UKACC International Conference on Control 2012
Cardiff, UK, 3-5 September 2012

Low Eigenvalue Sensitivity Eigenstructure Assignment to Linear Parameter Varying Systems

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Abstract— **This paper is concerned with the assignment of a desired eigenstructure to linear parameter-varying (LPV) systems as an extended version of the corresponding eigenstructure assignment problem for linear time-invariant systems. Based on a complete parametric solution of parametric generalized Sylvester matrix equation, a controller design method is proposed to guarantee a low sensitivity of the closed-loop eigenvalues. The observer state feedback structure is considered for output feedback control design. An example of control for a satellite attitude system is used to demonstrate the usefulness of the proposed approach.**

Keywords- Eigenstructure Assignment, Linear Parameter Varying systems, Low Eigenvalue Sensitivity, Sylvester Matrix Equation, Observer State Estimate Feedback

I. INTRODUCTION

Gain scheduling control has been widely used in practical applications [1, 2] to handle the nonlinearity of real systems. The classic gain scheduling approach consists in designing linear controllers for several operating points and then applying an interpolation strategy to obtain a global controller. Consequently, powerful tools for linear systems can be applied to nonlinear plants. In spite of the numerous applications, there has been no formal framework for gain scheduling until the early 1990s [3]. This framework gives heuristic rules to ensure global stability, but it does not provide a systematic design procedure. The linear parameter varying (LPV) approach to system modeling, estimation and control followed on from gain-scheduling as a strategy for attempting to model non-linear parametric variations using a time-varying linear systems approach [4, 5]

Eigenstructure assignment has been used in many applications and has been proven to be a useful tool both for analysis and design of linear time invariant (LTI) systems [6-8]. This method allows the designer to satisfy directly damping, settling time, and mode decoupling specifications by choosing the eigenvalues and eigenvectors. That is because the transient response of an LTI system is completely specified by the system eigenstructure. Generally, the eigenvalues determine the decay (or growth) rate of the response and the left and right eigenvectors fix the shape of the response [8]. Also, minimum eigenvalue sensitivity to model parameter variation and other performance requirements such as minimum gain control can be accommodated using explicit choices of the free controller parameters [8]. For years, many researchers have attempted to generalize the conventional notions of eigenvalues and eigenvectors for linear time-invariant systems to linear time-varying (LTV) systems [9-12]. Although existing eigenstructure assignment techniques and algorithms for LTV systems confirm the value of using this approach, eigenstructure assignment of LTV systems remains a difficult problem that is still in a state of development.

However, there are very few examples in the literature where eigenstructure assignment is applied within an LPV framework [13, 14]. In [13, 15], polynomial eigenstructure assignment of LTI systems was extended to solve the corresponding eigenstructure assignment problem for LPV systems using output feedback. In [14], a parametric approach for eigenstructure assignment, appropriate for LTI systems, was extended to LPV plants using state feedback. However, with the conditions given in [13], the choice of controller structure, the matching conditions and the solution of the controller are not unique and require much more additional criteria to constrain the order of controller gains. As discussed in [16], the method used in [14] to calculate the controller parameters is complex.

In this paper, a low eigenvalue sensitivity eigenstructure assignment method for LPV systems is proposed. A general complete parametric solution of the corresponding parametric generalized Sylvester matrix equation [14] is introduced. A parametric eigenstructure assignment is presented based on the proposed solution approach. Using the eigenstructure assignment design freedom, low eigenvalue sensitivity is achieved by projecting the desired eigenstructure into an allowable subspace. An observer-based state estimate feedback controller structure is chosen within an output feedback framework. An algorithm is proposed to calculate a state feedback controller with state observer. The remainder of this paper is organized as follows. Section II briefly introduces a parametric solution of the parametric generalized Sylvester matrix equation, and the eigenstructure assignment to LPV system is presented. In Section III, the definition of the overall eigenvalue sensitivity of matrix to LTI systems is reviewed and extended to LPV systems. The main results and an algorithm are also presented in Section III. Section IV gives an example of a satellite system to illustrate the application of the theory. Finally, conclusions are drawn up in Section V.

Fengming Shi acknowledges joint scholarship funding support from the CSC (China) and the Hull China Scholarship.

II. Parametric Eigenstructure Assignment for LPV systems

A. LPV systems

Reference [5] considers that LPV Systems are linear time-varying plants whose state-space matrices are fixed functions of some vector of varying parameters $\theta(t)$. LPV systems can be described by state-space equations of the form:

$$\left.\begin{aligned}\dot{\boldsymbol{x}}(\boldsymbol{t}) &= \boldsymbol{A}(\boldsymbol{\theta}(\boldsymbol{t}))\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B}(\boldsymbol{\theta}(\boldsymbol{t}))\boldsymbol{u}(\boldsymbol{t}) \\ \boldsymbol{y}(\boldsymbol{t}) &= \boldsymbol{C}(\boldsymbol{\theta}(\boldsymbol{t}))\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{D}(\boldsymbol{\theta}(\boldsymbol{t}))\boldsymbol{u}(\boldsymbol{t})\end{aligned}\right\} \tag{1}$$

where $x(t) \in R^n, u(t) \in R^r and\ y(t) \in R^m$ are the state vectors, the input vectors and measured output vector, respectively. $A(.), B(.), C(.), D(.)$, with corresponding dimensions, are known continuous function of a time-varying parameters vector $\theta(t)$ which satisfies:

$$\theta(t) = \left[\theta_1(t), \cdots \theta_{n_\theta}(t)\right]^T \in \Theta, \forall\, t \geq 0$$

where Θ is a compact set. The subscript t is omitted through the remainder of the paper without causing confusion.

From a practical point of view, LPV systems have at least two interesting interpretations. They can be viewed as LTI plants subject to time-varying parametric uncertainty $\theta(t)$. On the other hand, they can be models of linear time-varying plants or result from the linearization of nonlinear plants along the trajectories of the parameter θ. From the second view, the parameter $\theta(t)$ can be measured in real time during system operation. Consequently, the control strategy can exploit the available measurements of θ to increase performance. The LPV controller design approach proposed in this paper is based on the second view point.

B. Parametric Eigenstructure Assignment using state feedback

Consider an LPV system given in form of (1). A linear parameterised state feedback law:

$$u = K(\theta)x, K(\theta) \in R^{r\times n}$$

is applied, such that the closed-loop system is in the following form:

$$\dot{x} = (A(\theta) + B(\theta)K(\theta))x$$

Following [9, 14], the closed-loop self-conjugate eigenvalue set can be described as $\Lambda = \{\lambda_i(\theta): \lambda_i(\theta) \in C, i = 1,2,..\tilde{n}, 1 \leq \tilde{n} \leq n\}$, for which the algebraic and geometric multiplicities of the eigenvalue λ_i are denoted by q_i and r_i, respectively. Then in the Jordan form of the matrix $A_{cl}(\theta) = A(\theta) + B(\theta)K(\theta)$, there are r_i Jordan blocks, associated with the i-th eigenvalue λ_i , of orders p_{ij}, j = 1,2, ..., r_i. , p_{ij}, q_i *and* r_i satisfy the relations:

$$\sum_{j=1}^{r_i} p_{ij} = q_i, \sum_{i=1}^{\tilde{n}} q_i = n$$

Denoting the left and right eigenvectors and generalized eigenvectors of matrix $A_{cl}(\theta)$ associated with λ_i by $L_i(\theta)$ and $R_i(\theta)$, respectively, it follows that:

$$(\lambda_i(\theta)I - A_{cl}(\theta))R_{ij,k}(\theta) = -R_{ij,k-1}(\theta),$$

$$R_{ij,0}(\theta) = 0$$

$$(\lambda_i(\theta)I - A_{cl}(\theta))^T L_{ij,k}(\theta) = -L_{ij,k-1}(\theta),$$

$$L_{ij,0}(\theta) = 0$$

for $k = 1,2, \dots p_{ij}, j = 1,2, \dots r_i, and\ i = 1,2, \dots, \tilde{n}$
and

$$L^T(\theta)R(\theta) = I$$

Hence, the problem to assign a desired closed-loop eigenstructure to a system using a state feedback controller is to find a solution of the parametric equation:

$$A(\theta)R(\theta) + A(\theta)W(\theta) = R(\theta)F(\theta)$$

where $A(\theta), B(\theta)$ are the state space matrices, $R(\theta)$ is the desired right eigenvector matrix, $F(\theta)$ is the desired eigenvalues *diagnosis matrix* and $W(\theta)$ is an auxiliary matrix.

Theorem 1 is introduced to show how the eigenvectors and generalized eigenvectors can be parameterised.

Theorem 1 [14]

Let $[A(\theta)\ B(\theta)]$ be controllable, and matrix $B(\theta)$ be of full-column rank, then all the solutions of matrix equation:

$$A(\theta)R(\theta) + B(\theta)W(\theta) = R(\theta)F(\theta)$$

are given by:

$$\begin{bmatrix} R_{ij}^k \\ w_{ij}^k \end{bmatrix} =$$

$$\begin{bmatrix} N(\theta, \lambda_i(\theta)) & \cdots & \frac{1}{(k-1)!}\frac{d^{k-1}}{d\lambda^{k-1}}N(\theta, \lambda_i(\theta)) \\ D(\theta, \lambda_i(\theta)) & \cdots & \frac{1}{(k-1)!}\frac{d^{k-1}}{d\lambda^{k-1}}D(\theta, \lambda_i(\theta)) \end{bmatrix} \begin{bmatrix} f_{ij}^k(\theta) \\ \vdots \\ f_{ij}^1(\theta) \end{bmatrix} \tag{2}$$

$$k = 1, 2, \dots, p_{ij}, j = 1,2, \dots q_i, i = 1,2, \dots n'$$

where $f_{ij}^k \in C^r$ are arbitrarily chosen parameter vectors; N(θ, λ) and D(θ, λ) are right co-prime matrix polynomials satisfying:

$$[\lambda(\theta)I - A(\theta)]^{-1}B(\theta) = N(\theta, \lambda(\theta))D^{-1}(\theta, \lambda(\theta)) \tag{3}$$

From Theorem 1, it can be seen that the desired eigenvectors and generalized eigenvectors can be parameterised by (2) [14].

Remark 1:

- Theorem 1 concerns an extension of the eigenstructure assignment of LTI systems to an LPV modeling framework by introducing a solution of the parametric Sylvester matrix equation.
- Theorem 1 gives a clear analytical, complete, and explicit parametric solution expressed by the eigenvalues of the matrix $A_{cl}(\theta)$ and a group of free parameters, namely f_{ij} . By specially choosing the free parameters given in (2), solutions with desired properties can be obtained.

III. MAIN RESULT

It is known in [8, 17] that if a system has low sensitivity to perturbations and parameter variations in the system matrices then there may be a low chance of the closed-loop system becoming unstable compared with the case when controllers are used that are not based on sensitivity minimization. Hence, the eigenvalue sensitivity of a closed-loop system to modeling errors should be given suitable consideration. For the sake of simplicity, only the overall eigenvalue (i.e. Wilkinson) sensitivity is considered here [8, 17].

A. Overall Eigenvalue Sensitivity

The overall eigenvalue sensitivity of the matrix X is defined [8, 17] as:

$$\eta(R) = \|R\|_2 \|R^{-1}\|_2$$

where R is the right eigenvector matrix of the matrix X.

Similarly, in this study the overall eigenvalue sensitivity of the parameter varying matrix $X(\theta)$ is defined as:

$$\eta(R(\theta)) = \sup_{\theta \in \Theta} \|R(\theta)\|_2 \|R(\theta)^{-1}\|_2$$

where $R(\theta)$ is the right eigenvector matrix of the matrix $X(\theta)$.

Suppose that the right eigenvector matrix R is unitary, i.e., $R^T R = I$ then $\eta(R) = 1$. This indicates that if R is a unitary matrix the corresponding eigenvalues are perfectly conditioned and hence minimally sensitive to perturbations or parameter variations.

These observations provide the basis for the algorithms to be described in this paper. The objective of the paper is to show how to assign a set of closed-loop eigenvectors which match the columns of a unitary matrix as closely as possible. If this process is successful, a perfectly conditioned set of closed-loop eigenvalues results [8, 17].

B. Performance function

The desired eigenvectors must be projected into the allowable subspace which is optimal according to some performance function while the desired eigenvectors are not in the allowable subspace. As argued previously, to achieve overall low eigenvalue sensitivity, an LPV system performance function can be defined as:

$$J_p = \left(R_{d,i}(\theta) - R_i(\theta)\right)^T \widetilde{W}_i \left(R_{d,i}(\theta) - R_i(\theta)\right) \qquad (4)$$

where θ is the varying parameter and other symbols have the same meaning as in LTI system case.

If the right eigenvectors are parameterized as:

$$R_i(\theta) = P_{R,i}(\theta) W_i(\theta)$$

The solution that minimises the LPV performance function J_p is obtained by setting:

$$W_{o,i}(\theta) = \left(P_{R,i}^T(\theta)\widetilde{W}_i P_{R,i}(\theta)\right)^{-1} P_{R,i}^T(\theta)\widetilde{W}_i R_{d,i}(\theta) \qquad (5)$$

The least-squares best-fit LPV right eigenvectors can be computed by:

$$R_{o,i}(\theta) = P_{R,i}(\theta) W_{o,i}(\theta)$$

Now set $R_{d,}(\theta) = I$, the low eigenvalue sensitivity performance function of LPV system is:

$$J_{pl} = \left(R_{d,i}(\theta) - I_i(\theta)\right)^T \widetilde{W}_i (R_{d,i}(\theta) - I_i(\theta))$$

C. Results and Algorithms

Consider an LPV system described by (1), and the performance function is described as (4). The solution that minimises the performance function J_{pl} is obtained by setting:

$$\begin{bmatrix} f_{oij}^k(\theta) \\ \vdots \\ f_{oij}^1(\theta) \end{bmatrix} = \left(P_{R,ijk}^T(\theta)\widetilde{W}_i P_{R,ijk}(\theta)\right)^{-1} P_{R,ijk}^T(\theta)\widetilde{W}_i I_{,i}(\theta) \quad (6)$$

where:

$$P_{R,ijk}(\theta) = \left[N(\theta, \lambda_i(\theta)) \quad \cdots \quad \frac{1}{(k-1)!}\frac{d^{k-1}}{d\lambda^{k-1}} N(\theta, \lambda_i(\theta))\right], \quad (7)$$

$$\mathrm{k} = 1, 2, \ldots, \mathrm{p}_{ij}, \mathrm{j} = 1, 2, \ldots \mathrm{q}_i, \mathrm{i} = 1, 2, \ldots \mathrm{n}'$$

The least-squares best-fit LPV right eigenvector can be computed by

$$R_{oij}^k(\theta) = P_{R,ijk} \begin{bmatrix} f_{oij}^k(\theta) \\ \vdots \\ f_{oij}^1(\theta) \end{bmatrix} = \left[N(\theta, \lambda_i(\theta)) \quad \cdots \quad \frac{1}{(k-1)!}\frac{d^{k-1}}{d\lambda^{k-1}} N(\theta, \lambda_i(\theta))\right] \begin{bmatrix} f_{oij}^k(\theta) \\ \vdots \\ f_{oij}^1(\theta) \end{bmatrix},$$

The corresponding auxiliary matrix can be computed as:

$$w_{oij}^k(\theta) = \left[D(\theta, \lambda_i(\theta)) \quad \cdots \quad \frac{1}{(k-1)!}\frac{d^{k-1}}{d\lambda^{k-1}} D(\theta, \lambda_i(\theta))\right] \begin{bmatrix} f_{oij}^k(\theta) \\ \vdots \\ f_{oij}^1(\theta) \end{bmatrix}$$

The following introduces a non-iterative method which involves the direct projection of a unitary matrix into the allowable eigenvector subspace.

Algorithm 1

Step 1: Chose a set of desired closed-loop eigenvalues $\Lambda = \{\lambda_i(\theta): \lambda_i(\theta) \in C, i = 1,2,..ñ,\ 1 \le ñ \le n\}$

Step 2: Get $N\ (\theta, \lambda\ (\theta))$ and $D\ (\theta, \lambda\ (\theta))$ satisfying (3) by applying right co-prime factorization

Step 3: Project each column of the unitary matrix $U = [U_1 U_2 \cdots U_3]$ into each of the allowable eigenvector subspaces corresponding to each closed-loop eigenvalue using (6) and (7). For each column U_i of U, this produces a total of n achievable right eigenvectors $R_{ij}(\theta), j = 1, .. n$;

Step 4: Calculate the n^2 misalignment angles given by:

$$\alpha_{ij}(\theta) = \sup_{\theta \in \Theta} \cos^{-1}\left(\frac{|U_j^T R_{ij}(\theta)|}{\|U_j\|_2 \|R_{ij}(\theta)\|_2}\right)$$

Step 5: Choose the assignment from the $n!$ possibilities which have the smallest sum of misalignment angles α_{ij}.

Step 6: Calculate the controller $K(\theta) = W(\theta)R(\theta)^{-1}$

Remark 2

- The algorithm is an LPV-extended version of the existing right eigenstructure assignment scheme via the Sylvester matrix equation for LTI systems.
- The above algorithm will assign the closed-loop eigenvectors as close to a unitary matrix as possible to achieve optimum sensitivity.
- In the algorithm, if the rank of the control input matrix $B(\theta)$ is equal to the rank of the system matrix $A(\theta)$the desired right eigenvectors (a unitary matrix) as well as the desired eigenvalues can be achieved exactly.
- For a special case, if the desired eigenvalue is constant $\Lambda = \{\lambda_i: \lambda_i \in C, i = 1,2,..ñ,\ 1 \le ñ \le n\}$, and if the achieved eigenvectors are parameter-independent, then the closed-loop system could be time invariant.
- As suggested in [8] $n!$ could be very large when n is large. Hence, $n < 7$ is suggested to use in the proposed approach.
- For the Step 4, grid method [18, 19] is proposed to tackle the high dimensionality and nonlinear nature of the optimization problem.

IV. AN EXAMPLE

Now, an example is given of a satellite attitude control problem to show how to calculate the controller for a given LPV system to achieve low eigenvalue sensitivity. The example is given in [14]. Consider an LPV system:

$$\left.\begin{array}{l}\dot{x} = A(\theta)x + B(\theta)u \\ y = Cx + Du\end{array}\right\} \quad (8)$$

where

$$A(\theta) = \begin{bmatrix} 0 & a12 & 0 & 0 & a15 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a34 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ a51 & 0 & 0 & 0 & 0 & a56 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B(\theta) = \begin{bmatrix} b11 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b32 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b53 \\ 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = 0$$

The coefficients of $A(\theta)\ and\ B(\theta)$ are defined as:

$$a12 = 4(\mathrm{Iz} - \mathrm{Iy})\frac{w_0^2}{\mathrm{Ix}}, a15 = 4(\mathrm{Ix} + \mathrm{Iz} - \mathrm{Iy})\frac{w_0}{\mathrm{Ix}},$$

$$a51 = (\mathrm{Iy} - \mathrm{Iz} - \mathrm{Ix})\frac{w_0}{\mathrm{Iz}}, a56 = (\mathrm{Ix} - \mathrm{Iy})\frac{w_0^2}{\mathrm{Iz}},$$

$$a34 = 3(\mathrm{Iz} - \mathrm{Ix})\frac{w_0^2}{\mathrm{Iy}}, b11 = \frac{1}{\mathrm{Ix}}, b32 = \frac{1}{\mathrm{Iy}}, b53 = \frac{1}{\mathrm{Iz}},$$

$$w_0 = 1.1 * 10^{-3}, \mathrm{Ix} = 661.4723, \mathrm{Iy} \in [2620 \quad 3700],$$

$$\mathrm{Iz} \in [2780 \quad 3850]$$

θ is defined as $\theta = \{\theta_1, \theta_2\} = \{\mathrm{Iy}, \mathrm{Iz}\}$, where Iy and Iz are the moment of inertia parameters.

A. Controller design

The desired eigenvalues are set to be the same as in [18]:

$$\Lambda = \{-1.00 \pm 2.063i \quad -1.023 \pm 2.354i, \quad -1.592 \pm 1.676i\}$$

Using elementary transformations and the rational matrix factorization method [20, 21], it follows that:

$$N(\theta, s) = \begin{bmatrix} s & 0 & 0 \\ 1 & 0 & 0 \\ 0 & s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & s \\ 0 & 0 & 1 \end{bmatrix},$$

$$D(\theta, s) = \begin{bmatrix} \frac{s^2 - a12}{b11} & 0 & \frac{s * a15}{b11} \\ 0 & \frac{s^2 - a34}{b32} & 0 \\ \frac{s * a51}{b53} & 0 & \frac{s^2 - a56}{b53} \end{bmatrix}$$

The desired eigenvector matrix is set to be the identity matrix I_6 to force the closed-loop system to have low eigenvalue sensitivity to parameter uncertainty.

Using the above algorithm, the desired eigenvector Identity Matrix (I_6) is projected to the allowable subspace. The calculated controller is:

$$K = \begin{bmatrix} \frac{3.185}{b11} & k_{12} & 0 & 0 & \frac{a15}{b11} & 0 \\ 0 & 0 & \frac{2.0466}{b32} & k_{24} & 0 & 0 \\ \frac{a51}{b53} & 0 & 0 & 0 & \frac{2}{b53} & k_{36} \end{bmatrix} \quad (9)$$

$$k_{12} = \frac{5.3454-a12}{b11}, k_{24} = \frac{6.588+a34}{b32}, k_{36} = \frac{5.2539+a56}{b53}$$

When the designed state feedback controller (9) is applied to the original model (8); the closed-loop matrix is obtained as in (10).

$$A - BK = \begin{bmatrix} -3.185 & a_{cl} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2.0466 & -6.588 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & -5.2539 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (10)$$

$$a_{cl} = 2 * a12 - 5.3454$$

B. Observer design

Based on the Separation Principle, an observer state estimate feedback is used to achieve a form of output feedback control. The observer design can be achieved by recognizing the duality between the state feedback control and state estimation problems. A full order observer for the LPV system is considered with the following structure:

$$\left.\begin{aligned} \dot{\hat{x}} &= A(\theta)\hat{x} + B(\theta)u + L(\theta)(y - \hat{y}) \\ \hat{y} &= C\hat{x} + Du \end{aligned}\right\} \quad (11)$$

where $\hat{x} \in R^n$ is the estimated state and $L(\theta)$ is the designed observer gain.

To make the estimated states converge to real system states fast enough, the real part of the observer eigenvalues should be large enough. So, the desired eigenvalues for the observer system are chosen as:

$$\Lambda = \{-3 \pm 3i, \quad -6 \pm 3i, \quad -9 \pm 6i\}$$

Using the proposed procedure, the obtained observer gain is:

$$L = \begin{bmatrix} 72.04 & 18.03 & 0 & 0 & \frac{a15}{b11} & 0 \\ 0 & 0 & \frac{2.046}{b32} & \frac{6.588 + a34}{b32} & 0 & 0 \\ \frac{a51}{b53} & 0 & 0 & 0 & \frac{2}{b53} & \frac{5.254 + a56}{b53} \end{bmatrix}$$

C. Simulation Result

In the simulation model, the initial conditions are set to be $x(0) = [0, 0.0175, 0, 0.0175, 0, 0.0175]^T$. The simulation results are shown in Fig. 1. From the time responses it can be seen that their steady-state errors converge to zero asymptotically. And from the result of the closed-loop system matrix, it can be seen that the modes are decoupled from each other. This is because the system eigenstructure is also considered which would make the system more insensitive to parameter perturbations.

V. Conclusion

In this paper, a low eigenvalue sensitivity eigenstructure assignment approach is presented for LPV systems via observer/state feedback, based on the complete parametric solution of a parametric generalized Sylvester matrix equation. Furthermore, for the sake of practical applications, an example is used to demonstrate the usefulness and advantage of the proposed LPV control scheme. The results show that the closed-loop system transient response performance requirements are satisfied and low eigenvalue sensitivity to perturbation and parameter variation is achieved. The dynamic output feedback controller should be considered in the case where the Separation Principle breaks down.

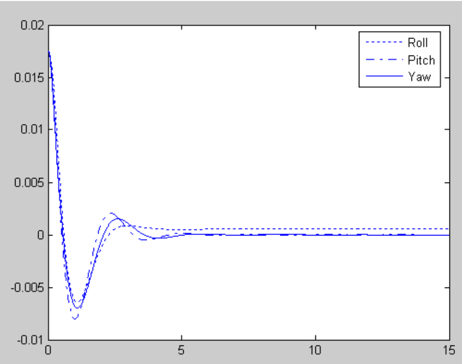

Figure 1. Observer/state feedback closed-loop time response

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