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New stability criteria for linear systems with time-varying interval delay

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Abstract— **This paper is concerned with the problem of stability of systems with time-varying delay in a given interval. A novel Lyapunov-Krasovskii functional is proposed to obtain new stability conditions. Some triple integral terms are introduced in the Lyapunov-Krasovskii functional and the information on the lower bound on the delay are sufficiently used. New delay-dependent stability criteria are derived using integral inequalities and formulated in terms of linear matrix inequality (LMI). Comparing numerical examples show that the proposed criteria yield a larger upper bound on the delay for a given lower bound on the delay than existing results.**

I. INTRODUCTION

During the past few years, time-delay systems have been an active research area. Much attention has been paid to the stability and stabilization of time-delay systems. In a practical system, time-delay often deteriorates the performance of the system and even causes instability. Especially, in networked control systems, there exist time-delays in both the forward channel and the feedback channel, which poses a negative effect on the stability and performance of the systems and makes the systems difficult to analyze and synthesize [1], [2], [3], [4], [5].

Stability criteria for time-delay systems in the literature can be roughly classified into two categories. One is delay-independent and the other is delay-dependent. Generally speaking, delay-dependent stability conditions are less conservative than delay-independent ones. So, many researchers specialize in developing less conservative stability criteria for time-delay systems and some important results have been obtained [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16]. Based on some model transformations, some stability conditions for time-delay systems have been obtained in [17], [18]. A descriptor system method was proposed in [19], [20], [21] where a time-delay system is presented in the form of a descriptor system. Combining the descriptor system method with Park's inequality [22] or Moon et. al's inequality [23] can yield much less conservative results. In order to further reduce the conservatism of stability criteria, a free-weighting matrices method was proposed by He et. al [24], [25], [26]. Some free-weighting matrices are introduced by Leibniz–Newton formula to estimate the upper bound of the derivative of the Lyapunov-Krasovskii functional. Numerical examples illustrated that this method can yield less conservative results than the descriptor system method. In order to reduce the decision variables in the stability criteria, Jensen's inequality [27] was used to derive stability results for time-delay systems. It has been proved that results obtained by Jensen's inequality are generally equivalent to those obtained by descriptor system method or free-weighting matrices method [28]. For the systems with time-varying delay, [29], [30] reported that some useful terms are ignored when estimating the upper bound of the derivative of the Lyapunov functional, which can introduce significant conservatism. Inspired by this observation, some less conservative results were proposed in [29], [30], [31] by taking into these useful terms account.

In the literature, the time-varying delay is often assumed belong to a given interval, that is,

$$0 < h_1 \leqslant d(t) \leqslant h_2 \tag{1}$$

However, the information on the lower bound of the delay is not sufficiently used in the Lyapunov functional. For example, $\int_{t-h_2}^{t} x^T(s)Qx(s)ds$ and $\int_{t-d(t)}^{t} x^T(s)Rx(s)ds$ are often used as a part of the Lyapunov functional. The integral upper limits of these terms are all t but not $t-h_1$, which may cause some conservatism just as proved in our previous work [32]. Similarly, some double integral terms such as $\int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)dsd\theta$ are often used as a part of the Lyapunov functional. The inner integral upper limit is t but not $t-h_1$ which may also introduce some additional conservatism. Observing this fact, a new Lyapunov functional is proposed in this paper where the information on the lower bound of the delay is sufficiently used, that is, some terms like $\int_{-h_2}^{-h_1}\int_{t+\theta}^{t-h_1} \dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)dsd\theta$ are used in the Lyapunov functional. Furthermore, it has been shown in [32] that introducing some triple-integral terms in Lyaounov functional can significantly reduce the conservatism of the obtained results. In this paper, a triple integral term like $\int_{-h_2}^{-h_1}\int_{\theta}^{-h_1}\int_{t+\lambda}^{t-h_1} \dot{x}^{\mathrm{T}}(s)R\dot{x}(s)dsd\lambda d\theta$ is introduced in the Lyapunov functional. It should be noted that the upper limits of s, λ and θ are $t-h_1$, $-h_1$ and $-h_1$, respectively. In this paper, a novel Lyapunov-Krasovskii functional which contains some new triple-integral terms and sufficiently uses the information on the lower bound of the delay is proposed. Some new delay-dependent stability criteria are obtained using some integral inequalities. Numerical examples illustrates

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that results in this paper are significant improvements over existing ones.

Notations: Throughout this paper, the superscripts '-1' and 'T' stand for the inverse and transpose of a matrix, respectively; $\mathbb{R}^n$ denotes an n-dimensional Euclidean space; $\mathbb{R}^{m\times n}$is the set of all $m\times n$ real matrices; $P>0$ means that the matrix P is symmetric positive definite; I is an appropriately dimensional identity matrix.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following linear system with time-varying interval delay:

$$\begin{aligned} \dot{x}(t) &= Ax(t)+A_1x(t-\tau(t)), \quad t>0 \\ x(t) &= \phi(t), \quad t\in[-\tau_2,\ 0] \end{aligned} \tag{2}$$

where $x(t)\in\mathscr{R}^n$ is the state vector; $A\in\mathscr{R}^{n\times n}$ and $A_1\in\mathscr{R}^{n\times n}$ are constant system matrices with appropriate dimensions; The initial condition $\phi(t)$ is a continuously differentiable vector-valued function; $\tau(t)$ is a time-varying differentiable function and satisfies

$$0<\tau_1\leqslant\tau(t)\leqslant\tau_2 \tag{3}$$

$$\dot{\tau}(t)\leqslant\mu \tag{4}$$

where $0<\tau_1<\tau_2$, and $0\leqslant\mu$ are constants.

Before moving on, the following integral inequalities are introduced.

Lemma 1: [27], [32] For any constant matrix $Z>0$ and scalars $\tau_2>\tau_1>0$ such that the following integrations are well defined, then

(1)

$$\begin{aligned} &-\int_{t-\tau_2}^{t-\tau_1} x^{\mathrm{T}}(s)Zx(s)ds \\ &\qquad \leqslant -\tau_{12}^{-1}\int_{t-\tau_2}^{t-\tau_1} x^{\mathrm{T}}(s)dsZ\int_{t-\tau_2}^{t-\tau_1} x(s)ds \end{aligned}$$

(2)

$$\begin{aligned} &-\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t-\tau_1} x^{\mathrm{T}}(s)Zx(s)dsd\theta \\ &\leqslant -2\tau_{12}^{-2}\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t-\tau_1} x^{\mathrm{T}}(s)dsd\theta Z\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t-\tau_1} x(s)dsd\theta \end{aligned}$$

where $\tau_{12}=\tau_2-\tau_1$.

The objective of this paper is to derive less conservative delay-dependent stability conditions for system (2). Using the obtained results, one can obtain a larger maximum upper bound of the delay for a given lower bound of the delay.

III. MAIN RESULTS

In this section, some less conservative stability criteria are developed. Before presenting the main results, we define $\xi(t)=col\{x(t),x(t-\tau(t)),x(t-\tau_1),x(t-\tau_2),\dot{x}(t-\tau_1),\dot{x}(t-\tau_2),\ \int_{t-\tau_1}^{t}x(s)ds,\ \int_{t-\tau(t)}^{t-\tau_1}x(s)ds,\ \int_{t-\tau_2}^{t-\tau(t)}x(s)ds\}$, and e_i $(i=1,2,\cdots,9)$ are block entry matrices. For example, $e_7^{\mathrm{T}}=[0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ I\ \ 0\ \ 0]$.

Theorem 1: Given scalars $0<\tau_1<\tau_2$, and $0\leqslant\mu$, if there exist matrices $P=[P_{ij}]_{5\times5}>0$, $Q=[Q_{ij}]_{2\times2}>0$, $X=[X_{ij}]_{2\times2}>0$, $S>0$, $Z_j>0$, $j=1,\cdots,4$, $R_1>0$, and $R_2>0$ with appropriate dimensions such that the following LMIs hold, then system (2) with a time-varying delay satisfying (3) and (4) is asymptotically stable.

$$\begin{aligned} \Theta_1 =\ & \Phi P\Upsilon^{\mathrm{T}}+\Upsilon P\Phi^{\mathrm{T}}+\Lambda+\Gamma^{\mathrm{T}}Y\Gamma+\Gamma^{\mathrm{T}}Q_{12}^{\mathrm{T}}e_1^{\mathrm{T}} \\ & +e_1Q_{12}\Gamma-\begin{bmatrix} e_4 & e_6 \end{bmatrix}X\begin{bmatrix} e_4^{\mathrm{T}} \\ e_6^{\mathrm{T}} \end{bmatrix} \\ & +\begin{bmatrix} e_3 & e_5 \end{bmatrix}(X-Q)\begin{bmatrix} e_3^{\mathrm{T}} \\ e_5^{\mathrm{T}} \end{bmatrix} \\ & -(e_1-e_3)Z_1(e_1^{\mathrm{T}}-e_3^{\mathrm{T}}) \\ & -2(e_2-e_4)Z_2(e_2^{\mathrm{T}}-e_4^{\mathrm{T}})-(e_3-e_2)Z_2(e_3^{\mathrm{T}}-e_2^{\mathrm{T}}) \\ & -e_8Z_4e_8^{\mathrm{T}}-2e_9Z_4e_9^{\mathrm{T}}-(\tau_1e_1-e_7)R_1(\tau_1e_1^{\mathrm{T}}-e_7^{\mathrm{T}}) \\ & -(\tau_{12}e_3-e_8-e_9)R_2(\tau_{12}e_3^{\mathrm{T}}-e_8^{\mathrm{T}}-e_9^{\mathrm{T}})<0 \end{aligned} \tag{5}$$

$$\begin{aligned} \Theta_2 =\ & \Phi P\Upsilon^{\mathrm{T}}+\Upsilon P\Phi^{\mathrm{T}}+\Lambda+\Gamma^{\mathrm{T}}Y\Gamma+\Gamma^{\mathrm{T}}Q_{12}^{\mathrm{T}}e_1^{\mathrm{T}} \\ & +e_1Q_{12}\Gamma-\begin{bmatrix} e_4 & e_6 \end{bmatrix}X\begin{bmatrix} e_4^{\mathrm{T}} \\ e_6^{\mathrm{T}} \end{bmatrix} \\ & +\begin{bmatrix} e_3 & e_5 \end{bmatrix}(X-Q)\begin{bmatrix} e_3^{\mathrm{T}} \\ e_5^{\mathrm{T}} \end{bmatrix} \\ & -(e_1-e_3)Z_1(e_1^{\mathrm{T}}-e_3^{\mathrm{T}}) \\ & -(e_2-e_4)Z_2(e_2^{\mathrm{T}}-e_4^{\mathrm{T}})-2(e_3-e_2)Z_2(e_3^{\mathrm{T}}-e_2^{\mathrm{T}}) \\ & -2e_8Z_4e_8^{\mathrm{T}}-e_9Z_4e_9^{\mathrm{T}}-(\tau_1e_1-e_7)R_1(\tau_1e_1^{\mathrm{T}}-e_7^{\mathrm{T}}) \\ & -(\tau_{12}e_3-e_8-e_9)R_2(\tau_{12}e_3^{\mathrm{T}}-e_8^{\mathrm{T}}-e_9^{\mathrm{T}})<0 \end{aligned} \tag{6}$$

where

$$\begin{aligned} \Phi &= [e_1\ \ e_3\ \ e_4\ \ e_7\ \ e_8+e_9] \\ \Upsilon &= \left[\Gamma^{\mathrm{T}}\ \ e_5\ \ e_6\ \ e_1-e_3\ \ e_3-e_4\right] \\ \Lambda &= \mathrm{diag}\{Q_{11}+\tau_1Z_3,\ -(1-\mu)S,\ +S+\tau_{12}^2Z_4,\ 0 \\ &\quad \tau_{12}^2Z_2+\frac{\tau_{12}^4}{4}R_2,\ 0,\ -Z_3,\ 0,\ 0\} \\ \Gamma &= [A\ \ A_1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0] \\ Y &= Q_4+\tau_1^2Z_1+\frac{\tau_1^4}{4}R_1 \end{aligned}$$

Proof: Choose a Lyapunov functional as follows:

$$\begin{aligned} V(x_t) =\ & \rho^{\mathrm{T}}(t)P\rho(t)+\int_{t-\tau_1}^{t}\zeta^{\mathrm{T}}(s)Q\zeta(s)ds \\ & +\int_{t-\tau_2}^{t-\tau_1}\zeta^{\mathrm{T}}(s)X\zeta(s)ds \\ & +\int_{t-\tau(t)}^{t-\tau_1}x^{\mathrm{T}}(s)Sx(s)ds \\ & +\int_{-\tau_1}^{0}\int_{t+\theta}^{t}\tau_1\dot{x}^{\mathrm{T}}(s)Z_1\dot{x}(s)dsd\theta \\ & +\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t-\tau_1}\tau_{12}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)dsd\theta \\ & +\int_{-\tau_1}^{0}\int_{t+\theta}^{t}\tau_1x^{\mathrm{T}}(s)Z_3x(s)dsd\theta \\ & +\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t-\tau_1}\tau_{12}x^{\mathrm{T}}(s)Z_4x(s)dsd\theta \end{aligned}$$

$$+\int_{-\tau_1}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t}\frac{\tau_1^2}{2}\dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)dsd\lambda d\theta$$
$$+\int_{-\tau_2}^{-\tau_1}\int_{\theta}^{-\tau_1}\int_{t+\lambda}^{t-\tau_1}\frac{\tau_{12}^2}{2}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)dsd\lambda d\theta \quad (7)$$

where $\rho(t) = col\{x(t), x(t-\tau_1), x(t-\tau_2), \int_{t-\tau_1}^{t}x(s)ds, \int_{t-\tau_2}^{t-\tau_1}x(s)ds\}$, $\zeta(s) = col\{x(s), \dot{x}(s)\}$.

Taking the derivative of $V(x_t)$ along the trajectory of system (2) yields

$$\begin{aligned}\dot{V}(x_t) = {} & 2\rho^{\mathrm{T}}(t)P\dot{\rho}(t)\\ & +\zeta^{\mathrm{T}}(t)Q\zeta(t)-\zeta^{\mathrm{T}}(t-\tau_1)Q\zeta(t-\tau_1)\\ & +\zeta^{\mathrm{T}}(t-\tau_1)X\zeta(t-\tau_1)-\zeta^{\mathrm{T}}(t-\tau_2)X\zeta(t-\tau_2)\\ & +x^{\mathrm{T}}(t-\tau_1)Sx(t-\tau_1)\\ & -(1-\mu)x^{\mathrm{T}}(t-\tau(t))Sx(t-\tau(t))\\ & +\tau_1^2\dot{x}^{\mathrm{T}}(t)Z_1\dot{x}(t)-\tau_1\int_{t-\tau_1}^{t}\dot{x}^{\mathrm{T}}(s)Z_1\dot{x}(s)ds\\ & +\tau_{12}^2\dot{x}^{\mathrm{T}}(t-\tau_1)Z_2\dot{x}(t-\tau_1)\\ & -\tau_{12}\int_{t-\tau_2}^{t-\tau_1}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)ds\\ & +\tau_1^2x^{\mathrm{T}}(t)Z_1x(t)-\tau_1\int_{t-\tau_1}^{t}x^{\mathrm{T}}(s)Z_3x(s)ds\\ & +\tau_{12}^2x^{\mathrm{T}}(t-\tau_1)Z_4x(t-\tau_1)\\ & -\tau_{12}\int_{t-\tau_2}^{t-\tau_1}x^{\mathrm{T}}(s)Z_4x(s)ds\\ & +\frac{\tau_1^4}{4}\dot{x}^{\mathrm{T}}(t)R_1\dot{x}(t)\\ & -\frac{\tau_1^2}{2}\int_{-\tau_1}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)dsd\theta\\ & +\frac{\tau_{12}^4}{4}\dot{x}^{\mathrm{T}}(t-\tau_1)R_2\dot{x}(t-\tau_1)\\ & -\frac{\tau_{12}^2}{2}\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t-\tau_1}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)dsd\theta \end{aligned} \quad (8)$$

Using Lemma 1, it can be obtained that

$$\begin{aligned}-\tau_1\int_{t-\tau_1}^{t}\dot{x}^{\mathrm{T}}(s)Z_1\dot{x}(s)ds\\ \le -\xi^{\mathrm{T}}(t)(e_1-e_3)Z_1(e_1^{\mathrm{T}}-e_3^{\mathrm{T}})\xi(t)\end{aligned} \quad (9)$$

$$\begin{aligned}-\tau_1\int_{t-\tau_1}^{t}x^{\mathrm{T}}(s)Z_3x(s)ds\\ \le -\int_{t-\tau_1}^{t}x^{\mathrm{T}}(s)dsZ_3\int_{t-\tau_1}^{t}x(s)ds\end{aligned} \quad (10)$$

$$\begin{aligned}-\frac{\tau_1^2}{2}\int_{-\tau_1}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)dsd\theta\\ \le -\xi^{\mathrm{T}}(t)(\tau_1e_1-e_7)R_1(\tau_1e_1^{\mathrm{T}}-e_7^{\mathrm{T}})\xi(t)\end{aligned} \quad (11)$$

$$\begin{aligned}-\frac{\tau_{12}^2}{2}\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t-\tau_1}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)dsd\theta\\ \le -\xi^{\mathrm{T}}(t)(\tau_{12}e_3-e_8-e_9)R_2(\tau_{12}e_3^{\mathrm{T}}-e_8^{\mathrm{T}}-e_9^{\mathrm{T}})\xi(t)\end{aligned} \quad (12)$$

let $\alpha = (\tau(t)-\tau_1)/\tau_{12}$ and use the similar method as in [31]

$$\begin{aligned}-\tau_{12}\int_{t-\tau_2}^{t-\tau_1}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)ds\\ = -\tau_{12}\int_{t-\tau_2}^{t-\tau(t)}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)ds\\ -\tau_{12}\int_{t-\tau(t)}^{t-\tau_1}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)ds\\ = -(\tau_2-\tau(t))\int_{t-\tau_2}^{t-\tau(t)}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)ds\\ -(\tau(t)-\tau_1)\int_{t-\tau_2}^{t-\tau(t)}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)ds\\ -(\tau(t)-\tau_1)\int_{t-\tau(t)}^{t-\tau_1}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)ds\\ -(\tau_2-\tau(t))\int_{t-\tau(t)}^{t-\tau_1}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)ds\\ \le -\xi^{\mathrm{T}}(t)(e_2-e_4)Z_2(e_2^{\mathrm{T}}-e_4^{\mathrm{T}})\xi(t)\\ -\xi^{\mathrm{T}}(t)(e_3-e_2)Z_2(e_3^{\mathrm{T}}-e_2^{\mathrm{T}})\xi(t)\\ -\alpha\xi^{\mathrm{T}}(t)(e_2-e_4)Z_2(e_2^{\mathrm{T}}-e_4^{\mathrm{T}})\xi(t)\\ -(1-\alpha)\xi^{\mathrm{T}}(t)(e_3-e_2)Z_2(e_3^{\mathrm{T}}-e_2^{\mathrm{T}})\xi(t)\end{aligned} \quad (13)$$

Similarly,

$$\begin{aligned}-\tau_{12}\int_{t-\tau_2}^{t-\tau_1}x^{\mathrm{T}}(s)Z_4x(s)ds\\ \le -\xi^{\mathrm{T}}(t)\left[e_8^{\mathrm{T}}Z_4e_8+e_9^{\mathrm{T}}Z_4e_9\right]\xi(t)\\ -\alpha\xi^{\mathrm{T}}(t)e_9^{\mathrm{T}}Z_4e_9\xi(t)\\ -(1-\alpha)\xi^{\mathrm{T}}(t)e_8^{\mathrm{T}}Z_4e_8\xi(t)\end{aligned} \quad (14)$$

From (8)-(14), one can obtain

$$\dot{V}(x_t) \leqslant \xi^{\mathrm{T}}(t)\left[\alpha\Theta_1+(1-\alpha)\Theta_2\right]\xi(t) \quad (15)$$

Since $0 \le \alpha \le 1$, $\alpha\Theta_1+(1-\alpha)\Theta_2$ is a convex combination of Θ_1 and Θ_2. Therefore, $\alpha\Theta_1+(1-\alpha)\Theta_2 < 0$ is equivalent to $\Theta_1 < 0$ and $\Theta_2 < 0$. If (5)-(6) are satisfied, then system (2) is asymptotically stable. ■

Remark 1: A new kind of augmented Lyapunov functional is proposed in this paper to develop new delay-interval-dependent stability criteria. Being distinguished from existing Lyapunov functionals, the one in this paper contains some triple-integral terms which has been proved able to reduce the conservatism of the obtained results effectively. Furthermore, the information on the lower bound of the delay is sufficiently used in the Lyapunov functional by including the terms $\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t-\tau_1}\tau_{12}\dot{x}^{\mathrm{T}}(s)Z_2\dot{x}(s)dsd\theta$ and $\int_{-\tau_2}^{-\tau_1}\int_{\theta}^{-\tau_1}\int_{t+\lambda}^{t-\tau_1}\frac{\tau_{12}^2}{2}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)dsd\lambda d\theta$. It can be seen that the integral upper limits of these terms are $t-\tau_1$ or $-\tau_1$. To the best knowledge of the authors', this kind of Lyapunov functional has been never used in the literature. Numerical examples will be given in the next section to show that such a kind of Lyapunov functional can yield less conservative results.

Remark 2: Using some integral inequalities and the idea of the convex combination, new delay-interval-dependent stability criteria are obtained without introducing any free-weighting matrices. Therefore, the method proposed in this paper may involve much less variables than the well-known free-weighting matrices method and the descriptor system method.

In some circumstances, the information on the derivative of the delay may not be always available or the delay is not differentiable. For this case, the following corollary can be derived from Theorem 1 by setting $S = 0$.

Corollary 1: Given scalars $0 < \tau_1 < \tau_2$, and $0 \leqslant \mu$, if there exist matrices $P = [P_{ij}]_{5\times5} > 0$, $Q = [Q_{ij}]_{2\times2} > 0$, $X = [X_{ij}]_{2\times2} > 0$, $Z_j > 0$, $j = 1, \cdots, 4$, $R_1 > 0$, and $R_2 > 0$ with appropriate dimensions such that the following LMIs hold, then system (2) with a time-varying delay satisfying (3) is asymptotically stable.

$$\begin{aligned}\hat{\Theta}_1 = {} & \Phi P \Upsilon^{\mathrm{T}} + \Upsilon P \Phi^{\mathrm{T}} + \hat{\Lambda} + \Gamma^{\mathrm{T}} Y \Gamma + \Gamma^{\mathrm{T}} Q_{12}^{\mathrm{T}} e_1^{\mathrm{T}} \\ & + e_1 Q_{12} \Gamma - \begin{bmatrix} e_4 & e_6 \end{bmatrix} X \begin{bmatrix} e_4^{\mathrm{T}} \\ e_6^{\mathrm{T}} \end{bmatrix} \\ & + \begin{bmatrix} e_3 & e_5 \end{bmatrix} (X - Q) \begin{bmatrix} e_3^{\mathrm{T}} \\ e_5^{\mathrm{T}} \end{bmatrix} \\ & - (e_1 - e_3) Z_1 (e_1^{\mathrm{T}} - e_3^{\mathrm{T}}) \\ & - 2(e_2 - e_4) Z_2 (e_2^{\mathrm{T}} - e_4^{\mathrm{T}}) - (e_3 - e_2) Z_2 (e_3^{\mathrm{T}} - e_2^{\mathrm{T}}) \\ & - e_8 Z_4 e_8^{\mathrm{T}} - 2 e_9 Z_4 e_9^{\mathrm{T}} - (\tau_1 e_1 - e_7) R_1 (\tau_1 e_1^{\mathrm{T}} - e_7^{\mathrm{T}}) \\ & - (\tau_{12} e_3 - e_8 - e_9) R_2 (\tau_{12} e_3^{\mathrm{T}} - e_8^{\mathrm{T}} - e_9^{\mathrm{T}}) < 0 \qquad (16)\end{aligned}$$

$$\begin{aligned}\hat{\Theta}_2 = {} & \Phi P \Upsilon^{\mathrm{T}} + \Upsilon P \Phi^{\mathrm{T}} + \hat{\Lambda} + \Gamma^{\mathrm{T}} Y \Gamma + \Gamma^{\mathrm{T}} Q_{12}^{\mathrm{T}} e_1^{\mathrm{T}} \\ & + e_1 Q_{12} \Gamma - \begin{bmatrix} e_4 & e_6 \end{bmatrix} X \begin{bmatrix} e_4^{\mathrm{T}} \\ e_6^{\mathrm{T}} \end{bmatrix} \\ & + \begin{bmatrix} e_3 & e_5 \end{bmatrix} (X - Q) \begin{bmatrix} e_3^{\mathrm{T}} \\ e_5^{\mathrm{T}} \end{bmatrix} \\ & - (e_1 - e_3) Z_1 (e_1^{\mathrm{T}} - e_3^{\mathrm{T}}) \\ & - (e_2 - e_4) Z_2 (e_2^{\mathrm{T}} - e_4^{\mathrm{T}}) - 2(e_3 - e_2) Z_2 (e_3^{\mathrm{T}} - e_2^{\mathrm{T}}) \\ & - 2 e_8 Z_4 e_8^{\mathrm{T}} - e_9 Z_4 e_9^{\mathrm{T}} - (\tau_1 e_1 - e_7) R_1 (\tau_1 e_1^{\mathrm{T}} - e_7^{\mathrm{T}}) \\ & - (\tau_{12} e_3 - e_8 - e_9) R_2 (\tau_{12} e_3^{\mathrm{T}} - e_8^{\mathrm{T}} - e_9^{\mathrm{T}}) < 0 \qquad (17)\end{aligned}$$

where Φ, Υ, Y and Γ are the same as those defined in Therom 1, and $\hat{\Lambda} = \mathrm{diag}\{Q_{11} + \tau_1 Z_3,\ 0,\ \tau_{12}^2 Z_4,\ 0,\ \tau_{12}^2 Z_2 + \frac{\tau_{12}^4}{4} R_2,\ 0,\ -Z_3,\ 0,\ 0\}$.

Remark 3: When there are norm-bounded uncertainties in system (2), Theorem 1 and Corollary 1 can be extended to deal with this case following a similar method as in [9], [23].

IV. NUMERICAL EXAMPLES

In this section, some numerical examples are given to show the effectiveness of the proposed method, that is, the method in this paper can yield less conservative results than exiting ones.

Example 1: Consider the following system [30], [31] with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$

For various μ and unknown μ, the maximum upper bounds of the delay (MUBDs), τ_2, for given lower bound, τ_1, are listed in Table I and II along with those obtained in [31], [32]. It is easy to see from Table I and II that our method can give less conservative results than those obtained in [31], [32].

TABLE I

MUBDs WITH GIVEN τ_1 FOR DIFFERENT μ FOR EXAMPLE 1

τ_1	Methods	$\mu = 0.3$	$\mu = 0.5$	$\mu = 0.9$
2	[31]	2.6972	2.5048	2.5048
	[32]	3.0129	2.5663	2.5663
	Theorem 1	3.0168	2.6116	2.6116
3	[31]	3.2591	3.2591	3.2591
	[32]	3.3408	3.3408	3.3408
	Theorem 1	3.3932	3.3932	3.3932
4	[31]	4.0744	4.0744	4.0744
	[32]	4.1690	4.1690	4.1690
	Theorem 1	4.2054	4.2054	4.2054
5	[31]	—	—	—
	[32]	5.0275	5.0275	5.0275
	Theorem 1	5.0440	5.0440	5.0440

TABLE II

MUBDs FOR VARIOUS τ_1 AND UNKNOWN μ FOR EXAMPLE 1

Methods	τ_1	2	3	4	5
[31]	τ_2	2.5048	3.2591	4.0744	—
[32]	τ_2	2.5663	3.3408	4.1690	5.0275
Corollary 1	τ_2	2.6116	3.3932	4.2054	5.0440

Example 2: Consider the following system [30], [31] with

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}.$$

Given different lower bounds, our objective is to calculate MUBDs which keep the above system asymptotically stable. Table III lists the results for $\mu = 0.6$ and Table IV lists results for unknown μ comparing those obtained in [31], [32]. It can be seen that the results obtained in this paper are better than those in [31], [32].

Example 3: Consider the following system [29] with

$$A = \begin{bmatrix} -0.5 & -2 \\ 1 & -1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -0.5 & -1 \\ 0 & 0.6 \end{bmatrix}.$$

For various μ, the MUBDs for given lower bound, τ_1, are listed in Table V. In Table V results in [31], [32] are also listed. It is easy to see that our results are less conservative than those in [31], [32].

V. CONCLUSIONS

In this paper, the problem of the stability of linear systems with time-varying interval delays has been investigated. New delay-interval-dependent criteria have been developed by introducing a new Lyapunov-Krasovskii functional and

TABLE III

MUBDs FOR VARIOUS τ_1 AND $\mu = 0.6$ FOR EXAMPLE 2

Methods	τ_1	0.3	0.5	0.8	1
[31]	τ_2	1.0715	1.2191	1.4539	1.6169
[32]	τ_2	1.0717	1.2198	1.4558	1.6198
Theorem 1	τ_2	1.0948	1.2588	1.5135	1.6867

TABLE IV

MUBDs FOR VARIOUS τ_1 AND UNKNOWN μ FOR EXAMPLE 2

Methods	τ_1	1	2	3	4	5
[31]	τ_2	1.6169	2.4798	3.3894	4.3250	5.2773
[32]	τ_2	1.6198	2.4884	3.4030	4.3424	5.2970
Corollary 1	τ_2	1.6867	2.5750	3.4878	4.4193	5.3654

TABLE V

MUBDS WITH GIVEN τ_1 FOR DIFFERENT μ FOR EXAMPLE 3

τ_1	Methods	$\mu = 0.2$	$\mu = 0.5$	$\mu = 0.7$
0.3	[31]	1.3831	1.1000	0.9513
	[32]	1.7022	1.3043	1.0713
	Theorem 1	1.7856	1.3261	1.1333
0.5	[31]	1.3843	1.1000	1.0289
	[32]	1.8580	1.3940	1.1780
	Theorem 1	1.9808	1.4216	1.2326
0.7	[31]	1.3863	1.1117	1.1115
	[32]	2.0148	1.4665	1.2898
	Theorem 1	2.1623	1.4933	1.3365
1	[31]	1.3918	1.2493	1.2493
	[32]	2.2024	1.5214	1.4743
	Theorem 1	2.3897	1.5709	1.5383

using the integral inequality technique and the idea of the convex combination. Due to the new construction of the introduced Lyapunov-Krasovskii functional and the sufficient use of the information on the delay interval, our results are less conservative than the existing ones. Some numerical examples have illustrated that the method proposed in this paper is efficient.

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