_k \quad (2)$$

$$x_k = \phi_k, \forall k \in \{-n, -n+1, \cdots, 0\} \quad (3)$$

where $x_k \in R^p$ is the state vector at the k th instant, $y_k \in R^m$

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is the measurement output vector. ϕ_k is a vector-valued initial function, $f_i(\cdot,\cdot): R^p \times R \to R^p$ is an unknown nonlinear function, n is the state delay of the system. $\sigma(k): Z^+ \to \underline{N} = \{1,2,\cdots,N\}$ is a switching signal. Moreover, $\sigma(k) = i$ means that the i th subsystem is activated. $u_k \in R^l$ is the control input of the i th subsystem at the k th instants. $A_i \in R^{p\times p}, A_{di} \in R^{p\times p}, B_i \in R^{p\times l}, C_i \in R^{m\times p}$ for $i \in \underline{N}$ are known real-valued matrices with appropriate dimensions, δ denotes the delta operator, the definition can be seen in [11], ie, $\delta x_k = (x_{k+1} - x_k)/T$, where T is a sample period.

We construct the following discrete-time switched system using delta operator to estimate the state of system (1)-(3):

$$\delta\hat{x}_k = A_{\sigma(k)}\hat{x}_k + A_{d\sigma(k)}\hat{x}_{k-n} + B_{\sigma(k)}u_k + f_{\sigma(k)}(\hat{x}_k,k) + L_{\sigma(k)}(y_k - \hat{y}_k) \tag{4}$$

$$\hat{y}_k = C_{\sigma(k)}\hat{x}_k \tag{5}$$

$$\hat{x}_k = 0, \forall k \in \{-n,-n+1,\cdots,0\} \tag{6}$$

where $\hat{x}_k \in R^p$ is the estimated state vector of x_k, $\hat{y}_k \in R^m$ is the observer output vector, $L_i \in R^{p\times m}$ for $i \in \underline{N}$ is the observer gain.

Let $\tilde{x}_k = x_k - \hat{x}_k$ be the estimated state error. By the definition of the delta operator, we have $\delta\tilde{x}_k = \delta x_k - \delta\hat{x}_k$, then we can obtain the following error system from (1)-(6):

$$\delta\tilde{x}_k = (A_{\sigma(k)} - L_{\sigma(k)}C_{\sigma(k)})\tilde{x}_k + A_{d\sigma(k)}\tilde{x}_{k-n} + f_{\sigma(k)}(x_k,k) - f_{\sigma(k)}(\hat{x}_k,k) \tag{7}$$

$$\tilde{x}_k = \phi_k, \forall k \in \{-n,-n+1,\cdots,0\} \tag{8}$$

If the state observer gain variations could not be avoided, a kind of non-fragile state observer will be designed as follows:

$$\delta\hat{x}_k = A_{\sigma(k)}\hat{x}_k + A_{d\sigma(k)}\hat{x}_{k-n} + B_{\sigma(k)}u_k + f_{\sigma(k)}(\hat{x}_k,k) + (L_{\sigma(k)} + \Delta L_{\sigma(k)})(y_k - \hat{y}_k) \tag{9}$$

$$\hat{y}_k = C_{\sigma(k)}\hat{x}_k \tag{10}$$

$$\hat{x}_k = 0, \forall k \in \{-n,-n+1,\cdots,0\} \tag{11}$$

where $\Delta L_i \in R^{p\times m}$ are uncertain real-valued matrix functions representing norm-bounded parameter uncertainties.

According to system (1)-(3) and (9)-(11), the dynamic equations of error switched system for non-fragile observer can be prescribed:

$$\delta\tilde{x}_k = [A_{\sigma(k)} - (L_{\sigma(k)} + \Delta L_{\sigma(k)})C_{\sigma(k)}]\tilde{x}_k + A_{d\sigma(k)}\tilde{x}_{k-n} + f_{\sigma(k)}(x_k,k) - f_{\sigma(k)}(\hat{x}_k,k) \tag{12}$$

$$\tilde{x}_k = \phi_k, \forall k \in \{-n,-n+1,\cdots,0\} \tag{13}$$

Without loss of generality, we make the following assumptions.

Assumption 1 $f_i(x_k,k)$ for $i \in \underline{N}$ are nonlinear functions satisfying:

$$\|f_i(x_k,k) - f_i(\hat{x}_k,k)\| \le \|U_i(x_k - \hat{x}_k)\| \tag{14}$$

where U_i are known real constant matrices.

Assumption 2 The gain perturbations ΔL_i are of the norm-bounded form:

$$\Delta L_i = H_i F_{ik} E_i \tag{15}$$

where H_i, E_i which denote the structure of the uncertainties are known real constant matrices with proper dimensions, and F_{ik} are unknown time-varying matrices which satisfy:

$$F_{ik}^T F_{ik} \le I$$

The unknown matrices F_{ik} contain the uncertain parameters in the linear part of the subsystem and the matrices H_i, E_i specify how the unknown parameters in F_{ik} affect the elements of the nominal matrices L_i.

The parameter uncertainty structure in equation (15) has been widely used and can represent parameter uncertainty in many physical cases. Equation (15) is called additive gain variations.

Lemma 1 [18] Let U, V, W and X be real matrices of appropriate dimensions with X satisfying $X = X^T$, then for all $V^TV \le I$

$$X + UVW + W^TV^TU^T < 0$$

if and only if there exists a scalar $\varepsilon > 0$ such that

$$X + \varepsilon UU^T + \varepsilon^{-1}W^TW < 0$$

Lemma 2 [19] Consider the following system

$$x_{k+1} = f_k(x_k) \tag{16}$$

If there is a function $V: Z^+ \times R^n \to R$ such that:

(1) V is a positive-definite function, decreasing and radially unbounded.

(2) $\Delta V(k,x_k) = V(k+1,x_{k+1}) - V(k,x_k) < 0$ is negative definite along the solution of (16).

then system (16) is asymptotically stable.

Lemma 3[20] For any constant positive semi-definite symmetric matrix W, two positive integers r and r_0 satisfying $r \ge r_0 \ge 1$, the following inequality holds

$$\left(\sum_{i=r_0}^{r} x(i)\right)^T W \left(\sum_{i=r_0}^{r} x(i)\right) \le \rho \sum_{i=r_0}^{r} x^T(i)Wx(i)$$

where $\rho = r - r_0 + 1$.

The objective of this paper is to design non-fragile observer gain L_i for delta operator time delay switched nonlinear systems (1)-(3) such that the estimated state error dynamics is asymptotically convergent.

III. MAIN RESULTS

A. Stability analysis

In this subsection, we investigate the stability of the following delta operator time delay switched nonlinear system

$$\delta x_k = A_{\sigma(k)}x_k + A_{d\sigma(k)}x_{k-n} + f_{\sigma(k)}(x_k,k) \tag{17}$$

where $\|f_i(x_k,k)\| \le \|U_ix_k\|$ for $i \in \underline{N}$. Define the indicator function:

$$\xi(k) = (\xi_1(k), \xi_2(k), \cdots, \xi_N(k))^T \tag{18}$$

with $i \in \{1,2,\cdots,N\}$, where

$$\xi_i(k)=\begin{cases}1 & \text{when the } i\text{th subsystem is activated}\\ 0 & \text{others}\end{cases},$$

then system (17) can be written as

$$\delta x_k=\sum_{i=1}^{N}\xi_i(k)(A_i x_k + A_{di}x_{k-n}+f_{\sigma(k)}(x_k,k))$$
$$\forall i \in \{1,2,\cdots,N\}$$

Theorem 1 Consider system (17). If there exist symmetric positive definite matrices $P_i>0$, $R_i^{(r)}>0$, $R_i^{(s)}>0$, $Q_i^{(s)}>0$, $Q_i^{(n)}>0$ and scalar $\varepsilon_i>0$, $i\in\{1,2,\cdots,N\}$, $r=1,2,\cdots,s-1$, $s=1,2,\cdots,n-1$, such that

$$P_i<\varepsilon_i I \tag{19}$$

$$R_j^{(r+1)}<R_i^{(r)},\ r=1,2,\cdots,s-1 \tag{20}$$

$$Q_j^{(s+1)}<Q_i^{(s)},\ s=1,2,\cdots,n-1 \tag{21}$$

$$\begin{bmatrix} A_i^T P_j + P_j A_i + TA_i^T P_j A_i + \varepsilon_i (T+1) U_i^T U_i + \frac{1}{T}(P_j-\frac{1}{2}P_i)-\frac{1}{n}R_i^{(s)}+Q_j^{(1)} & A_i^T P_i & (TA_i^T+I)P_j A_{di}+\frac{1}{n}R_i^{(s)} \\ * & -P_i+nT^2R_j^{(1)} & P_i A_{di} \\ * & * & TA_{di}^T P_j A_{di}-Q_i^{(n)}-\frac{1}{n}R_i^{(s)} \end{bmatrix}<0 \tag{22}$$

$\forall i,j\in\{1,2,\cdots,N\}$, then system (17) is asymptotically stable under arbitrary switching.

Proof Consider the following switched Lyapunov-Krasovskii functional:

$$V(k,x_k)=V_1(k,x_k)+V_2(k,x_k)+V_3(k,x_k) \tag{23}$$

with

$$V_1(k,x_k)=\frac{1}{2}x_k^T P(\xi(k))x_k=\frac{1}{2}x_k^T\left(\sum_{i=1}^{N}\xi_i(k)P_i\right)x_k$$

$$V_2(k,x_k)=T\sum_{s=1}^{n}x_{k-s}^T Q^{(s)}(\xi(k))x_{k-s}=T\sum_{s=1}^{n}\sum_{i=1}^{N}x_{k-s}^T\xi_i(k)Q_i^{(s)}x_{k-s}$$

$$V_3(k,x_k)=T\sum_{s=1}^{n}\sum_{r=1}^{s}e_{k-r}^T R^{(r)}(\xi(k))e_{k-r}=T\sum_{s=1}^{n}\sum_{r=1}^{s}\sum_{i=1}^{N}e_{k-r}^T\xi_i(k)R_i^{(r)}e_{k-r}$$

with P_i, $Q_i^{(s)}$, $R_i^{(r)}$, $i\in\{1,2,\cdots,N\}$ being symmetric positive-definite matrices and $e_k=x_k-x_{k+1}$.

Then switched Lyapunov-Krasovskii functional in delta domain has the following form:

$$\delta V_1(k,x_k)=\frac{V_1(k+1,x_{k+1})-V_1(k,x_k)}{T}$$
$$=\frac{x_{k+1}^T P(\xi(k+1))x_{k+1}-x_k^T P(\xi(k))x_k}{2T}$$
$$=\frac{(x_k+T\delta x_k)^T P(\xi(k+1))(x_k+T\delta x_k)-x_k^T P(\xi(k))x_k}{2T} \tag{24}$$
$$\le x_k^T[A_i^T P_j+P_jA_i+TA_i^T P_j A_i+\varepsilon_i T U_i^T U_i+\frac{1}{T}(P_j-\frac{1}{2}P_i)]x_k$$
$$+2x_k^T[(TA_i^T+I)P_jA_{di}]x_{k-n}+x_{k-n}^T(TA_{di}^TP_jA_{di})x_{k-n}$$

$$\delta V_2(k,x_k)=\frac{1}{T}[T\sum_{s=1}^{n}x_{k-s+1}^T Q^{(s)}(\xi(k+1))x_{k-s+1}-T\sum_{s=1}^{n}x_{k-s}^T Q^{(s)}(\xi(k))x_{k-s}]$$
$$=x_k^TQ_j^{(1)}x_k-x_{k-n}^TQ_i^{(n)}x_{k-n}+\sum_{s=1}^{n-1}x_{k-s}^T(Q_j^{(s+1)}-Q_i^{(s)})x_{k-s}$$

By (21), we can obtain that

$$\delta V_2(k,x_k)<x_k^TQ_j^{(1)}x_k-x_{k-n}^TQ_i^{(n)}x_{k-n} \tag{25}$$

$$\delta V_3(k,x_k)=\frac{1}{T}[T\sum_{s=1}^{n}\sum_{r=1}^{s}e_{k-r+1}^T R^{(r)}(\xi(k+1))e_{k-r+1}-T\sum_{s=1}^{n}\sum_{r=1}^{s}e_{k-r}^T R^{(r)}(\xi(k))e_{k-r}]$$
$$=ne_k^TR_j^{(1)}e_k+\sum_{s=2}^{n}\sum_{r=1}^{s-1}e_{k-r}^TR_j^{(r+1)}e_{k-r}-\sum_{s=2}^{n}\sum_{r=1}^{s-1}e_{k-r}^TR_i^{(r)}e_{k-r}-\sum_{s=1}^{n}e_{k-s}^TR_i^{(s)}e_{k-s}$$

By (20), we can obtain that

$$\delta V_3(k,x_k)\le ne_k^TR_j^{(1)}e_k-\frac{1}{n}(\sum_{s=1}^{n}e_{k-s})^TR_i^{(s)}(\sum_{s=1}^{n}e_{k-s})$$
$$=-\frac{1}{n}(x_{k-n}-x_k)^TR_i^{(s)}(x_{k-n}-x_k)+nT^2\delta x_k^TR_j^{(1)}\delta x_k \tag{26}$$

If $\delta V(k,x_k)<0$ holds under arbitrary switching signal, it follows that this has to hold for special configuration $\xi_i(k)=1$, $\xi_{h\neq i}(k)=0$, $\xi_j(k+1)=1$, $\xi_{g\neq j}(k+1)=0$ and for all $x_k\in R^p$. By (24), (25) and (26), we have

$$\delta V(k,x_k)=\delta V_1(k,x_k)+\delta V_2(k,x_k)+\delta V_3(k,x_k)$$
$$\le x_k^T[A_i^TP_j+P_jA_i+TA_i^TP_jA_i+\varepsilon_iTU_i^TU_i$$
$$+\frac{1}{T}(P_j-\frac{1}{2}P_i)]x_k+x_{k-n}^T(TA_{di}^TP_jA_{di})x_{k-n} \tag{27}$$
$$+2x_k^T[(TA_i^T+I)P_jA_{di}]x_{k-n}+x_k^TQ_j^{(1)}x_k+nT^2\delta x_k^TR_j^{(1)}\delta x_k$$
$$-x_{k-n}^TQ_i^{(n)}x_{k-n}-\frac{1}{n}(x_{k-n}-x_k)^TR_i^{(s)}(x_{k-n}-x_k)$$

Notice that

$$0=-2\delta x_k^TP_i(\delta x_k-A_ix_k-A_{di}x_{k-n}-f_i(x_k,k)) \tag{28}$$

Combining (27) and (28) leads to

$$\delta V(k,x_k)\le\eta^T\begin{bmatrix} A_i^TP_j+P_jA_i+TA_i^TP_jA_i+\varepsilon_i(T+1)U_i^TU_i+\frac{1}{T}(P_j-\frac{1}{2}P_i)-\frac{1}{n}R_i^{(s)}+Q_j^{(1)} & A_i^TP_i & (TA_i^T+I)P_jA_{di}+\frac{1}{n}R_i^{(s)} \\ * & -P_i+nT^2R_j^{(1)} & P_iA_{di} \\ * & * & TA_{di}^TP_jA_{di}-Q_i^{(n)}-\frac{1}{n}R_i^{(s)} \end{bmatrix}\eta$$

where $\eta = \begin{bmatrix} x_k^T & \delta x_k^T & x_{k-n}^T \end{bmatrix}^T$. From (22), we have $\delta V(k, x_k) < 0$. By Lemma 2, the system (17) is asymptotically stable. The proof is completed.

Remark 1 When time delay $n \equiv 0$ and $f_{\sigma(k)}(x_k, k) \equiv 0$, system (17) becomes as follows:

$$\delta x_k = \tilde{A}_{\sigma(k)} x_k$$

where $\tilde{A}_{\sigma(k)} = A_{\sigma(k)} + A_{d\sigma(k)}$. From (22), we know that the matrix inequality imply that the following inequality holds:

$$\tilde{A}_i^T P_j + P_j \tilde{A}_i + T \tilde{A}_i^T P_j \tilde{A}_i + \frac{1}{T}(P_j - P_i) < 0$$

The above inequality is just a sufficient condition of stability for the delta operator switched system without time delay, which can be expressed in [14].

Remark 2 When sample period $T = 0$ and $f_{\sigma(k)}(x_k, k) \equiv 0$, system (17) becomes a continuous-time switched linear system as follows:

$$\dot{x}(t) = A_{\sigma(t)} x(t) + A_{d\sigma(t)} x(t-d) \tag{29}$$

We can obtain sufficient condition of stability for system (29) by Theorem 1.

Corollary 1 Consider system (29). If there exist symmetric positive definite matrices $P > 0$, $R > 0$, $Q > 0$, such that

$$\begin{bmatrix} A_i^T P + P A_i - \frac{1}{d} R + Q & A_i^T P & P A_{di} + \frac{1}{d} R \\ * & -2P & P A_{di} \\ * & * & -Q - \frac{1}{d} R \end{bmatrix} < 0 \tag{30}$$

$\forall i \in \{1, 2, \cdots, N\}$, then system (29) is asymptotically stable under arbitrary switching.

Remark 3 Let $\bar{A}_{\sigma(k)} = A_{\sigma(k)} + I$. When sample period $T = 1$ and $f_{\sigma(k)}(x_k, k) \equiv 0$, system (17) becomes a discrete-time switched linear system as follows:

$$x_{k+1} = \bar{A}_{\sigma(k)} x_k + A_{d\sigma(k)} x_{k-n} \tag{31}$$

We can obtain sufficient condition of stability for system (31) by Theorem 1.

Corollary 2 Consider system (31). If there exist symmetric positive definite matrices $P_i > 0$, $R_i^{(r)} > 0$, $R_i^{(s)} > 0$, $Q_i^{(s)} > 0$, $Q_i^{(n)} > 0$, $i \in \{1, 2, \cdots, N\}$, $r = 1, 2, \cdots, s-1$, $s = 1, 2, \cdots, n-1$, such that

$$R_j^{(r+1)} < R_i^{(r)}, \ r = 1, 2, \cdots, s-1 \tag{32}$$

$$Q_j^{(s+1)} < Q_i^{(s)}, \ s = 1, 2, \cdots, n-1 \tag{33}$$

$$\begin{bmatrix} \bar{A}_i^T P_j + P_j \bar{A}_i + \bar{A}_i^T P_j \bar{A}_i - P_j - P_i - \frac{1}{n} R_i^{(s)} + Q_j^{(1)} & \bar{A}_i^T P_i & \bar{A}_i^T P_j A_{di} + \frac{1}{n} R_i^{(s)} \\ * & -2P_i + n R_j^{(1)} & P_i A_{di} \\ * & * & A_{di}^T P_j A_{di} - Q_i^{(n)} - \frac{1}{n} R_i^{(s)} \end{bmatrix} < 0 \tag{34}$$

$\forall i, j \in \{1, 2, \cdots, N\}$, then system (31) is asymptotically stable.

B. Observer Design

Now we consider system (7)-(8). The following theorem presents sufficient conditions for the existence of asymptotic stability of system (7)-(8).

Theorem 2 Consider system (7)-(8), if there exist a set of matrices $X_i > 0$, $V_i^{(s)} > 0$, $V_i^{(n)} > 0$, $Z_i^{(r)} > 0$, $Z_i^{(s)} > 0$, $V_{ij}^{(s)} > 0$, $V_{ij}^{(n)} > 0$, $Z_{ij}^{(r)} > 0$, $Z_{ij}^{(s)} > 0$, $\varepsilon_i > 0$, $i \in \{1, 2, \cdots, N\}$, $r = 1, 2, \cdots, s-1$, $s = 1, 2, \cdots, n-1$, and W_i such that the following LMIs have feasible solutions

$$X_i > \varepsilon_i^{-1} I \tag{35}$$

$$Z_{ij}^{(r+1)} < Z_i^{(r)}, \ r = 1, 2, \cdots, s-1 \tag{36}$$

$$V_{ij}^{(s+1)} < V_i^{(s)}, \ s = 1, 2, \cdots, n-1 \tag{37}$$

$$\begin{bmatrix} -\frac{1}{2T} X_i + V_{ij}^{(1)} - \frac{1}{n} Z_i^{(s)} & (A_i X_i - W_i)^T & \frac{1}{n} Z_i^{(s)} & T(A_i X_i - W_i)^T + X_i & X_i U_i^T \\ * & -X_i + n T^2 Z_{ij}^{(1)} & A_{di} X_i & 0 & 0 \\ * & * & -V_i^{(n)} - \frac{1}{n} Z_i^{(s)} & T X_i A_{di}^T & 0 \\ * & * & * & -T X_j & 0 \\ * & * & * & * & -\frac{1}{\varepsilon_i (T+1)} I \end{bmatrix} < 0 \tag{38}$$

where $i, j \in \{1, 2, \cdots, N\}$, then the state observer gain L_i satisfying the following equations:

$$L_i C_i = W_i X_i^{-1} \tag{39}$$

can guarantees system (7)-(8) is asymptotically stable under arbitrary switching.

Proof Denote

$$M_{ij} = \begin{bmatrix} (A_i - L_i C_i)^T P_j + P_j (A_i - L_i C_i) + T (A_i - L_i C_i)^T P_j (A_i - L_i C_i) + \varepsilon_i (T+1) U_i^T U_i + \frac{1}{T}(P_j - \frac{1}{2} P_i) - \frac{1}{n} R_i^{(s)} + Q_j^{(1)} & (A_i - L_i C_i)^T P_i & [T(A_i - L_i C_i)^T + I] P_j A_{di} + \frac{1}{n} R_i^{(s)} \\ * & -P_i + n T^2 R_j^{(1)} & P_i A_{di} \\ * & * & T A_{di}^T P_j A_{di} - Q_i^{(n)} - \frac{1}{n} R_i^{(s)} \end{bmatrix}$$

By Theorem 1, if $M_{ij} < 0$, then the system (7)-(8) is asymptotically stable. By Schur Complement, we have $M_{ij} < 0$ is equivalent to

$$\begin{bmatrix} -\frac{1}{2T}P_i+Q_j^{(1)}-\frac{1}{n}R_i^{(s)} +\varepsilon_i(T+1)U_i^TU_i & (A_i-L_iC_i)^TP_i & \frac{1}{n}R_i^{(s)} & T(A_i-L_iC_i)^T+I \\ * & -P_i+nT^2R_j^{(1)} & P_iA_{di} & 0 \\ * & * & -Q_i^{(n)}-\frac{1}{n}R_i^{(s)} & TA_{di}^T \\ * & * & * & -TP_j^{-1} \end{bmatrix}<0 \quad (40)$$

Using $diag\{P_i^{-1} \quad P_i^{-1} \quad P_i^{-1} \quad I\}$ to pre- and post- multiply the left term of (40) respectively, and denoting $X_i = P_i^{-1}$, $W_i = L_iC_iX_i$, $Z_i^{(s)} = P_i^{-1}R_i^{(s)}P_i^{-1}$, $V_i^{(n)} = P_i^{-1}Q_i^{(n)}P_i^{-1}$, $V_{ij}^{(1)} = P_i^{-1}Q_j^{(1)}P_i^{-1}$, $Z_{ij}^{(1)} = P_i^{-1}R_j^{(1)}P_i^{-1}$, we can obtain

$$\begin{bmatrix} -\frac{1}{2T}X_i+V_{ij}^{(1)}-\frac{1}{n}Z_i^{(s)} +\varepsilon_i(T+1)X_iU_i^TU_iX_i & X_i(A_i-L_iC_i)^T & \frac{1}{n}Z_i^{(s)} & TX_i(A_i-L_iC_i)^T+X_i \\ * & -X_i+nT^2Z_{ij}^{(1)} & A_{di}X_i & 0 \\ * & * & -V_i^{(n)}-\frac{1}{n}Z_i^{(s)} & TX_iA_{di}^T \\ * & * & * & -TX_j \end{bmatrix}<0 \quad (41)$$

Using Schur Complement again, we can obtain (38). By the MFL method, $M_{ij}<0$, $\forall i,j \in \{1,2,\cdots,N\}$ can guarantee that the system (7)-(8) is asymptotically stable under arbitrary switching. The proof is completed.

The procedure of observer design of system (7)-(8) can be summarized as follows:

The Procedure of Observer Design

Step 1. If there exist a set of positive definite symmetric matrices $X_i>0$, $V_i^{(s)}>0$, $V_i^{(n)}>0$, $Z_i^{(r)}>0$, $Z_i^{(s)}>0$, $V_{ij}^{(s)}>0$, $V_{ij}^{(n)}>0$, $Z_{ij}^{(r)}>0$, $Z_{ij}^{(s)}>0$, $\varepsilon_i>0$, $i\in\{1,2,\cdots,N\}$, $r=1,2,\cdots,s-1$, $s=1,2,\cdots,n-1$ and W_i, LMIs (35)-(38) have feasible solutions, then go to Step 2.

Step 2. If the equations $L_iC_i = W_iX_i^{-1}$ have feasible solutions for unknown matrices L_i, then the observer can be constructed as (4)-(6), where L_i are the observer gain.

Remark 4 For the matrices equations (39), by the matrix theory we can know that the sufficient and necessary condition for the equations having feasible solutions is

$$rank[C_i] = rank[C_i^T \vdots (W_iX_i^{-1})^T]$$

However, when $rank[C_i] \neq rank[C_i^T \vdots (W_iX_i^{-1})^T]$, the equations don not exist feasible solutions. In particular, if $rank[C_i]=p$, that is to say the output matrices C_i is column filled rank matrices, then the equations (39) have feasible solutions. But $rank[C_i]\neq p$ implies that the equations (39) may have not feasible solutions. When there do not exist feasible solutions for the equations (39), we can construct the matrices B_i such that $rank[C_i] = rank[C_i^T \vdots B_i^T]$. Let $W_iX_i^{-1}=B_i$, substituting $W_i = B_iX_i$ to (38), if there exists feasible solutions for LMIs (35)-(38), then the observer gain L_i can be obtain from the equations $L_iC_i=B_i$. Specially, when the matrices B_i are chosen as C_i and LMIs (35)-(38) have feasible solutions, L_i can be constructed as $L_i = I$ obviously, where I denotes the identity matrix.

C. Non-fragile Observer Design

Now consider systems (12)-(13), if there exist additive gain variations in state observer, we can obtain the following result.

Theorem 3 Consider system (12)-(13), if there exist a set of matrices $X_i>0$, $V_i^{(s)}>0$, $V_i^{(n)}>0$, $Z_i^{(r)}>0$, $Z_i^{(s)}>0$, $V_{ij}^{(s)}>0$, $V_{ij}^{(n)}>0$, $Z_{ij}^{(r)}>0$, $Z_{ij}^{(s)}>0$, $\varepsilon_i>0$, $\mu_i>0$, $i\in\{1,2,\cdots,N\}$, $r=1,2,\cdots,s-1$, $s=1,2,\cdots,n-1$ and W_i such that the following LMIs hold:

$$X_i > \varepsilon_i^{-1}I \quad (42)$$

$$Z_{ij}^{(r+1)} < Z_i^{(r)},\ r=1,2,\cdots,s-1 \quad (43)$$

$$V_{ij}^{(s+1)} < V_i^{(s)},\ s=1,2,\cdots,n-1 \quad (44)$$

$$\begin{bmatrix} -\frac{1}{2T}X_i+V_{ij}^{(1)}-\frac{1}{n}Z_i^{(s)} & (A_iX_i-W_i)^T & \frac{1}{n}Z_i^{(s)} & T(A_iX_i-W_i)^T+X_i & X_iU_i^T & -X_iC_i^TE_i^T \\ * & -X_i+nT^2Z_{ij}^{(1)}+\mu_iH_iH_i^T & A_{di}X_i & \mu_iTH_iH_i^T & 0 & 0 \\ * & * & -V_i^{(n)}-\frac{1}{n}Z_i^{(s)} & TX_iA_{di}^T & 0 & 0 \\ * & * & * & -TX_j+\mu_iT^2H_iH_i^T & 0 & 0 \\ * & * & * & * & -\frac{1}{\varepsilon_i(T+1)}I & 0 \\ * & * & * & * & * & -\mu_iI \end{bmatrix}<0 \quad (45)$$

where $i\in\{1,2,\cdots,N\}$, then the non-fragile state observer gain L_i satisfying the following equations:

$$L_iC_i = W_iX_i^{-1} \quad (46)$$

can guarantees system (12)-(13) is asymptotically stable under arbitrary switching.

Proof Denote

$$T_{ij} = \begin{bmatrix} -\frac{1}{2T}X_i+V_{ij}^{(1)}-\frac{1}{n}Z_i^{(s)} & X_i[A_i-(L_i+\Delta L_i)C_i]^T & \frac{1}{n}Z_i^{(s)} & \rightarrow \\ * & -X_i+nT^2Z_{ij}^{(1)} & A_{di}X_i \\ * & * & -V_i^{(n)}-\frac{1}{n}Z_i^{(s)} \\ * & * & * \\ * & * & * \end{bmatrix}$$

$$
\begin{array}{cc}
TX_i[A_i-(L_i+\Delta L_i)C_i]^T+X_i & X_iU_i^T \\
0 & 0 \\
TX_iA_{di}^T & 0 \\
-TX_j & 0 \\
* & -\frac{1}{\varepsilon_i(T+1)}I
\end{array}\Bigg]
$$

Substituting (15) into the above equation leads to

$$T_{ij}=T_{ij}^{(1)}+T_i$$

where

$$
T_{ij}^{(1)}=\left[\begin{array}{ccc}
-\frac{1}{2T}X_i+V_{ij}^{(1)}-\frac{1}{n}Z_i^{(s)} & [(A_i-L_iC_i)X_i]^T & \frac{1}{n}Z_i^{(s)} \\
* & -X_i+nT^2Z_{ij}^{(1)} & A_{di}X_i \\
* & * & -V_i^{(n)}-\frac{1}{n}Z_i^{(s)} \\
* & * & * \\
* & * & *
\end{array}\right. \rightarrow
$$

$$
\left.\begin{array}{cc}
T[(A_i-L_iC_i)X_i]^T+X_i & X_iU_i^T \\
0 & 0 \\
TX_iA_{di}^T & 0 \\
-TX_j & 0 \\
* & -\frac{1}{\varepsilon_i(T+1)}I
\end{array}\right]
$$

$$
T_i=\begin{bmatrix}
0 & -(H_iF_{ik}E_iC_iX_i)^T & 0 & -T(H_iF_{ik}E_iC_iX_i)^T & 0 \\
* & 0 & 0 & 0 & 0 \\
* & * & 0 & 0 & 0 \\
* & * & * & 0 & 0 \\
* & * & * & * & 0
\end{bmatrix}
$$

$$
=\begin{bmatrix}0\\ H_i\\ 0\\ TH_i\\ 0\end{bmatrix}F_{ik}\left[-E_iC_iX_i \quad 0 \quad 0 \quad 0 \quad 0\right]+\left(\begin{bmatrix}0\\ H_i\\ 0\\ TH_i\\ 0\end{bmatrix}F_{ik}\left[-E_iC_iX_i \quad 0 \quad 0 \quad 0 \quad 0\right]\right)^T
$$

According to Lemma 1, we have

$$
T_{ij}\le T_{ij}^{(1)}+\mu_i\begin{bmatrix}0\\ H_i\\ 0\\ TH_i\\ 0\end{bmatrix}\begin{bmatrix}0\\ H_i\\ 0\\ TH_i\\ 0\end{bmatrix}^T \qquad (47)
$$

$$
+\mu_i^{-1}\left[-E_iC_iX_i \quad 0 \quad 0 \quad 0 \quad 0\right]^T\left[-E_iC_iX_i \quad 0 \quad 0 \quad 0 \quad 0\right]
$$

Denote Z_{ij} is the right term of inequality (47), by Schur Complement, $Z_{ij}<0$ are equivalent to

$$
\left[\begin{array}{ccc}
-\frac{1}{2T}X_i+V_{ij}^{(1)}-\frac{1}{n}Z_i^{(s)} & [(A_i-L_iC_i)X_i]^T & \frac{1}{n}Z_i^{(s)} \\
* & -X_i+nT^2Z_{ij}^{(1)}+\mu_iH_iH_i^T & A_{di}X_i \\
* & * & -V_i^{(n)}-\frac{1}{n}Z_i^{(s)} \\
* & * & * \\
* & * & * \\
* & * & *
\end{array}\right. \rightarrow
$$

$$
\left.\begin{array}{ccc}
T[(A_i-L_iC_i)X_i]^T+X_i & X_iU_i^T & -X_iC_i^TE_i^T \\
\mu_iTH_iH_i^T & 0 & 0 \\
TX_iA_{di}^T & 0 & 0 \\
-TX_j+\mu_iT^2H_iH_i^T & 0 & 0 \\
* & -\frac{1}{\varepsilon_i(T+1)}I & 0 \\
* & 0 & -\mu_iI
\end{array}\right]<0 \qquad (48)
$$

Denote $W_i=L_iC_iX_i$, (48) is equivalent to (45). By Theorem 1, we conclude that system (12)-(13) is asymptotically stable. The proof is completed.

If the state observer gain variations could not be avoided, then the non-fragile observer for system (1)-(3) can be designed according to the following procedure.

The Procedure of Non-fragile Observer Design

Step 1. If there exist a set of positive definite symmetric matrices $X_i>0$, $V_i^{(s)}>0$, $V_i^{(n)}>0$, $Z_i^{(r)}>0$, $Z_i^{(s)}>0$, $V_{ij}^{(s)}>0$, $V_{ij}^{(n)}>0$, $Z_{ij}^{(r)}>0$, $Z_{ij}^{(s)}>0$, $\varepsilon_i>0$, $\mu_i>0$, $i\in\{1,2,\cdots,N\}$, $r=1,2,\cdots,s-1$, $s=1,2,\cdots,n-1$ and W_i, LMIs (42)-(45) have feasible solutions, then go to Step 2.

Step 2. If the equations $L_iC_i=W_iX_i^{-1}$ have feasible solutions for unknown matrices L_i, then the observer can be constructed as (9)-(11), where L_i are the observer gain.

IV. NUMERICAL EXAMPLE

In this section, we present an example to illustrate the effectiveness of the proposed approach. Consider system (1)-(3) with parameters as follows:

$$A_1=\begin{bmatrix}0.1 & 0.2\\ 0.3 & -0.1\end{bmatrix},\ A_2=\begin{bmatrix}-0.1 & 0\\ 0.1 & 0.2\end{bmatrix},$$

$$A_{d1}=\begin{bmatrix}0.1 & 0.1\\ 0 & 0.2\end{bmatrix},\ A_{d2}=\begin{bmatrix}0 & 0.1\\ 0 & 0\end{bmatrix},$$

$$E_1=\begin{bmatrix}-0.1 & 0\\ 0.1 & 0\end{bmatrix},\ E_2=\begin{bmatrix}0.1 & 0.1\\ 0 & 0.1\end{bmatrix},$$

$$U_1=\begin{bmatrix}0.1 & 0\\ 0 & 0\end{bmatrix}, U_2=\begin{bmatrix}0 & 0\\ -2 & 0\end{bmatrix}$$

$$H_1=H_2=\begin{bmatrix}0.2 & 0.1\\ 0 & 0\end{bmatrix},$$

$$C_1=C_2=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$$

and the uncertain time-varying parameter matrices $F_{1,k} = F_{2,k} = \begin{bmatrix} \sin k & 0 \\ 0 & \sin k \end{bmatrix}$. The sampling period $T = 0.5$, and the time delay $n = 2$, then using the Matlab LMI Control Toolbox to solve the LMIs in (42) to (45), we obtain the solution as follows:

$$X_1 = \begin{bmatrix} 76.8668 & 0.1346 \\ 0.1346 & 84.1706 \end{bmatrix},\ X_2 = \begin{bmatrix} 36.2793 & -0.0023 \\ -0.0023 & 76.7278 \end{bmatrix}$$

$$W_1 = \begin{bmatrix} 90.1949 & 18.2174 \\ 23.3830 & 64.1468 \end{bmatrix},\ W_2 = \begin{bmatrix} 28.0449 & -1.0833 \\ 3.6168 & 76.6194 \end{bmatrix}$$

Therefore, by Theorem 3 a desired non-fragile observer gain can be solved from the following equations:

$$L_1 C_1 = W_1 X_1^{-1} \text{ and } L_2 C_2 = W_2 X_2^{-1}$$

The we can obtain that

$$L_1 = \begin{bmatrix} 1.1730 & -0.9584 \\ 0.3029 & 0.4587 \end{bmatrix},\ L_2 = \begin{bmatrix} 0.7730 & -0.7871 \\ 0.0998 & 0.8988 \end{bmatrix}.$$

V. Conclusion

In this paper, we focus on both stability and non-fragile observer design for delta operator time delay switched nonlinear systems. By using the MLF method and choosing Lyapunov-Krasovskii functional in delta domain appropriately, sufficient conditions for the existence of non-fragile observer have been given in terms of LMIs. The result is dependent of sampling period and time delay and can be developed for non-fragile observer problems. Future work will focus on using the proposed design method to solve the problem of stabilization based on non-fragile observer for time delay switched nonlinear systems via delta operator approach.

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