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Adaptive sliding mode control for spacecraft attitude maneuvers with reduced or eliminated reaching phase

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***Abstract*—This paper aims to present an improved adaptive sliding mode control (ASMC) design for rigid spacecraft attitude maneuvers. An adaptive scheme is proposed for the switching gain calculation when the upper bound of the system uncertainty is unknown in advance. Unlike existing ASMC design, which may result in an over-adaptation of the upper bound when the initial system trajectory is located far from the sliding surface, this paper presents a novel ASMC strategy by introducing a decay term in the sliding function to reduce or eliminate the unrelated factor in the adaptation scheme. Consequently, a lower-chattering control signal is achieved. Simulation results are presented to illustrate the effectiveness of the proposed strategy.**

***Index Terms*—attitude maneuver, adaptive sliding mode control, over-adaptation, global sliding mode, chattering suppression.**

I. Introduction

As a subclass of variable structure control systems, sliding mode control (SMC) is a nonlinear control method that is well known for its robust performance. In the pase decades, SMC has been extensively studied in many practical control systems. SMC can offer many good properties, such as insensitivity to parameter variation, external disturbance rejection, and fast dynamic response, which make it a potential approach for spacecraft attitude control. In [1], the attitude regulation problem was studied and the sliding function was determined by solving an optimal control problem. A smoothing model-reference SMC algorithm was presented in [2], where a desired quaternion error response was predefined for the attitude control system. To reduce the static error, an integral term was added in the sliding function and modified Rodrigues parameters (MRPs) were used instead of quaternion for the non-redundancy in [3]. Moreover, in [4], a nonlinear sliding function was defined according to the properties related to the attitude kinematics.

However, for the SMC design mentioned above, a prior knowledge of the system uncertainty upper bound is required. When such a bound is unavailable in advance, conservative method is generally adopted, where the switching gain is selected sufficiently large. It is well known that the chattering level is directly determined by the switching gain. Hence, such a conservative method may aggravate the chattering problem which could excite the unmodelled dynamics and may lead to instability. Further investigations have proceeded along two lines. On the one hand, technologies are studied for the chattering reduction. Higher-order sliding mode control has been recently proposed to reduce the chattering problem while keeping the main advantages of conventional SMC ([5]). In [6], the control chattering is reduced by low-pass filtering the control signal. In particular, in [7], three methods were presented for the chattering suppression. Nonetheless, there are no constructive conditions for the switching gain selection in those algorithms and generally a prior knowledge of the bound of the system uncertainty and/or the system states is needed.

On the other hand, attention has also been focused on eliminating the requirement of the prior knowledge of the uncertainty bound. One way is using the disturbance observer (DOB) technique, as suggested in [8] and [9]. However, the DOB based SMC algorithms usually assume that the model uncertainty is generated by a linear exogenous system [10], which is hard to satisfy due to the complexity and unpredictability of the uncertainty. The other effective approach is to integrate adaptive scheme into SMC designs. By updating the switching gain adaptively, the upper bound of model uncertainty is not required to be known in advance. At the first stage, it is generally assumed that the norm of uncertainty was bounded by a linear function of the state-norm. Correspondingly, adaptive laws were designed for the linear function parameters, as suggested in [11], [12], [13]. In particular, in [13], an ASMC algorithm was proposed for the attitude stabilization of a rigid spacecraft, where the lumped uncertainty is assumed to be bounded by a linear function of the norms of angular velocity and quaternion. Afterwards, in [14] the lumped uncertainty was assumed to be bounded by an unknown constant and consequently a simple adaptive law was proposed for the switching gain calculation. Subsequent results can be found in many other applications such as internal combustion engines ([15]), induction servomotor ([16]), planetary gear-type inverted-pendulum ([17]), etc. The major problem of the ASMC algorithms mentioned above is their over-adaptation for the switching gain with respect to the uncertainty bound, which results in the serious chattering phenomena and unnecessary energy consumption.

Considering the shortcomings of the chattering suppression techniques and current ASMC design, it is necessary to put

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forward a novel ASMC strategy, which does not need a prior knowledge of the upper bound and has the chattering suppression ability. Such a problem was recently investigated in [18], where two new methodologies for the ASMC design were proposed. Unfortunately,those two methods only reduce the switching gain in the sliding phase without considering the reaching phase. Similar to the ASMC design mentioned before, which will be called the ASMC-I algorithm in the following contents, the switching gain is also overestimated in [18] due to its failure in accounting for the initial system error in the adaptation scheme. With this in mind, this paper tries to present an improved ASMC design principle by addressing the attitude control problem for a rigid spacecraft in the presence of inertia matrix uncertainty and external disturbance. The key feature of the proposed ASMC algorithm, referred as the ASMC-II algorithm, is that a decay function is introduced to reduce or eliminate the impact of initial system error in the adaptation scheme. By this modification, the ASMC-II algorithm can give a more accurate estimation of the uncertainty bound and generate a lower-chattering control signal. A large angle attitude reorientation maneuver is employed in the simulation, where the simulation results demonstrate the effectiveness of the proposed strategy.

II. Preliminary and Problem Statement

Rigid spacecraft attitude control for large angle maneuvers poses a difficult problem, including nonlinear characteristics in both the dynamics and the kinematics, modelling uncertainty, and persistent external disturbance. In this paper, we will resolve this problem in the ASMC framework. Before moving on, some notations and assumptions are presented here. Three coordinate frames are used in this paper, which are the inertia reference frame $\mathcal{F}_I$, body-fixed frame $\mathcal{F}_B$, and the desired frame $\mathcal{F}_D$. Unless otherwise specified, all the quantities are expressed in their corresponding frames. And it is assumed that the spacecraft attitude and angular velocity are available and the dynamics of actuator is neglected.

A. Mathematical Model

Consider a thruster control rigid spacecraft, whose dynamics is described as follows:

$$J\dot{\omega}_b + \omega_b^\times J\omega_b = T_b + T_d \tag{1}$$

where $J \in \mathbb{R}^{3\times3}$ is the spacecraft inertia matrix, $\omega_b = [\omega_{b1} \quad \omega_{b2} \quad \omega_{b3}]^T \in \mathbb{R}^3$ is the angular velocity vector of $\mathcal{F}_B$ with respect to $\mathcal{F}_I$. The superscript $(\cdot)^\times$ on ω_b denotes the skew-symmetric matrix operator performing the cross product between two vectors, e.g.,

$$\alpha^\times \beta = \alpha \times \beta \tag{2}$$

where α and β are two vectors in $\mathbb{R}^3$. $T_b = [T_{b1} \quad T_{b2} \quad T_{b3}]^T \in \mathbb{R}^3$ is the vector of control torque provided by the thrusters, $T_d = [T_{d1} \quad T_{d2} \quad T_{d3}]^T \in \mathbb{R}^3$ is the external disturbance vector, including gravitational torque, aerodynamic torque, radiation torque, and other environmental and non-environmental torques. Furthermore, the inertia matrix uncertainty is taken into account. Let $J = \hat{J} + \Delta J$ with ΔJ the uncertainty caused by the change in mass properties and $\hat{J} = \mathrm{diag}(J_1, J_2, J_3)$ the nominal inertia matrix. Then the attitude dynamics is given by:

$$\hat{J}\dot{\omega}_b + \omega_b^\times \hat{J}\omega_b = T_b + T_d - \Delta J\dot{\omega}_b - \omega_b^\times \Delta J\omega_b \tag{3}$$

According to the structural feature in (3), one can merge all the elements caused by inertia matrix uncertainty and external disturbance as the lumped uncertainty, i.e., let $d = [d_1 \quad d_2 \quad d_3]^T \in \mathbb{R}^3$ with $d = T_d - \Delta J\dot{\omega}_b - \omega_b^\times \Delta J\omega_b$. Correspondingly, the attitude dynamics is rewritten as:

$$\hat{J}\dot{\omega}_b + \omega_b^\times \hat{J}\omega_b = T_b + d \tag{4}$$

From (4), it is clear that the lumped uncertainty is matched to the system. Without loss of generality, it is assumed that d is bounded by an unknown upper bound, e.g., $\|d\|_\infty < d_{\max}$ with $\|\cdot\|_\infty$ the vector infinite-norm.

By introducing the shadow MRPs, the MRPs set can provide a nonsingular, bounded, minimal attitude description. Hence, MRPs are utilized in this paper instead of quaternion, whose kinematics is:

$$\dot{\sigma}_b = M(\sigma_b)\omega_b \tag{5}$$

where $\sigma_b = [\sigma_{b1} \quad \sigma_{b2} \quad \sigma_{b3}]^T \in \mathbb{R}^3$ denotes the inertial MRPs vector of $\mathcal{F}_B$ with respect to $\mathcal{F}_I$. $M : \mathbb{R}^3 \to \mathbb{R}^{3\times3}$ such that $M(\sigma_b)$ is the Jacobian matrix with $M(\sigma_b) = \dfrac{(1-\|\sigma_b\|^2)I_3 + 2\sigma_b^\times + 2\sigma_b\sigma_b^T}{4}$, $\|\cdot\|$ is the vector 2-norm and I_3 is the 3×3 identity matrix. Moreover, $M^T(\sigma_b)M(\sigma_b) = m(\sigma_b)I_3$ with $m : \mathbb{R}^3 \to \mathbb{R}$ such that $m(\sigma_b) = (1 + \|\sigma_b\|^2)^2/16$. The transition matrix from $\mathcal{F}_I$ to $\mathcal{F}_B$ in terms of MRPs is given by:

$$R(\sigma_b) = I_3 + \frac{8\sigma_b^\times\sigma_b^\times - 4(1-\|\sigma_b\|^2)\sigma_b^\times}{(1+\|\sigma_b\|^2)^2} \tag{6}$$

A typical Rest-to-Rest attitude maneuver is studied in this paper. The objective is reorienting the spacecraft from an arbitrary stationary attitude to a desired attitude with zero angular velocity. The attitude variables of the desired frame, $\mathcal{F}_D$, are denoted by $\sigma_d \in \mathbb{R}^3$ and $\omega_d \in \mathbb{R}^3$. Then, the error attitude variables are defined as follows:

$$\sigma_e = \sigma_b \oplus \sigma_d^* \tag{7}$$

$$\omega_e = \omega_b - R(\sigma_e)\omega_d \tag{8}$$

where $\sigma_e = [\sigma_{e1} \quad \sigma_{e2} \quad \sigma_{e3}]^T \in \mathbb{R}^3$, $\oplus$ is the MRPs production operator characterizing the successive rotations. For two MRPs expressed in their corresponding frames, e.g., $\sigma_1 \in \mathbb{R}^3$ and $\sigma_2 \in \mathbb{R}^3$, it is operated as follows:

$$\sigma_1 \oplus \sigma_2 = \frac{(1-\|\sigma_2\|^2)\sigma_1 + (1-\|\sigma_1\|^2)\sigma_2 - 2\sigma_1^\times\sigma_2}{1 + \|\sigma_2\|^2\|\sigma_1\|^2 - 2\sigma_2^T\sigma_1} \tag{9}$$

σ_d^* is the inverse of σ_d, which is extracted from the inverse of $R(\sigma_d)$ and $\sigma_d^* = -\sigma_d$. $R(\sigma_e)$ and $R(\sigma_d)$ are the transition matrices from $\mathcal{F}_D$ to $\mathcal{F}_B$ and from $\mathcal{F}_I$ to $\mathcal{F}_D$, their expressions can be obtained by replacing σ_b by σ_e and σ_d in (6). As $\omega_d = 0$, $\omega_e = \omega_b$. Therefore, the error attitude dynamics is expressed same as (4). With respect to the error attitude kinematics, following lemma is introduced.

Lemma 1: If the attitude variables pairs (σ_b, ω_b) and (σ_d, ω_d) satisfy the MRPs kinematics formulation described in (5), then the error attitude variables pair (σ_e, ω_e) also satisfies that MRPs kinematics formulation.

Proof: The proof is based on the successive rotations in terms of transition matrix. See [20] for further details. ■

Then, the system is governed by the following equations:

$$\begin{cases} \hat{J}\dot{\omega}_b &= T_b + d - \omega_b^{\times}\hat{J}\omega_b \\ \dot{\sigma}_e &= M(\sigma_e)\omega_b \end{cases} \tag{10}$$

B. Problem Statement

Our aim can be summarized as follows: find a SMC algorithm to steer the attitude variables pair (σ_b, ω_b) from $(\sigma_b(0), 0)$ to $(\sigma_d, 0)$ in the presence of the lumped uncertainty, and find an adaptive law to update the estimation of the unknown $d_{\max}$ for the switching gain calculation which has the chattering suppression ability.

III. Main Results

A. AMSC-I Algorithm Design

In this section, we will briefly apply the ASMC-I algorithm for the attitude control problem under consideration. First, the sliding function defined in [4] is given by:

$$S = \omega_b + \lambda \frac{M^T(\sigma_e)}{m(\sigma_e)}\sigma_e \tag{11}$$

where $S = [s_1 \quad s_2 \quad s_3]^T \in \mathbb{R}^3$ and the corresponding sliding surface is determined by $S = 0$, $\lambda > 0$ is the sliding function gain. In the following derivations, $M(\sigma_e)$ and $m(\sigma_e)$ will be denoted by M and m for clarity.

By a left multiplication of (11) with M and using the fact that $M^T M = mI_3$, one has:

$$MS = \dot{\sigma}_e + \lambda\sigma_e \tag{12}$$

When the sliding mode occurs, i.e., $S = 0$ holds, it is easy to conclude that an exponential convergence of the error MRPs, i.e., $\sigma_e(t) = \mathrm{e}^{-\lambda(t-t_r)}\sigma_e(t_r)$, can be obtained if a proper SMC law is designed, where t_r is the time of arrival at the sliding surface. Such a SMC algorithm can be derived by producing a negative definite derivative of the following Lyapunov function:

$$V = \frac{1}{2}S^T\hat{J}S \tag{13}$$

On the basis of [4], one can get the following SMC algorithm:

$$T_b = \omega_b^{\times}\hat{J}\omega_b - \lambda\hat{J}\frac{\left(4M - 2\sigma_e\sigma_e^T\right)\omega_b}{1 + \|\sigma_e\|^2} - \Gamma\mathrm{sgn}(S) \tag{14}$$

where $\Gamma = \mathrm{diag}(\gamma_1, \gamma_2, \gamma_3)$ is the switching gain matrix with its elements $\gamma_i > d_{\max}$ $(i = 1, 2, 3)$ to guarantee the system stability and $\mathrm{sgn}(\cdot)$ is the sign function.

During the above derivations, the sliding function gain λ can be determined according the desired system response in the sliding phase. In order to determine the switching gain in the absence of a prior knowledge of $d_{\max}$, the ASMC-I algorithm can be applied.

Consider the modified Lyapunov function:

$$V = \frac{1}{2}S^T\hat{J}S + \frac{1}{2c}\tilde{d}^2 \tag{15}$$

where $c > 0$ is the adaptive gain, $\tilde{d} = \hat{d} - d_{\max}$ is the estimation error with $\hat{d}$ the estimation of $d_{\max}$. According to the ASMC-I design principle in [14], the adaptive switching gain law for the attitude control is:

$$\hat{d} = c\int_0^t \|S\|_1 d\tau \tag{16}$$

where $\|\cdot\|_1$ is the vector 1-norm.

Correspondingly, the ASMC-I algorithm is given by:

$$T_b = \omega_b^{\times}\hat{J}\omega_b - \lambda\hat{J}\frac{\left(4M - 2\sigma_e\sigma_e^T\right)\omega_b}{1 + \|\sigma_e\|^2} - \hat{d}\mathrm{sgn}(S) \tag{17}$$

B. Over-adaptation in ASMC-I Algorithm

From the ASMC-I algorithm design in Section III-A, one can see that the ASMC-I algorithm is based on conventional SMC algorithm. It is well known that the system trajectory employing the SMC algorithm consists two parts, the reaching phase and the sliding phase, as illustrated in Fig. 1.

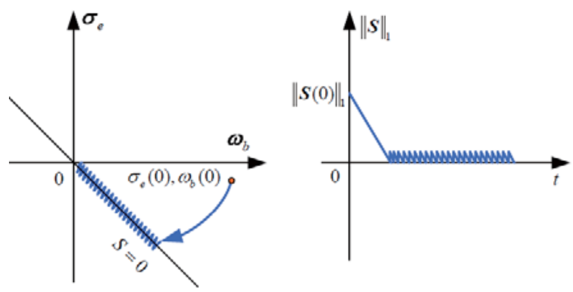

Fig. 1. System trajectory and sliding function response using SMC algorithm in (14)

Recalling the adaptive law in (16), it is obvious that the basic idea of the ASMC-I technique lies in that $d_{\max}$ can be adjusted by the deviation from the sliding surface. From (16), the integral action starts from the very beginning and any departure from the sliding surface, $S = 0$, will result in the increase of the switching gain. In other words, the switching gain adaptation depends on the initial value of the sliding function besides the lumped uncertainty. However, the initial system trajectory is generally located far from the sliding surface as shown in Fig. 1. Hence, $\hat{d}$ would increase quickly at the beginning due to a large $\|S(0)\|_1$ and the resulting $\hat{d}$ is much larger than $d_{\max}$, which leads to an over-adaptation of the switching gain and correspondingly the serious chattering problem in the ASMC-I algorithm.

Moreover, if we divide the adaptive law in (16) into two parts, i.e.,

$$\hat{d} = c\int_{0}^{t_r} \|S\|_1 d\tau + c\int_{t_r}^{t} \|S\|_1 d\tau \tag{18}$$

Then, it is obvious that the first integral term in (18) deals with the deviation in the reaching phase, which is mainly caused by the initial system error; while the second term handles the departure in the sliding phase, which is mainly affected by the lumped disturbance.

C. ASMC-II Algorithm Design

To address the over-adaptation problem in the ASMC-I design, it is natural to reduce or eliminate the proportion of the first integral term in (18). With this in mind, we present the ASMC-II algorithm. First, the sliding function in (11) is modified as

$$S(t) = \omega_b - f(t)\xi + \lambda\frac{M^T}{m}\left[\sigma_e - f(t)\rho\right] \tag{19}$$

where $S(t) = [s_1(t) \quad s_2(t) \quad s_3(t)]^T \in \mathbb{R}^3$, $f(t)$ is a continuous, strictly decreasing function on $t \in [0,\infty)$ with its initial value $f(0) \in [0,1]$ and its final value $f_f = 0$, $\xi = [\xi_1 \quad \xi_2 \quad \xi_3]^T \in \mathbb{R}^3$, $\rho = [\rho_1 \quad \rho_2 \quad \rho_3]^T \in \mathbb{R}^3$ are the coefficients related to the initial system states, and $\rho = \sigma_e(0)$, $\xi = \omega_b(0)$.

According to the above definition, we can find that the initial value of the sliding function is reduced to a small value or even becomes zero by the additional decay function. Therefore, if a proper SMC algorithm is designed to achieve the sliding motion, the sliding surface related to (19) is a new kind of sliding surface, which is illustrated in Fig. 2.

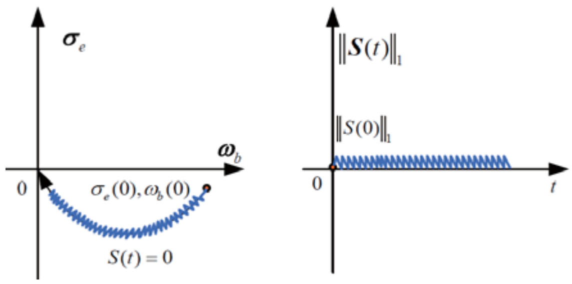

Fig. 2. System trajectory and sliding function response using sliding function in (19)

Therefore, if the sliding function in (19) is used for the switching gain adaptation, the unrelated effect of the initial system error can be reduced or eliminated in the adaptation scheme and the upper bound of the lumped uncertainty can be estimated more precisely, which is the motivation for the ASMC-II algorithm.

Remark 1: Actually, the sliding function defined in (19) is an extension of the time-varying sliding function investigated in [21] and [22]. In particular, in [21], it was proved that a *global sliding mode* would be achieved by using the sliding function like (19) with $f(t)$ selected as the exponential decay function $f(t) = \mathrm{e}^{-\kappa t}$ and $\kappa > 0$. However, due to the sensor noise, the initial system error can not be entirely cancelled. Furthermore, if the total time-varying sliding mode case, i.e., $S(0) = 0$, is used for the switching gain adaptation, the system cannot provide enough information for the adaptation due to the fact that there is no departure from the sliding function at the initial time, which will slow down the adaptation procedure. Therefore, in this paper, we introduce a weight in the decay function, e.g., let $f(t) = p\mathrm{e}^{-\kappa t}$ with $p \in [0,1]$ in the following derivations, which will produce an initial departure from the sliding surface purposefully to speed up the adaptive process. When $p = 0$, the sliding function in (19) turns out to be the sliding function in (11); whereas if $p = 1$, it becomes the total time-varying sliding function studied in [21].

For the attitude reorientation control problem, we have $\omega_b(0) = 0$, i.e., $\xi = 0$. For simplicity, the κ in the decay function is selected same as the sliding function gain λ, i.e., let $\kappa = \lambda$. Then the sliding function is specified as:

$$S(t) = \omega_b + \lambda\frac{M^T}{m}\left(\sigma_e - p\mathrm{e}^{-\lambda t}\rho\right) \tag{20}$$

Then, we are ready to present the following theorem:

Theorem 1: For the system governed by (10), by adopting the sliding function in (20) and the ASMC-II algorithm in (21), the system trajectory will converge to the sliding function as $t \to \infty$.

$$\begin{cases} T_b = & \lambda p\mathrm{e}^{-\lambda t}\hat{J}\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{M^T\rho}{m}\right) - \lambda\hat{J}\frac{\left(4M - 2\sigma_e\sigma_e^T\right)\omega_b}{1+\|\sigma_e\|^2} \\ & +\omega_b^\times\hat{J}\omega_b - \lambda^2 p\mathrm{e}^{-\lambda t}\hat{J}\frac{M^T\rho}{m} - \hat{d}\mathrm{sgn}(S(t)) \\ \hat{d} = & c\int_0^t \|S(t)\|_1 d\tau \end{cases} \tag{21}$$

where

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{M^T\rho}{m}\right) =& 8\frac{\sigma_e^T\rho M\omega_b - (\rho - \mu + \epsilon)^\times M\omega_b}{(1+\|\sigma_e\|^2)^2} \\ & - \frac{4M^T\rho\sigma_e^T M\omega_b}{(1+\|\sigma_e\|^2)m}\end{aligned}$$

where $\mu = [\sigma_{e3}\rho_2 \quad \sigma_{e1}\rho_3 \quad \sigma_{e2}\rho_1]^T \in \mathbb{R}^3$ and $\epsilon = [\sigma_{e2}\rho_3 \quad \sigma_{e3}\rho_1 \quad \sigma_{e1}\rho_2]^T \in \mathbb{R}^3$.

Proof 1: Considering the following Lyapunov function:

$$V = \frac{1}{2}S^T(t)\hat{J}S(t) + \frac{1}{2c}\tilde{d}^2 \tag{22}$$

The time derivative of the above Lyapunov function along the closed-loop system trajectory is:

$$\begin{aligned}\dot{V} =& S^T(t)\hat{J}\dot{S}(t) + \frac{\hat{d} - d_{\max}}{c}\dot{\hat{d}} \\ =& S^T(t)\left(d - \hat{d}\mathrm{sgn}(S(t))\right) + (\hat{d} - d_{\max})\|S(t)\|_1 \\ =& S^T(t)d - \hat{d}\|S(t)\|_1 + \hat{d}\|S(t)\|_1 - d_{\max}\|S(t)\|_1 \\ =& -\sum_{i=1}^{3}(d_{\max}|s_i(t)| - d_i s_i(t)) \le 0\end{aligned}$$

Let $\chi = \sum_{i=1}^{3} (d_{\max}|s_i(t)| - d_i s_i(t))$ and it is obvious that χ is uniformly continuous. By integrating the above equation from zero to t, one has:

$$\int_0^t \dot{V} d\tau \leq -\int_0^t \chi d\tau \Rightarrow V(0) \geq \int_0^t \chi d\tau \tag{23}$$

Taking the limits as $t \to \infty$ on both sides of (23) gives:

$$\infty > V(0) \geq \lim_{t\to\infty} \int_0^t \chi d\tau \tag{24}$$

On the basis of Barbalat lemma, one can obtain

$$\lim_{t\to\infty} \chi = 0 \tag{25}$$

which implies that $\lim_{t\to\infty} S(t) = 0$.

Remark 2: Above proof implies that the ASMC-II algorithm can only guarantee the asymptotic stability of the sliding function but not the attitude variables, i.e., the system trajectory will converge to the sliding surface in infinite time. However, from the adaptation law in (21), one can see that the switching gain $\hat{d}$ will keep increasing if $S(t) \neq 0$. When $\hat{d}$ increases up to a value large enough to suppress the lumped uncertainty, e.g., $\hat{d} > d_{\max} + \delta$ with δ a sufficiently small positive scalar, the sliding mode will start in finite time. Similarly, denote the arrival time as t_r. By a left multiplication of (20) with M, following 3-dimensional first-order vector differential equation can be obtained:

$$\dot{\sigma}_e + \lambda\sigma_e = \lambda p e^{-\lambda t}\rho \tag{26}$$

The analytical solution for σ_e is:

$$\sigma_e(t) = \mathrm{e}^{-\lambda(t-t_r)}(\lambda p\rho t + \sigma_e(t_r)) \tag{27}$$

It is obvious $\lim_{t\to\infty} \sigma_e(t) = 0$ and $\lim_{t\to\infty} \omega_b(t) = 0$ from (20). Hence, the attitude control system in (10) with the ASMC-II algorithm in (21) is globally asymptotically stable.

Remark 3: Recalling the adaptive law in (21), $\hat{d}$ will become unbounded due to the fact that the sliding function is not identically equal to zero, which may be caused by the finite switching frequency or measurement noise. For implementation in practice, the adaptive law has to be modified to get a bounded switching gain, such as the σ modification method in [12]. Here, the approach proposed in [18] will be used, where the adaptive law in (21) is modified as

$$\hat{d} = \begin{cases} c\int_0^t \|S(t)\|_1 \mathrm{sgn}(\|S(t)\|_1 - \eta)d\tau & \text{if} \quad \hat{d} > \varrho \\ \int_0^t \varrho d\tau & \text{if} \quad \hat{d} \leq \varrho \end{cases} \tag{28}$$

where $\varrho > 0$ is a very small scalar to ensure $\hat{d}$ is positive and $\eta > 0$ is carefully chosen to deal with the trade-off in control accuracy and bounded switching gain. Further details on η tuning can refer to [18].

IV. Numerical Simulation

In this section, a comparison of the ASMC-I and ASMC-II algorithms is employed for a large angle attitude maneuver to test the effectiveness of the proposed strategy.

The inertia matrix for the controller design is given by $\hat{J} = \mathrm{diag}(48, 25, 61.8)$ (kg.m) and the uncertainty is 10% of the nominal value. The external disturbance is $T_d = [0.02\sin(0.01t) \quad 0.02\cos(0.01t) \quad 0.04\sin(0.01t)]^T$ (N.m). The initial attitude variables are $\sigma_b(0) = [-0.2 \quad 0.3 \quad 0.1]^T$ and $\omega_b(0) = [0 \quad 0 \quad 0]^T$ (rad/s). The desired attitude is $\sigma_d = [0.1 \quad 0.2 \quad -0.3]^T$ with the desired angular velocity $\omega_d = [0 \quad 0 \quad 0]^T$ (rad/s). For comparison, same control parameters are selected for both the ASMC-I algorithm and the ASMC-II algorithm, where $c = 1$, $\lambda = 0.25$ and the initial value of $\hat{d}$ is zero. The weight p is selected as 0.8. The simulation results are shown in Fig.3–Fig.6.

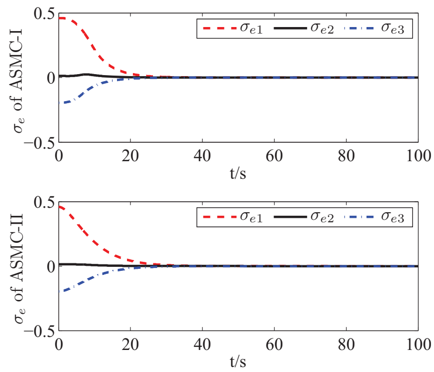

Fig. 3. Error MRPs responses of ASMC-I and ASMC-II

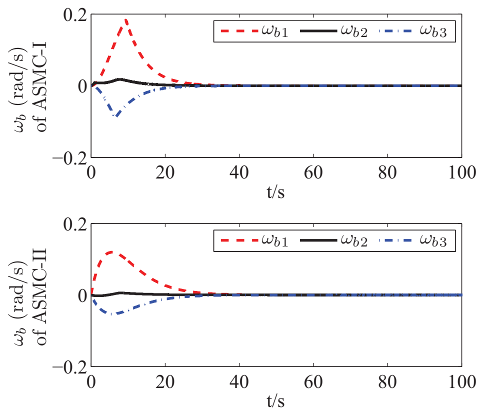

Fig. 4. Angular velocities of ASMC-I and ASMC-II

The maneuver evolutions controlled by ASMC-I and ASMC-II are compared in Fig. 3 and Fig. 4 with the corresponding control torque commands shown in Fig. 5. From Fig. 3, it is clear that both the ASMC-I and the ASMC-II can accomplish the attitude reorientation in the absence of the

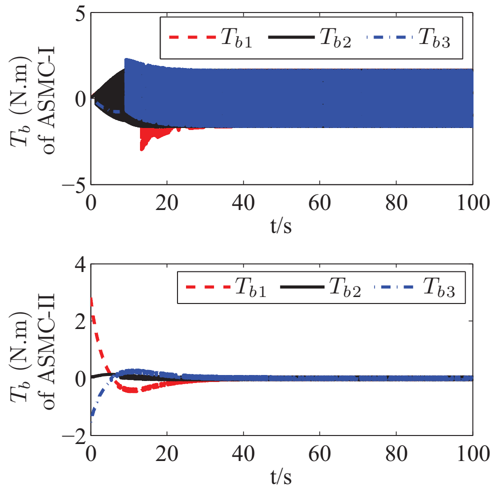

Fig. 5. Control torque commands of ASMC-I and AMSC-II

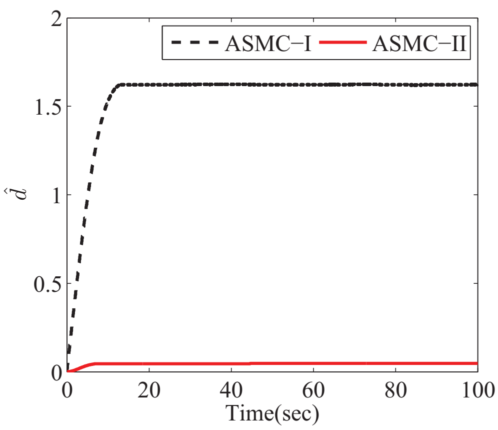

Fig. 6. Adaptive switching gains of ASMC-I and ASMC-II

prior knowledge of $d_{\max}$ and the system responses are similar. Fig. 4 shows the angular velocity response comparison, where the angular velocity controlled by the ASMC-II is smoother than that controlled by the ASMC-I. Nonetheless, there is a significant difference in the control torque commands as shown in Fig. 5. According to Fig. 5, it is obvious that the chattering in ASMC-I is much more serious than that in ASMC-II, which verified the effectiveness of the proposed strategy. Moreover, as shown in Fig. 6, the adaptive switching gain generated by ASMC-II is much smaller than the ASMC-I case, where $\hat{d} \approx 1.62$ for ASMC-I and $\hat{d} \approx 0.046$ for ASMC-II, which verified the chattering suppression ability of ASMC-II.

V. Conclusion

The attitude control problem of a rigid spacecraft involving inertia matrix uncertainty and external disturbance has been considered. An effective solution has been presented to address the over-adaptation problem in current ASMC design. Such an improvement is achieved by reducing or eliminating the influence caused by initial system error on the switching gain adaptation. It has been shown by theoretical analysis and simulation results that the proposed strategy can produce a much smaller switching gain as compared with current ASMC design and achieve a smoother system response.

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