UKACC International Conference on Control 2012
Cardiff, UK, 3-5 September 2012

Stability Analysis of Dynamic Quantized Feedback System With Packet Loss

Mu Li, Jian Sun, Lihua Dou
School of Automation
Beijing Institute of Technology
Beijing 100081, China
limu.bit@gmail.com, helios1225@yahoo.com.cn, doulihua@bit.edu.cn

Abstract—**This article is concerned with stability analysis of a linear discrete-time dynamic quantizer system with packet loss. First, several modifications are made to the original dynamic quantizer to make it easier to realize. Then communication channel subject to packet loss of Bernoulli distribution from the quantizer to the plant input is considered. Moreover, based on Lyapunov function approach, a sufficient condition for mean square stability of the closed-loop system is derived. Finally, a numerical simulation is given for effectiveness of the proposed method.**

Keywords-Discrete-time; Dynamic quantizer; Mean square stability; Linear discrete system; Pactket loss;

I. INTRODUCTION

In the past few years, quantized control has been of great significance in the research field of control systems. In practical discrete-time systems, owing to limited network bandwidth, extensive use of encoders and decoders, command-driven actuators and discrete-level sensors, it becomes necessary for the signals to be quantized before transmission.

Numerous results on this subject have been obtained in recent years. Early studies of quantized control systems mainly focus on construction of static quantizers which can guarantee the stability of the system. For example, global asymptotic stabilization of continuous-time systems is considered in [1], [9], where a special uniform quantizer with scaling factor is used for quantization. Stabilization of the given system with the coarsest quantization density is analyzed in [2], [3], [7] using the sector bound approach. Furthermore, stabilization problem for systems with one-dimensional input using quantized feedback with a memory structure is analyzed in [4]-[6], focusing on the tradeoff between static quantizer complexity and system performance. And the least amount of information needs to be communicated between the quantizer and the controller in order to stabilize an unstable linear system is addressed in [8], [10]-[13]. Moreover, the coarsest quantization density to stabilize the system with networked packet losses is considered in [17]-[18], and input-to-state stability of systems with time-varying delays is analyzed in [19] in terms of LMIs.

For early works of quantized control systems, the parameters of quantizers stay invariant as the system evolves, which will generate large quantized error. This means that part of the system performance has to be sacrificed to cope with such a quantized error. For this reason, in [14], [15], a novel optimal dynamic quantizer was given whose parameters can vary as the system evolves, which is able to handle such problem. The optimal dynamic quantizer in [14], [15] is constructed to minimize the quantized error according to the output of the plant, which can sacrifice fewer system's performance. In further study of this work in [16], the stability problem was considered under a dynamic quantized LFT system in terms of the poles/zeros. However, these works cannot deal with networked problems such as packet losses and time-delays effectively.

In this paper, stability of optimal dynamic quantized system is analyzed based on Lyapunov function approach. Besides, networked packet loss of the closed-loop system is also taken into consideration, which cannot be effectively solved in the LFT form. Moreover, some modifications are made to the original dynamic quantizer. The static part of the dynamic quantizer is replaced by the static quantizer with scaling factor and saturation value, which makes it easier to be realized and can guarantee smaller quantized error when the state comes close to the equilibrium point.

The whole paper is organized as follows. In section 2, we review the original dynamic quantized system and introduce the improved dynamic quantizers used in this paper. Next in section 3, main result on stability analysis is derived. A numerical simulation is given in section 4 and section 5 concludes this article.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Original Dynamic Quantized System

Consider the discrete-time systems shown in Fig.1, where the linear plant P is given by

$$P:\begin{cases}x(k+1)=Ax(k)+Bv(k)\\ y(k)=Cx(k)\end{cases} \tag{1}$$

where $x\in R^n$ is the state, $v\in R^m$ is the control input, $y\in R^p$ is the output, $A\in R^{n\times n}$, $B\in R^{n\times m}$, and $C\in R^{p\times n}$ are system matrices. The initial state of the plant P is

This work was supported by the National Natural Science Foundation of China (Grant No. 61104097, No. 61120106010), Research Fund for the Doctoral Program of Higher Education of China (Grant No. 20111101120027).

$x(0) = x_0$ for $x_0 \in R^n$, the pair (A, B) is stabilizable and A is unstable. Assume the following assumption holds:

Assumption 1. The plant P satisfies that the dimensions of v and y are the same $(m = p)$ and the matrix CB is nonsingular.

The original optimal dynamic quantizer Q in Fig.1 (Σ^{**}) is given by [14]

$$Q: \begin{cases} \xi(k+1) = A\xi(k) - Bu(k) + Bv(k) \\ v(k) = q(-(CB)^{-1}CA\xi(k) + u(k)) \end{cases} \tag{2}$$

where $\xi \in R^n$, $u \in R^m$, and $v \in R^m$ are the state, input, and output of this quantizer respectively, A, B and C of the quantizer are the same as the plant P, $q: R^m \to R^m$ is a static uniform quantizer.

The static uniform quantizer q in (2) satisfies that

$$q(x) = \left\lfloor \frac{x}{\Delta} + \frac{1}{2} \right\rfloor \Delta \tag{3}$$

that is

$$abs(q(x) - x) = abs(x - q(x)) \le \frac{\Delta}{2} \tag{4}$$

where $\tilde{a} = \lfloor a \rfloor$ denotes the biggest integer satisfying $\tilde{a} \le a$, and Δ is the quantized interval of the quantizer.

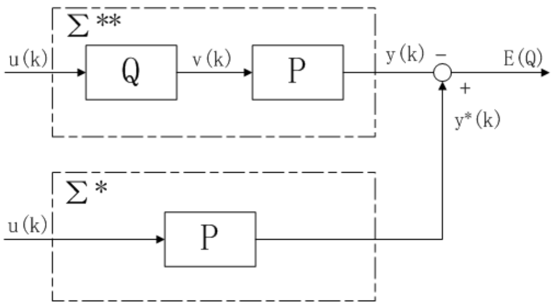

Figure 1. Two controlled plants: (Σ^*) usual controlled plant; (Σ^{**}) controlled plant with dynamic quantizer

Remark 1. The dynamic quantizer Q in Fig.1 (Σ^{**}) has been proved to be an optimal quantizer according to the output of the plant in [14]. Which indicates the parameters of the dynamic quantizer Q are chosen such that the output error $E(Q) = \sup_{k \in Z_+} \| y(k) - y^*(k) \|$ between the system Σ^{**} and Σ^* is minimized. For the static quantizer q, the quantized error is $\frac{\Delta}{2}$ as (4) indicates. It is clear that such open-loop dynamic quantized system cannot always guarantee stability of the whole system for the quantizer itself may not be stable.

B. Closed-loop System

In this paper, consider that packet losses occur with probability α in the input channel of the plant as is shown in Fig.2. The plant is described as

$$P: \begin{cases} x(k+1) = Ax(k) + B\theta(k)v(k) \\ y(k) = Cx(k) \end{cases} \tag{5}$$

where $\theta(k)$ is a Bernoulli random variable with a probability distribution given by

$$\Pr(\theta(k) = i) = \begin{cases} \alpha, & i = 0, \\ 1 - \alpha, & i = 1, \end{cases} \quad 0 \le \alpha < 1 \tag{6}$$

Consider the closed-loop system Σ in Fig.2 using sate feedback control law $u(k)$ which is given by

$$u(k) = Kx(k) \tag{7}$$

where $K \in R^{m \times n}$ is the feedback gain.

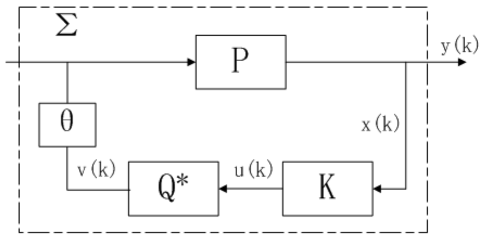

Figure 2. Dynamic quantized system Σ with packet loss

In this paper we replace static uniform quantizer q in (2) with q_μ

$$q_\mu(x) = \begin{cases} \mu(k)M, & if \ \frac{x}{\mu(k)} > M - \frac{1}{2}\Delta \\ -\mu(k)M, & if \ \frac{x}{\mu(k)} \le -M + \frac{1}{2}\Delta \\ \left\lfloor \frac{x}{\Delta\mu(k)} + \frac{1}{2} \right\rfloor \Delta\mu(k), & if \ \frac{|x|}{\mu(k)} \le M - \frac{1}{2}\Delta \end{cases} \tag{8}$$

where $\mu(k)$ is the scaling factor, M is the saturation value and Δ is the sensitivity.

Obviously, the following conditions can be obtained according to the quantizer:

Ⅰ.If $|x| \le M\mu(k)$, then $|q_\mu(x) - x| \le \frac{\Delta\mu(k)}{2}$;

Ⅱ.If $|x| > M\mu(k)$, then $|q_\mu(x)| > M\mu(k) - \frac{\Delta\mu(k)}{2}$.

Remark 2. The uniform quantizer q_μ here is different from quantizer q in that it brings in a scaling factor $\mu(k)$, and a saturation value M . The former can guarantee q_μ not saturate by adjusting scaling factor $\mu(k)$ properly, which will be considered in next section. The latter makes the static part of the dynamic quantizer easier to realize. Moreover, it is clear that by bringing in this scaling factor $\mu(k)$, when the quantizer does not meet with saturation, the quantized error varies according to $\mu(k)$.

Therefore, an improved dynamic quantizer Q^* in Fig.2 is given by

$$Q^*:\begin{cases}\xi(k+1)=A\xi(k)-Bu(k)+Bv(k)\\ v(k)=q_\mu(-(CB)^{-1}CA\xi(k)+u(k))\end{cases} \tag{9}$$

Remark 3. It should be noticed that although the static quantizer q_μ used here is different from q in [14], the dynamic quantizer Q^* is still the optimal quantizer for the system Σ . It has been proved in [14] that for the dynamic quantizer (2) with static quantizer (3), minimum value of output error $E(Q)$ can be given as

$$E(Q)=\|abs(CB)\|\frac{\Delta}{2}$$

where for the matrix $M:=\{M_{ij}\}$, $abs(M):=\{|M_{ij}|\}$.

When it comes to q_μ for our system, the quantized error becomes $\frac{\Delta\mu(k)}{2}$, obviously, similar conclusion can be expressed as

$$E(Q)=\|abs(CB)\|\frac{\Delta\overline{\mu}}{2}$$

where $\overline{\mu}=\sup_{k\in Z_+}(\mu(k))$ is upper bound of $\mu(k)$. As a result, the improved dynamic quantizer is still optimal dynamic quantizer for the system.

Therefore, the closed-loop system Σ in Fig.2 can be written as

$$\begin{cases}\begin{bmatrix}x(k+1)\\ \xi(k+1)\end{bmatrix}=\begin{bmatrix}A+BK & -\theta B(CB)^{-1}CA\\ 0 & A-B(CB)^{-1}CA\end{bmatrix}\begin{bmatrix}x(k)\\ \xi(k)\end{bmatrix}\\ \qquad\qquad +\begin{bmatrix}\theta B\\ B\end{bmatrix}\left[q_\mu(\phi)-\phi\right]\\ y(k)=[C\quad 0]\begin{bmatrix}x(k)\\ \xi(k)\end{bmatrix}\end{cases} \tag{10}$$

where $\phi=-(CB)^{-1}CA\xi(k)+Kx(k)$.

Rewrite the closed-loop system Σ as

$$\begin{cases}z(k+1)=\overline{A}z(k)+\overline{B}e(\Gamma z(k))\\ y(k)=\overline{C}z(k)\end{cases} \tag{11}$$

where $z(k)=\left[x(k)\quad \xi(k)\right]^T$, $e(\Gamma z(k))=q_\mu(\phi)-\phi$ denotes the quantized error, and $\Gamma=\left[-(CB)^{-1}CA\quad K\right]$.

Matrices $\overline{A}\in R^{2n\times 2n}$, $\overline{B}\in R^{2n\times m}$, $\overline{C}\in R^{p\times 2n}$ are defined as

$$\overline{A}=\begin{bmatrix}A+\theta BK & -\theta B(CB)^{-1}CA\\ 0 & A-B(CB)^{-1}CA\end{bmatrix},\overline{B}=\begin{bmatrix}\theta B\\ B\end{bmatrix},\overline{C}=[C\quad 0].$$

When $\theta(k)=1$, we let

$$A_1=\overline{A}=\begin{bmatrix}A+BK & -B(CB)^{-1}CA\\ 0 & A-B(CB)^{-1}CA\end{bmatrix},B_1=\overline{B}=\begin{bmatrix}B\\ B\end{bmatrix}.$$

When $\theta(k)=0$, we can get

$$A_2=\overline{A}=\begin{bmatrix}A & 0\\ 0 & A-B(CB)^{-1}CA\end{bmatrix},B_2=\overline{B}=\begin{bmatrix}0\\ B\end{bmatrix}.$$

Definition 1. System (11) is said to be mean square stable if

$$\lim_{k\to\infty}E\left\{|z(k)|^2\right\}=0 \tag{12}$$

for any initial state $z(0)\in R^{2n}$.

Lemma 1. The following inequality holds for any positive definite matrix G and matrices E and F

$$E^TGF+F^TGE\le E^TGE+F^TGF \tag{13}$$

III. STABILITY ANALYSIS

The following theorem presents a sufficient condition for the stability of the closed-loop system Σ :

Theorem 1. For a given feedback gain K and packet loss probability α , if there exists a positive definite symmetric matrix P satisfying that

$$(1-\alpha)A_1^TPA_1+\alpha A_2^TPA_2-\frac{1}{2}P<0 \tag{14}$$

then the system (11) is mean square stable under control law $u(k)=Kx(k)$.

Proof: The closed-loop system (11) can be expressed as

$$\begin{cases}z(k+1)=(1-\alpha)\left(A_1z(k)+B_1e(\Gamma z(k))\right)\\ \qquad\qquad +\alpha\left(A_2z(k)+B_2e(\Gamma z(k))\right)\\ y(k+1)=\overline{C}z(k)\end{cases} \tag{15}$$

Choose a Lyapunov function, $V(z(k))=z^T(k)Pz(k)$, where $P\in R^{2n\times 2n}$ is a positive definite matrix, $\Delta V(z(k))$ is given by the following expression

$$\Delta V(z(k))=E\{V(z(k+1))\}-V(z(k))$$

by using Lemma 1, we can get

$$\begin{aligned}
&\Delta V(z(k)) \\
&= E\left\{z^T(k+1)Pz(k+1)\right\} - z^T(k)Pz(k) \\
&= (1-\alpha)[z^T(k)(A_1^T PA_1 - P)z(k) + z^T(k)A_1^T PB_1 e(\Gamma z(k)) \\
&\quad + e^T(\Gamma z(k))B_1^T PA_1 z(k) + e^T(\Gamma z(k))B_1^T PB_1 e(\Gamma z(k))] + \\
&\quad + \alpha[z^T(k)(A_2^T PA_2 - P)z(k) + z^T(k)A_2^T PB_2 e(\Gamma z(k)) \\
&\quad + e^T(\Gamma z(k))B_2^T PA_2 z(k) + e^T(\Gamma z(k))B_2^T PB_2 e(\Gamma z(k))] \\
&\le 2z^T(k)[(1-\alpha)A_1^T PA_1 + \alpha A_2^T PA_2 - \frac{1}{2}P]z(k) \\
&\quad + 2e^T(\Gamma z(k))[(1-\alpha)B_1^T PB_1 + \alpha B_2^T PB_2]e(\Gamma z(k)) \\
&\le 2z^T(k)[(1-\alpha)A_1^T PA_1 + \alpha A_2^T PA_2 - \frac{1}{2}P]z(k) \\
&\quad + 2e^T(\Gamma z(k))B^{*T} PB^* e(\Gamma z(k))
\end{aligned} \tag{16}$$

where $B^* = \left\{\overline{B} : \max(B_1^T PB_1, B_2^T PB_2)\right\}$.

Define $D = -\left[(1-\alpha)A_1^T PA_1 + \alpha A_2^T PA_2 - \frac{1}{2}P\right]$, if the matrix D is positive definite, (14) can be obtained directly, and (16) can be rewritten as

$$\begin{aligned}
&\Delta V(z(k)) \\
&\le -2z^T(k)Dz(k) + 2e^T(\Gamma z(k))B^{*T} PB^* e(\Gamma z(k)) \qquad (17) \\
&\le -2\left[\lambda_{\min}(D)|z(k)|^2 - \left\|B^{*T} PB^*\right\|\Delta^2\mu^2(k)\right]
\end{aligned}$$

The last expression of (17) is negative if the state of Σ is outside the ball

$$H = \left\{z(k) : z(k) \le \Theta\Delta\mu(k)\right\} \tag{18}$$

where $\Theta = \sqrt{\dfrac{\left\|B^{*T} PB^*\right\|}{\lambda_{\min}(D)}}$.

Define the scaling factor Ω as

$$\Omega = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\sqrt{\Theta^2 + \varepsilon}\,\|\Gamma\|\Delta M^{-1} \tag{19}$$

where $\varepsilon > 0$ is a fixed real number.

We will analyze the control strategy by two stages according to the variation of the scaling factor $\mu(k)$ as [1], [9]:

The "zooming-out" stage of the quantizer.

Set $u(k) = 0$, $\mu(0) = 1$, $\mu(k) = \|A\|^k$ and increase k fast enough, then a positive integer k can be found such that

$$\left|\frac{\Gamma z(k)}{\mu(k)}\right| \le M\sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}} - 2\Delta$$

In the view of condition Ⅰ in the former section

$$\left|q_\mu\left(\frac{\Gamma z(k)}{\mu(k)}\right)\right| \le \left|\frac{\Gamma z(k)}{\mu(k)}\right| + \Delta \le M\sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}} - \Delta$$

Define

$$k_0 = \min\left\{k \ge 1 : \left|q_\mu\left(\frac{\Gamma z(k)}{\mu(k)}\right)\right| \le M\sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}} - \Delta\right\} \tag{20}$$

Hence it follows that

$$\left|\frac{\Gamma z(k_0)}{\mu(k_0)}\right| \le \left|q_\mu\left(\frac{\Gamma z(k_0)}{\mu(k_0)}\right)\right| + \Delta \le M\sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}}$$

Therefore

$$\left|\Gamma z(k_0)\right| \le M\mu(k_0)\sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}}$$

Which means

$$\left|z(k_0)\right| \le \frac{M}{\|\Gamma\|}\mu(k_0)\sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}}$$

Hence $z(k_0)$ belongs to an ellipsoid

$$R_1 = \left\{z(k) : z^T(k)Pz(k) \le \frac{M^2}{\|\Gamma\|^2}\mu^2(k)\lambda_{\min}(P)\right\} \tag{21}$$

It is obvious that $\Gamma z(k) \le M\mu(k)$ holds with $\mu(k) = \mu(k_0)$ for all $z(k) \in R_1$.

Take M and Δ in (19) properly to guarantee $\Omega < 1$, it follows that $R_1 \supset H$. Moreover, if $k \ge k_0$, $z(k)$ will never leave R_1.

The "zooming-in" stage of the quantizer.

Define

$$\tilde{\tau} = \frac{M^2\lambda_{\min}(P) - \Delta^2\Theta^2\|T\|^2\lambda_{\max}(P)}{\|T\|^2\lambda_{\min}(D)\Delta^2\varepsilon} \tag{22}$$

We can have $\tilde{\tau} > 0$ as $\Omega < 1$. Define $\tau = \lceil\tilde{\tau}\rceil$, where $\lceil\tilde{\tau}\rceil$ denotes the smallest integer satisfy $\tau \ge \tilde{\tau}$, $\tau \in Z_{0+}$.

Assume an inequality can be get according to $\tau \in Z_{0+}$

$$E[z^T(k_0+\tau)Pz(k_0+\tau)] \le \Delta^2\mu^2(k_0)(\Theta^2+\varepsilon)\lambda_{\max}(P) \tag{23}$$

Suppose (23) is not true, then we can have that

$$E[z^T(k_0+\tau)Pz(k_0+\tau)] > \Delta^2\mu^2(k_0)(\Theta^2+\varepsilon)\lambda_{\max}(P) \tag{24}$$

That is

$$E[\left|z(k_0+\tau)\right|^2] > \Delta^2\mu^2(k_0)(\Theta^2+\varepsilon) \tag{25}$$

for all $k \in [k_0, k_0+\tau]$.

Based on (18) and $\Omega < 1$, it is clear that

$$\begin{aligned}
&\Delta V[z(k_0+\tau-1)] \\
&= E[z^T(k_0+\tau)Pz(k_0+\tau)] - \\
&\quad E[z^T(k_0+\tau-1)Pz(k_0+\tau-1)] \qquad (26)\\
&\le -\lambda_{\min}(D)E[|z(k_0+\tau-1)|^2] + \lambda_{\min}(D)\Theta^2\Delta^2\mu^2(k_0) \\
&< -\lambda_{\min}(D)\Delta^2\mu^2(k_0)\varepsilon
\end{aligned}$$

Furthermore, it can be obtained that

$$\begin{aligned}
&\Delta V[z(k_0+\tau-i)] \\
&= E[z^T(k_0+\tau-i+1)Pz(k_0+\tau-i+1)] - \\
&\quad E[z^T(k_0+\tau-i)Pz(k_0+\tau-i)] \qquad (27)\\
&\le -\lambda_{\min}(D)E[|z(k_0+\tau-i)|^2] + \lambda_{\min}(D)\Theta^2\Delta^2\mu^2(k_0) \\
&< -\lambda_{\min}(D)\Delta^2\mu^2(k_0)\varepsilon
\end{aligned}$$

where $i \in \{1,2,3,\cdots,\tau\}$.

Then we have

$$\begin{aligned}
&E[z^T(k_0+\tau)Pz(k_0+\tau)] - z^T(k_0)Pz(k_0) \\
&< -\lambda_{\min}(D)\Delta^2\mu^2(k_0)\varepsilon\tau \\
&\le -\lambda_{\min}(D)\Delta^2\mu^2(k_0)\varepsilon\tilde{\tau} \qquad (28)\\
&= \lambda_{\max}(P)\Delta^2\Theta^2\mu^2(k_0) - \frac{M^2}{\|\Gamma\|^2}\lambda_{\min}(P)\mu^2(k_0)
\end{aligned}$$

However, the following inequality can be obtained from (21) and (24)

$$\begin{aligned}
&E[z^T(k_0+\tau)Pz(k_0+\tau)] - z^T(k_0)Pz(k_0) \\
&\ge \Delta^2\mu^2(k_0)(\Theta^2+\varepsilon)\lambda_{\max}(P) - \mu^2(k_0)\frac{M^2}{\|\Gamma\|^2}\lambda_{\min}(P) \qquad (29)\\
&\ge \lambda_{\max}(P)\Delta^2\Theta^2\mu^2(k_0) - \frac{M^2}{\|\Gamma\|^2}\lambda_{\min}(P)\mu^2(k_0)
\end{aligned}$$

Obviously (28) and (29) contradict with each other, which implies the validity of (23).

Based on (23) and $\Omega < 1$, it follows that

$$\begin{aligned}
E[z^T(k_0+\tau)Pz(k_0+\tau)] &\le \Delta^2\mu^2(k_0)(\Theta^2+\varepsilon)\lambda_{\max}(P) \\
&< (\Omega\mu(k_0))^2\frac{M^2}{\|\Gamma\|^2}\lambda_{\min}(P) \qquad (30)
\end{aligned}$$

Thus it is clear that $z(k_0+\tau)$ belongs to

$$R_2 = \left\{ z(k) : E[z^T(k)Pz(k)] \le (\Omega\mu(k))^2 \frac{M^2}{\|\Gamma\|^2}\lambda_{\min}(P) \right\} \quad (31)$$

Let $\mu(k) = \Omega\mu(k_0)$ for $k_0+\tau \le k \le k_0+2\tau$, a similar result can be obtained

$$E[z^T(k_0+2\tau)Pz(k_0+2\tau)] < (\Omega^2\mu(k_0))^2\frac{M^2}{\|\Gamma\|^2}\lambda_{\min}(P) \quad (32)$$

Let $\mu(k) = \Omega^{i-1}\mu(k_0)$ for $k_0+(i-1)\tau \le k \le k_0+i\tau$, it can be given as

$$E[z^T(k_0+i\tau)Pz(k_0+i\tau)] < (\Omega^i\mu(k_0))^2\frac{M^2}{\|\Gamma\|^2}\lambda_{\min}(P) \quad (33)$$

By repeating this procedure, we can obviously obtain that $\mu(k) \to 0$ as $k \to \infty$, and $\lim_{k\to\infty} E[|z(k)|^2] \to 0$, which means the closed-loop system Σ is mean square stable.

IV. A NUMERICAL EXAMPLE

In this section, a numerical example is given to show the effectiveness of the proposed method. Consider the following system

$$P : \begin{cases} x(k+1) = \begin{bmatrix} 1.1 & \\ & 0.5 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 2 \end{bmatrix}\theta(k)v(k) \\ y(k) = \begin{bmatrix} 1 & 0.2 \end{bmatrix} x(k) \end{cases}$$

where $\theta(k)$ is a Bernoulli random variable with probability distribution given by

$$\Pr(\theta(k) = i) = \begin{cases} 0.2, & i = 0 \\ 0.8, & i = 1 \end{cases}$$

which means the packet-loss rate is 0.2.

Obviously, the plant P is unstable as one of its eigenvalues is 1.1. And it is stabilizable as $rank[B\ AB] = 2$. Set the feedback gain $K = [-1.53\ \ 0.13]$. The dynamic quantizer is given as

$$Q : \begin{cases} \xi(k+1) = \begin{bmatrix} 1.1 & \\ & 0.5 \end{bmatrix}\xi(k) - \begin{bmatrix} -1.53 & 0.13 \\ -3.06 & 0.26 \end{bmatrix} x(k) \\ \qquad\qquad + \begin{bmatrix} 1 \\ 2 \end{bmatrix} v(k) \\ v(k) = q_\mu(\begin{bmatrix} -0.7857 & 0.0714 \end{bmatrix}\xi(k) \\ \qquad\qquad + \begin{bmatrix} -1.53 & 0.13 \end{bmatrix} x(k)) \end{cases}$$

whose parameters are determined by the plant P and the feedback gain K .

The closed-loop system Σ can be expressed as

$$\begin{cases} z(k+1) = \begin{bmatrix} -0.43 & 0.13 & -0.7857\theta & -0.0714\theta \\ -3.06 & 0.76 & -1.5714\theta & -0.1429\theta \\ & & 0.3143 & -0.0714 \\ & & -1.5714 & 0.3571 \end{bmatrix} z(k) \\ \qquad + \begin{bmatrix} \theta & 2\theta & 1 & 2 \end{bmatrix}^T e(\Gamma z(k)) \\ y(k+1) = \begin{bmatrix} 1 & 0.2 & 0 & 0 \end{bmatrix} z(k) \end{cases}$$

We can find there is a positive definite matrix

$$P = \begin{bmatrix} 7.7602 & -1.2269 & 2.1796 & -0.4196 \\ -1.2269 & 0.3699 & -0.5016 & 0.1113 \\ 2.1796 & -0.5016 & 9.8224 & 0.1117 \\ -0.4196 & 0.1113 & 0.1117 & 0.5190 \end{bmatrix}$$

satisfying (14), then the system is mean square stable under control law (7).

The static quantizer is given by

$$q_\mu(x) = \begin{cases} 4\mu(k), & if\ x > 3.95\mu(k) \\ -4\mu(k), & if\ x \le -3.95\mu(k) \\ \left\lfloor \dfrac{x}{\mu(k)} + 0.05 \right\rfloor \mu(k), & if\ |x| \le 3.95\mu(k) \end{cases}$$

with $M = 4$ and $\Delta = 0.1$.

Set $\varepsilon = 0.1$, we can get $\Omega = 0.7497 < 1$. Let the initial state of the system be $z(0) = [5\ \ 7\ \ 8\ \ 6]^T$. Then the trajectories of state $z(k)$ are shown in Fig.3, where $z(i)$ denotes the i-th component of $z(k)$. It is clear that system (11) is mean square stable as $z(k) \to 0$ when $k \to \infty$.

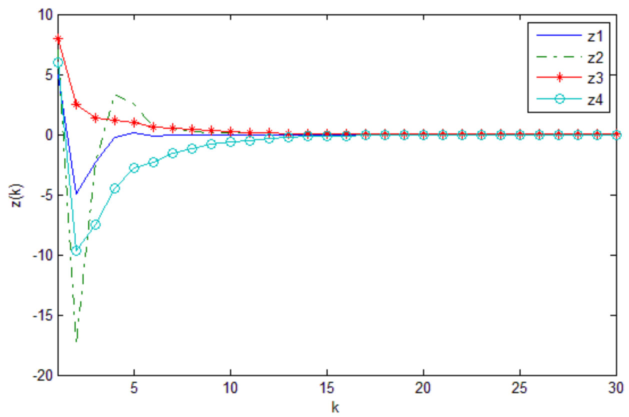

Figure 3. Trajectories of state $z(k)$

V. Conclusion

This paper has discussed the stability of the optimal dynamic quantizer system with packet loss subject to Bernoulli distribution. In order to make the quantizer more practical and have better performance, traditional optimal dynamic quantizer has been improved here by replacing its static part with another one which contains saturation value and scaling factor. The communication channel has been considered subject to packet loss from the quantizer to the plant input. Based on Lyapunov function approach, a sufficient condition for mean square stability of the closed-loop system has been given.

References

[1] R. W. Brockett and D. Liberzon, “Quantized feedback stabilization of linear systems”, IEEE Transactions on Automatic Control, Vol. 45, No. 7, pp. 1279–1289, 2000.

[2] N. Elia and S. K. Mitter, “Stabilization of linear systems with limited information”, IEEE Transactions on Automatic Control, Vol. 46, No. 9, pp. 1384–1400, 2001.

[3] M. Fu and L. Xie, “The sector bound approach to quantized feedback control”, IEEE Transactions on Automatic Control, Vol. 50, No. 11, pp. 1698–1711, 2005.

[4] F. Fagnani and S. Zampieri, “Performance evaluations of quantized stabilizers”, Proceedings of the 42nd IEEE Conference on Decision and Control, pp. 1897–1901, 2003.

[5] F. Fagnani and S. Zampieri, “Stability Analysis and Synthesis for Scalar Linear Systems With a Quantized Feedback”, IEEE Transactions on Automatic Control, Vol. 48, No. 9, pp. 1569–1584, 2003.

[6] F. Fagnani and S. Zampieri, “Quantized Stabilization of Linear Systems : Complexity Versus Performance”, IEEE Transactions on Automatic Control, Vol. 49, No. 9, pp. 1534–1548, 2004.

[7] H. Hatmovich, M.M.Seron and G.C.Goodwin, “Geometric characterization of multivariable quadratically stabilizing quantizers” , International Journal of Control, Vol. 79, No. 8, pp. 845–857, 2006.

[8] W. S. Wong and R. W. Brockett, “Systems with finite communication bandwidth constraints, ii: Stabilization with limited information feedback”.IEEE Transactions on Automatic Control, Vol. 44, 1049–1053, 1999.

[9] D. Liberzon, “Hybrid feedbackstabilization of systems with quantized signals”, Automatica, Vol. 39, pp. 1543–1554, 2003.

[10] G.N. Nair and R.J. Evans, “Stabilizability of stochastic linear systems with finite feedback data rates”, SIAM J. Control Optim., Vol. 43, No.2, pp. 413–436, 2004.

[11] S. Tatikonda and S. Mitter, “Control under communication constraints,” IEEE Trans. Automat. Control, Vol. 49, No.7, pp. 1056–1068, 2004.

[12] G.N.Nair, F. Fagnani, Sandro Zampieri, and R.J. Evans, “Feedback Control Under Data Rate Constraints: An Overview” , Proceedings of the IEEE, Vol. 95, No. 1, pp. 108–137, 2007.

[13] G.N. Nair and R.J. Evans, “Exponential stabilisability of finite dimensional linear systems with limited data rates” , Automatica, Vol. 44, No. 9, pp. 2364–2369, 2008.

[14] S. Azuma and T. Sugie, “Optimal dynamic quantizers for discrete valued input control” , Automatica, Vol. 44, No. 2, pp. 396–406, 2008.

[15] S. Azuma and T. Sugie, “An Analytical Solution to Dynamic Quantization Problem of Nonlinear Control Systems” , Proceedings of the 48th IEEE Conference on Decision and Control, pp. 3914–3919, 2009.

[16] S. Azuma and T. Sugie, “Stability analysis of optimally quantised LFT-feedback systems”, International Journal of Control, Vol. 83, No. 6, 1125–1135, 2010.

[17] H.Ishii, T.Koji , “The Coarsest Logarithmic Quantizers for Stabilization of Linear Systems with Packet Losses” , Proceedings of the 46th IEEE Conference on Decision and Control, pp. 2235–2240, 2007.

[18] H.Ishii, T.Koji , “Tradeoffs between quantization and packet loss in networked control of linear systems” , Automatica, Vol. 45, No. 12, pp. 2963–2970, 2009.

[19] E.Fridman, M.Dambrine, “On input-to-state stability of systems with time-delay: A matrix inequalities approach” , Automatica, Vol. 44, No. 9, pp. 2364–2369, 2008.