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Output regulation for switched linear systems with different coordinate transformations

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***Abstract*—This paper addresses the output regulation problem for switched linear systems. When each regulation equation has their own solution, we give a sufficient condition for the output regulation problem to be solvable. Firstly, we give the regulation equations of switched linear systems and the relation of the transformed states between two consecutive switching times. Secondly, the existence of a minimal average dwell-time for every switching sequence is assumed, and by virtue of an appropriate Lyapunov analysis, the output regulation is achieved. Our result is of much less conservativeness.**

I. INTRODUCTION

A switched system is a special kind of hybrid system that consists of a family of continuous time or discrete-time subsystems and a rule orchestrating the switching among the subsystems [1]. This type of systems has wide applications in many fields, for instance, stochastic control [2], fault tolerant cooperative control [3], power systems [4], autonomous aircraft control [5]. A lot of methods have been presented to study stability and stabilization problems for switched systems, such as the convex combination technique, the common Lyapunov function technique, the multiple Lyapunov function method, the switched Lyapunov function method and the average dwell-time approach [6]-[9].

Output regulation is an important and interesting problem in control theory. This problem aims to achieve asymptotic tracking and disturbance rejection for a class of reference inputs and disturbances, which generated by an exosystem, besides closed-loop stability. Thus, the problem of output regulation is more challenging than stabilization and has attracted much attention. The problem for non-switched linear systems was widely studied in [10]-[12]. For non-switched nonlinear systems, there are also many results for the problem [13]-[15].

It is well known that for a non-switched linear system, in order to obtain the solvability condition for the output regulation problem, we need to solve a group of regulator equations. However, for a switched linear system, the solvability of the output regulation problem is more difficult and complicated. In order to get sufficient conditions for the problem, we need to solve a family of groups of regulator equations. If this family of systems has a common solution, then one can give the solvability conditions for the problem of switched linear systems. For instance, the problem for a class of switched systems with disturbances is solved under the assumption that the output regulation problem of a convex combination system is solvable [16]. A necessary and sufficient condition for the output regulation problem of switched systems under an arbitrary switching signal to be solvable is obtained in [17]. In addition, the multiple Lyapunov function method is used to solve the problem of output regulation for a class of discrete-time switched systems in [18]. The optimal output regulation is also guaranteed for the discrete switched linear system in [19]-[20]. However, to the authors' best knowledge, the output regulation problem has not been investigated for different coordinate transformations, which motivates the present study.

In this paper, for reducing the conservativeness, we consider the regulation equations of each subsystem has its own solution. This means that each subsystem of the switched linear system has the different transformation and complicates the solvability of the problem. Based on this case, we fist give the relation of the transformed states between two consecutive switching times. Then, the existence of a minimal average dwell-time for every switching sequence is assumed, and by virtue of an appropriate Lyapunov analysis, the output regulation is achieved.

This paper is organized as follows. Section 2 presents the problem statement and preliminaries. In section 3, the sufficient conditions for the output regulation problem to be solvable are given based on full information feedback controllers and error feedback controllers. Conclusion is stated in Section 4.

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PROBLEM STATEMENTS AND PRELIMINARIES

Consider a switched linear system modelled by equations of the form

$$\begin{aligned} \dot{x} &= A_{\sigma(t)}x + B_{\sigma(t)}u + P_{\sigma(t)}\omega, \\ e &= C_{\sigma(t)}x + Q_{\sigma(t)}\omega, \end{aligned} \quad (1)$$

with the state $x \in R^n$, the control input $u \in R^m$, the switching signal $\sigma : [0, \infty) \to I_N = \{1, \cdots, N\}$ is a piecewise constant function of time, the error variable $e \in R^p$, the exogenous input variable $\omega \in R^r$ satisfying the following exosystem

$$\dot{\omega} = S\omega. \quad (2)$$

We are in a position to state the output regulation problem of the switched system (1).

Output regulation via full information. Given $\{A_i, B_i, P_i, C_i, Q_i, S\}$, $i \in I_N$, find full information controllers

$$u = K_i x + L_i \omega \quad (3)$$

and switching law $\sigma(t)$ such that:

1. the system (1) with the controllers (3) is asymptotically stable under the designed switching law $\sigma(t)$ without disturbance input.

2. for each (x^0, ω^0), the solution $(x(t), \omega(t))$ of

$$\begin{aligned} \dot{x} &= (A_{\sigma(t)} + B_{\sigma(t)}K_{\sigma(t)})x + (P_{\sigma(t)} + B_{\sigma(t)}L_{\sigma(t)})\omega, \\ \dot{\omega} &= S\omega \end{aligned} \quad (4)$$

satisfying $(x(0), \omega(0)) = (x^0, \omega^0)$ is such that

$$\lim_{t\to\infty}(C_{\sigma(t)}x + Q_{\sigma(t)}\omega) = 0.$$

Output regulation via error feedback. Given $\{A_i, B_i, P_i, C_i, Q_i, S\}$, $i \in I_N$, find error feedback controllers

$$\begin{aligned} \dot{\xi} &= F_i\xi + G_i e, \\ u &= H_i\xi \end{aligned} \quad (5)$$

and switching law $\sigma(t)$ such that:

1. the system (1) with the controllers (5) is asymptotically stable under the designed switching law $\sigma(t)$ without disturbance input.

2. for each (x^0, ξ^0, ω^0), the solution $(x(t), \xi(t), \omega(t))$ of

$$\begin{aligned} \dot{x} &= A_{\sigma(t)}x + B_{\sigma(t)}H_{\sigma(t)}\xi + P_{\sigma(t)}\omega, \\ \dot{\xi} &= G_{\sigma(t)}C_{\sigma(t)}x + F_{\sigma(t)}\xi + G_{\sigma(t)}Q_{\sigma(t)}\omega, \\ \dot{\omega} &= S\omega \end{aligned} \quad (6)$$

satisfying $(x(0), \xi(0), \omega(0)) = (x^0, \xi^0, \omega^0)$ is such that

$$\lim_{t\to\infty}(C_{\sigma(t)}x + Q_{\sigma(t)}\omega) = 0.$$

Remark 1. The system and the output regulation problem for non-switched system are given in [21]. In [16], the system model (1) and the description of output regulation problem are presented for switched linear systems.

In what follows, we assume the exosystem (2) satisfies the following assumption.

Assumption 1(A1) [22]. The system (2) is antistable, i.e. all the eigenvalues of S have nonnegative real part.

Before obtaining the main results, we give a definition of the average dwell time.

Definition 1 [23] [24]. For switching signal $\sigma(t)$ and any $T \geq t \geq 0$, let $N_\sigma(T, t)$ be the switching numbers of $\sigma(t)$ over the interval (t, T). If for any given $N_0 \geq 0$ and $\tau_a > 0$, we have $N_\sigma(t, T) \leq N_0 + (T - t)/\tau_a$, then τ_a and N_0 are called average dwell time and the chatter bound, respectively.

MAIN RESULTS

In this section, we will give the solvability conditions for the output regulation problem for the switched linear system (1) with the full information controllers and the error feedback controllers, where the regulation equations for the switched linear system (1) are given.

Now, we give a sufficient condition for the output regulation problem to be solvable based on the full information feedback.

Theorem 1. If there exist Π_i , Γ_i for $\forall i \in I_N$, satisfying the following equations

$$\begin{aligned} \Pi_i S &= A_i\Pi_i + B_i\Gamma_i + P_i, \\ 0 &= C_i\Pi_i + Q_i, \end{aligned} \quad (7)$$

and the system (1) with different coordinate transformation

$$\begin{aligned} \tilde{x}(t_k^-) &= x(t_k^-) - \Pi_{i_k}\omega(t_k^-), \\ \tilde{x}(t_k^+) &= x(t_k^-) - \Pi_{i_{k+1}}\omega(t_k^-), \end{aligned} \quad (8)$$

and

$$x(t_k^-) - \Pi_{i_{k+1}}\omega(t_k^-) = T_{i_k,i_{k+1}}(x(t_k^-) - \Pi_{i_k}\omega(t_k^-)), \quad (9)$$

where $T_{i_k,i_{k+1}}$, $k \in N$, are given matrices, then under the average dwell time

$$\tau_a > \tfrac{ln\mu}{\lambda_0}, \quad (10)$$

where

$$\mu = \sup_{1\leq i,j\leq N} \parallel P^{1/2}T_{i,j}P^{1/2} \parallel, \lambda_0 = \inf_{1\leq i\leq N} \tfrac{\lambda_{\min}(Q_i\prime)}{\lambda_{\max}(P)}, \quad (11)$$

and

$$(A_i + B_iK_i)^T P + P(A_i + B_iK_i) = -Q_i\prime, \quad (12)$$

the full information feedback controllers (3) solve the problem of output regulation for the switched system (1).

Proof. Set $L_i = \Gamma_i - K_i\Pi_i$, and consider the coordinate transformation $\tilde{x} = x - \Pi_i\omega$. According to (7), the system (4) is rewritten as

$$\begin{aligned} \dot{\tilde{x}} &= (A_{\sigma(t)} + B_{\sigma(t)}K_{\sigma(t)})\tilde{x} = \tilde{A}_{\sigma(t)}\,\tilde{x}, \\ \dot{\omega} &= S\omega, \\ e &= C_{\sigma(t)}\tilde{x}. \end{aligned} \quad (13)$$

Therefore, the output regulation problem for the switched system (1) is equivalently converted to the stabilization problem of the system (13). According to (9), we have the relation of the transformed states between the two consecutive switching times t_k^+ and t_k^- as follows

$$x(t_k^-) - \Pi_{i_{k+1}}\omega(t_k^-) = T_{i_k,i_{k+1}}(x(t_k^-) - \Pi_{i_k}\omega(t_k^-)).$$

We define the following Lyapunov function candidate

$$V(\tilde{x}) = \tilde{x}^T P \tilde{x}, \tag{14}$$

where P is a positive definite matrix satisfying (12). When $t \in [t_k, t_{k+1})$, we have

$$\begin{aligned} V(t) &\leq \exp\left(-\frac{\lambda_{\min}(Q_{i_{k+1}})'}{\lambda_{\max}(P)}(t_{k+1}-t_k)\right)V(t_k^+) \\ &\leq \exp(-\lambda_0(t_{k+1}-t_k))V(t_k^+). \end{aligned}$$

Because the coordinate transformations are different, $V(t_k^+) \neq V(t_k^-)$. By virtue of the properties of the positive definite matrices, there exist $P^{\frac{1}{2}} > 0$ such that $P = P^{\frac{1}{2}}P^{\frac{1}{2}}$, applying (9), we obtain

$$\begin{aligned} V(t_k^+) &= \| P^{1/2}\tilde{x}(t_k^+) \|^2 = \| P^{1/2}T_{i_k,i_{k+1}}\tilde{x}(t_k^-) \|^2 \\ &\leq \| P^{1/2}T_{i_k,i_{k+1}}P^{-1/2} \|^2 \| P^{1/2}\tilde{x}(t_k^-) \|^2 \\ &\leq \| P^{1/2}T_{i_k,i_{k+1}}P^{-1/2} \|^2 \, V(t_k^-) \\ &\leq \mu V(t_k^-). \end{aligned}$$

Based on the above inequalities, we have

$$V(t_{k+1}^+) \leq \mu \exp(-\lambda_0(t_{k+1}-t_k))V(t_k^+). \tag{15}$$

Repeating the inequality (15) from $k = 0$ to $k = N_\sigma - 1$ yields

$$V(t^-) \leq V(t_{N_\sigma}) \leq \mu^{N_{\sigma(t)}} \exp(-\lambda_0 t)V(0).$$

According to $N_\sigma(T,t) \leq N_0 + \frac{T-t}{\tau_a}$, we get $V(t^-) \leq \mu^{N_0} \exp\left(t\left(\frac{\ln\mu}{\tau_a} - \lambda_0 t\right)\right)V(0)$. Then, the system (4) without the disturbance input is asymptotically stable under the average dwell time (10). Meanwhile, we get

$$\lim_{t\to\infty}(C_{\sigma(t)}x + Q_{\sigma(t)}w) = \lim_{t\to\infty} C_{\sigma(t)}\tilde{x} = 0.$$

Therefore, we can conclude that the output regulation problem is solved.

Remark 2. Note that, each group of regulation equations (7) has their own solution for each subsystem, which leads to the switched system has different coordinate transformations. The above Theorem solves the problem of different transformations for the switched linear systems.

Next, we give another sufficient condition for the problem to be solvable via error feedback.

Theorem 2. If there exist $\Pi_i, \Sigma_i, H_i, \ F_i \ \bar{T}_{i,j}$ for $\forall i,j \in I_N$, satisfying the following equations

$$\begin{aligned} \Pi_i S &= A_i\Pi_i + B_iH_i\Sigma_i + P_i, \\ \Sigma_i S &= F_i\Sigma_i, \\ 0 &= C_i\Pi_i + Q_i, \end{aligned} \tag{16}$$

and the system (1) with different coordinate transformations

$$\begin{aligned} \tilde{\chi}(t_k^-) &= \chi(t_k^-) - \bar{\Pi}_{i_k}\omega(t_k^-), \\ \tilde{\chi}(t_k^+) &= \chi(t_k^-) - \bar{\Pi}_{i_{k+1}}\omega(t_k^-), \end{aligned} \tag{17}$$

and

$$\chi(t_k^-) - \bar{\Pi}_{i_k}\omega(t_k^-) = \bar{T}_{i,j}(\chi(t_k^-) - \bar{\Pi}_{i_{k+1}}\omega(t_k^-)), \tag{18}$$

where

$$\tilde{\chi} = \begin{pmatrix} x - \Pi_i\omega \\ \xi - \Sigma_i\omega \end{pmatrix}, \chi = \begin{pmatrix} x \\ \xi \end{pmatrix}, \bar{\Pi}_i = \begin{pmatrix} \Pi_i \\ \Sigma_i \end{pmatrix},$$

then under the average dwell time

$$\tau_a > \frac{ln\mu}{\lambda_0}, \tag{19}$$

where

$$\mu = \sup_{1\leq i,j\leq N} \| P^{1/2}\bar{T}_{i,j}P^{1/2} \|, \lambda_0 = \inf_{1\leq i\leq N} \frac{\lambda_{\min}(Q_i\prime)}{\lambda_{\max}(P)}, \tag{20}$$

and

$$\bar{A}_i^T P + P\bar{A}_i = -Q_i\prime, \tag{21}$$

the error feedback controllers (5) solve the problem of output regulation for the switched system (6), where $\bar{A}_i = \begin{pmatrix} A_i & B_iH_i \\ G_iC_i & F_i \end{pmatrix}$.

Proof. Based on (16), we can rewrite the regulation equations as follows

$$\begin{aligned} \bar{\Pi}_i S &= \bar{A}_i\bar{\Pi}_i + \bar{P}_i, \\ 0 &= \bar{C}_i\bar{\Pi}_i + Q_i, \end{aligned} \tag{22}$$

where

$$\bar{P}_i = \begin{pmatrix} P_i \\ G_iQ_i \end{pmatrix}, \bar{C}_i = \begin{bmatrix} C_i & 0 \end{bmatrix}.$$

Consider the transformations

$$\tilde{\chi} = \chi - \bar{\Pi}_i\omega.$$

According to (22), the system (6) is rewritten as

$$\begin{aligned} \dot{\tilde{\chi}} &= \bar{A}_{\sigma(t)}\tilde{\chi}, \\ \dot{w} &= Sw, \\ e &= \bar{C}_{\sigma(t)}\tilde{\chi}. \end{aligned} \tag{23}$$

For the switched linear system (23), we choose the Lyapunov function as follows

$$V(t) = \tilde{\chi}^T(t)P\tilde{\chi}(t).$$

Similar to the proof of Theorem 1, we can show that the problem of output regulation for the switched linear (6) is solved under the switching law (20).

II. Conclusion

In this paper, the output regulation problem for switched linear systems is investigated. The problem is difficult for switched systems, therefore, the results are few. The different coordinate transformations for switched systems is always a complicated issue. Because once the coordinate transformation is different, the Lyapunov function is discontinuous, and the decreasing of the Lyapunov function is difficult to verify. For switched linear systems, the problem can be solved by common coordinate transformation, but the conservativeness is always relatively large.

In this work, we address the output regulation problem with the transformations of switching instant between the consecutive time satisfying a relational expressions, based on this relation, we solve the output regulation problem. We easily know that when the disturbance and the state of the switched systems are linearly dependent, the relational expressions given in this paper is automatically satisfied. By virtue of this, we can get that the conditions are easy to implement.

The main contribution in this paper is that the output regulation problem for the switched linear systems with different coordinate transformations is considered. Sufficient conditions for the problem to be solvable are given based on the average dwell time method. For the regulation equations of the switched linear system, we suppose that each subsystem has its own solution, which reduce the conservativeness.

Output regulation for switched systems is a complicated and important problem. Further, studying a robust output regulation problem for switched linear systems and switched nonlinear systems is another task for future work.

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