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Active control of speed fluctuations in rotating machines using feedback linearization

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Abstract - **This paper presents a method for actively controlling torsional vibrations in rotating machines caused by angle-dependent parameters. The work is motivated by rotating machines with crank or cam gear mechanisms that cause fluctuations in the angular speed when the machine is driven by a constant load torque or when the speed is controlled with conventional controllers. A very general model for such a system is introduced and used to derive a control law by feedback linearization. With this control law, the speed fluctuations are completely eliminated and desired linear dynamics can be prescribed for the system. The method is tested in a simulation study with a model of a real industrial machine. Although the proposed method works well, the study is preliminary in the sense that the method has not been applied experimentally and its robustness has not been assessed.**

Active vibration control; Feedback linearization; Nonlinear control; Speed control; Torsional vibrations

I. MOTIVATION

There are different causes for torsional vibrations in drives. These vibrations can be the result of eccentric masses [1] or be excited by the propulsion itself, e.g. a combustion engine [2, 3, 4] or an electric drive [5, 6]. Another commonly occurring cause is the transformation of the rotational motion of the propulsion into a translational motion. This typically happens in a cam gear or a crank assembly and causes a periodically varying load torque.

As an example, a simple crank assembly, which only includes a single oscillating mass, is shown in Fig. 1. The torque equilibrium for this example leads to the equation of motion given as

$$\begin{aligned} T &= m \cdot \ddot{s}(\varphi) \frac{\mathrm{d}s}{\mathrm{d}\varphi} \\ &= m\left(\frac{\mathrm{d}^2 s}{\mathrm{d}\varphi^2} \dot{\varphi}^2 + \frac{\mathrm{d}s}{\mathrm{d}\varphi} \cdot \ddot{\varphi} \right) \frac{\mathrm{d}s}{\mathrm{d}\varphi} \\ &= m \frac{\mathrm{d}^2 s}{\mathrm{d}\varphi^2} \frac{\mathrm{d}s}{\mathrm{d}\varphi} \dot{\varphi}^2 + m \left(\frac{\mathrm{d}s}{\mathrm{d}\varphi} \right)^2 \ddot{\varphi} \ , \end{aligned} \tag{1}$$

where T is the driving torque and all the other variables are as shown in Fig. 1. The displacement s depends on the rotation angles α and φ and is given as

$$\begin{aligned} s &= l\left(1 - \cos\alpha\right) + r\left(1 - \cos\varphi\right) \\ &= l\left(1 - \sqrt{1 - \frac{r^2}{l^2} \sin^2 \varphi}\right) + r\left(1 - \cos\varphi\right) = s\left(\varphi\right). \end{aligned} \tag{2}$$

The introduction of $m \cdot \left(\mathrm{d}s / \mathrm{d}\varphi\right)^2$ as an angle depended parameter $J(\varphi)$ (which could be seen as an apparent, angle-dependent, moment of inertia) leads to

$$T = \frac{1}{2} \frac{\mathrm{d}J(\varphi)}{\mathrm{d}\varphi} \dot{\varphi}^2 + J(\varphi)\ddot{\varphi} \, . \tag{3}$$

Even if a constant driving torque is applied to the system, the internal dynamics lead to nonlinear vibrations. This is due to the dependency of the parameters in the model describing the system on the rotational angle.

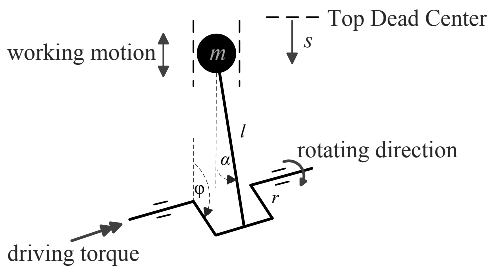

Fig. 1 Schematic diagram of the crankshaft

Such torsional vibrations are a well-known problem in industrial drives like rolling mill systems [7, 8], paper machines [9, 10] or turbo generators [11] and also in vehicles [2]. They lead to product quality problems like damage of machinery and a shorter lifecycle of the plant.

Possible approaches for the attenuation of these vibrations are to change the inertia or to use damping elements, but this often has a negative effect on the dynamics of the machines. Because of these disadvantages several active control approaches to attenuate torsional vibrations have been proposed. In [12], a damping filter is implemented and in [13] a digital PI/PID controller is used to achieve active damping of the vibrations. In [14], an adaptive sliding neuro-fuzzy approach is suggested and in [15] sliding mode and force dynamics are considered. Active damping based on learning control is presented in [16]. In [4] the main target is to reduce the torsional vibrations of a crankshaft by adjusting the fuel injection duration by minimizing a cost function.

Another way to control the dynamic system and attenuate the nonlinear behavior and prescribe linear system dynamics is to use the method of feedback linearization. In [17] and [18], feedback linearization control is used for improved fuel consumption in combustion engines by using feedback linearization, but the afore-mentioned dynamical connections are only regarded marginally. In other fields, vibration control is the main aspect as in [19] where feedback linearization is used for active vehicle suspension control to reduce the vibrations. To the best of the authors' knowledge, however, no applications of feedback linearization to the problem considered in this paper have been reported in the literature to date.

As mentioned above, the torsional vibrations can result from angle-dependent parameters in the system dynamics. The angle itself is also a state of the system. In order to make the method presented in this paper applicable to a wide range of systems, a very universal system description is introduced for systems with state-dependent parameters. The resulting description is very compact. With this description, it is very easy to derive a control law (which is given here only for systems with a relative degree of two, but can easily be extended).

The remainder of this paper is organized as follows. The system description is introduced in Sec. II. The control law is derived in Sec. III. The control approach has been tested in simulation studies for a model that corresponds to an industrial machine that is operated by an industrial partner with whom this study has been conducted. The obtained results are shown in Sec. IV. Some conclusions are given in Sec. V.

II. System Description

Real industrial drives are more complex systems than the simple example discussed in the introduction. Often, they are composed of more than one crank assembly or cam gear or even combine both types. Furthermore, elasticity, damping and gear ratios have to be taken into account.

Nevertheless, the final system description of such systems will still contain constant and angle-dependent parameters. Due to the rotational mode of operation, the angle-dependent parameters are periodic in the rotational angle (see for example, the parameter J_α for the example considered in Sec. IV shown in Fig. 2).

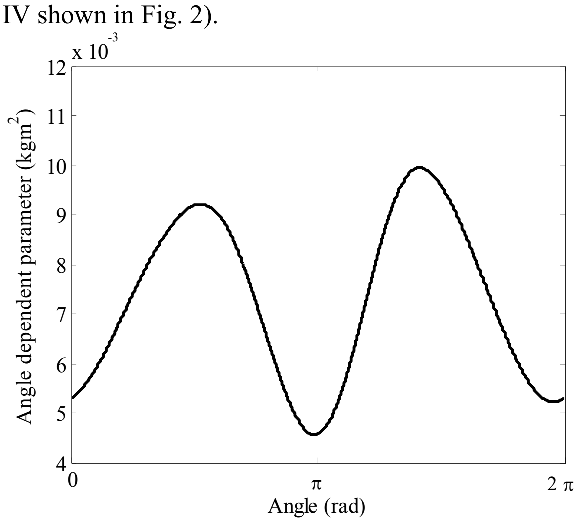

Fig. 2 Parameter J_α over the rotation angle φ_2

These angle-dependent parameters can then be expanded into a Fourier series. Here, the exponential Fourier series is used for an easier handling of the derivatives. Expanding system parameters into Fourier systems is quite common for describing rotational systems (see, for example, [2], [4] and [20]).

The system dynamics are described by the nonlinear state-space model

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x})\,\boldsymbol{x} + \boldsymbol{B}u\,, \tag{4}$$

where $\boldsymbol{x}$ is the state vector, u the input of the system and y the output. The "system matrix" $\boldsymbol{A}(\boldsymbol{x})$ is assumed to depend on linear combinations of the state vectors according to

$$\boldsymbol{A}(\boldsymbol{x}) = \boldsymbol{A}_0 + \bar{\boldsymbol{A}}\left(\boldsymbol{v}_1^{\mathrm{T}}\boldsymbol{x},\dots,\boldsymbol{v}_N^{\mathrm{T}}\boldsymbol{x}\right) + \tilde{\boldsymbol{A}}\left(\boldsymbol{v}_1^{\mathrm{T}}\boldsymbol{x},\dots,\boldsymbol{v}_N^{\mathrm{T}}\boldsymbol{x}\right). \tag{5}$$

The matrix $\boldsymbol{A}_0$ is constant (and therefore, corresponds to the "linear part" of the system dynamics), the matrix $\bar{\boldsymbol{A}}$ depends periodically on linear combinations of the state vector and the matrix $\tilde{\boldsymbol{A}}$ depends periodically and linearly on the linear combinations of the state vector.

The matrices $\bar{\boldsymbol{A}}(\boldsymbol{x})$ and $\tilde{\boldsymbol{A}}(\boldsymbol{x})$ in the model are expressed as

$$\begin{aligned}\bar{\boldsymbol{A}}(\boldsymbol{x}) &= \bar{\boldsymbol{A}}\left(\boldsymbol{v}_1^{\mathrm{T}}\boldsymbol{x},\dots,\boldsymbol{v}_N^{\mathrm{T}}\boldsymbol{x}\right)\\ &= \sum_{l=1}^{N}\sum_{\mu=-L}^{L}\bar{\boldsymbol{M}}_{l,\mu}\,\mathrm{e}^{\,\mathrm{j}\mu\,\boldsymbol{v}_l^{\mathrm{T}}\boldsymbol{x}}\\ &= \sum_{l=1}^{N}\begin{bmatrix}\bar{\boldsymbol{M}}_{l,-L} & \cdots & \bar{\boldsymbol{M}}_{l,L}\end{bmatrix}\left(\begin{bmatrix}\mathrm{e}^{\left(-L\,\mathrm{j}\boldsymbol{v}_l^{\mathrm{T}}\boldsymbol{x}\right)}\\ \vdots\\ \mathrm{e}^{\left(L\mathrm{j}\boldsymbol{v}_l^{\mathrm{T}}\boldsymbol{x}\right)}\end{bmatrix}\otimes\mathbf{I}_n\right)\end{aligned} \tag{6}$$

and

$$\begin{aligned}&\tilde{\boldsymbol{A}}(\boldsymbol{x}) =\\ &= \tilde{\boldsymbol{A}}\left(\boldsymbol{v}_1^{\mathrm{T}}\boldsymbol{x},\dots,\boldsymbol{v}_N^{\mathrm{T}}\boldsymbol{x}\right)\\ &= \sum_{h=1}^{N}\sum_{l=1}^{N}\sum_{\mu=-L}^{L}\tilde{\boldsymbol{M}}_{h,l,\mu}\,\mathrm{e}^{\,\mathrm{j}\mu\,\boldsymbol{v}_l^{\mathrm{T}}\boldsymbol{x}}\,\boldsymbol{v}_h^{\mathrm{T}}\boldsymbol{x}\\ &= \sum_{h=1}^{N}\sum_{l=1}^{N}\begin{bmatrix}\tilde{\boldsymbol{M}}_{h,l,-L} & \cdots & \tilde{\boldsymbol{M}}_{h,l,L}\end{bmatrix}\left(\begin{bmatrix}\mathrm{e}^{\left(-L\mathrm{j}\boldsymbol{v}_l^{\mathrm{T}}\boldsymbol{x}\right)}\\ \vdots\\ \mathrm{e}^{\left(L\mathrm{j}\boldsymbol{v}_l^{\mathrm{T}}\boldsymbol{x}\right)}\end{bmatrix}\otimes\left(\boldsymbol{v}_h^{\mathrm{T}}\boldsymbol{x}\mathbf{I}_n\right)\right).\end{aligned} \tag{7}$$

Here, $\otimes$ stands for the Kronecker product [21], L is the order of the Fourier series and N the number of the linear combinations of the state vector that the matrices depend upon. The linear combinations are expressed as $\boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}$ with $p = 1,\dots,N$.

The matrices $\bar{\boldsymbol{M}}_{l,\mu}$ and $\tilde{\boldsymbol{M}}_{h,l,\mu}$ result from expanding the periodic system parameters into Fourier series. Since the exponential Fourier series representation is used, it holds that

$$\operatorname{Re}\bar{\boldsymbol{M}}_{l,\mu} = \operatorname{Re}\bar{\boldsymbol{M}}_{l,-\mu}\,,\ \operatorname{Re}\tilde{\boldsymbol{M}}_{h,l,\mu} = \operatorname{Re}\tilde{\boldsymbol{M}}_{h,l,-\mu} \tag{8}$$

$$\operatorname{Im}\bar{\boldsymbol{M}}_{l,\mu} = -\operatorname{Im}\bar{\boldsymbol{M}}_{l,-\mu}\,,\ \operatorname{Im}\bar{\boldsymbol{M}}_{h,l,\mu} = -\operatorname{Im}\bar{\boldsymbol{M}}_{h,l,-\mu} \tag{9}$$

and

$$\operatorname{Im}\bar{\boldsymbol{M}}_{l,0} = \boldsymbol{0}\,,\ \operatorname{Im}\bar{\boldsymbol{M}}_{h,l,0} = \boldsymbol{0}\,. \tag{10}$$

III. Derivation of the Control Law

Consider a completely controllable SISO system of the form already introduced above, namely

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x})\,\boldsymbol{x} + \boldsymbol{B}u \tag{11}$$

with the output equation

$$y = \boldsymbol{C}\boldsymbol{x}\,. \tag{12}$$

The first step in the derivation of the control law in feedback linearization consists of differentiating the output y until the input signal u shows up in the derivative. The amount of times the output has to be differentiated is known as the relative degree. In the following, it is assumed that the relative degree is two.
The first derivative of y with respect to the time t is

$$\dot{y} = \boldsymbol{C}\,\dot{\boldsymbol{x}} = \boldsymbol{C}\boldsymbol{A}(\boldsymbol{x})\boldsymbol{x} + \boldsymbol{C}\boldsymbol{B}u\,. \tag{13}$$

Since the relative degree is two, it holds that

$$\boldsymbol{C}\boldsymbol{B} = 0\,. \tag{14}$$

The second derivative is then given by

$$\begin{aligned}\ddot{y} &= \boldsymbol{C}\boldsymbol{A}(\boldsymbol{x})\dot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{A}}(\boldsymbol{x})\boldsymbol{x}\\ &= \boldsymbol{C}\boldsymbol{A}(\boldsymbol{x})\big(\boldsymbol{A}(\boldsymbol{x})\boldsymbol{x} + \boldsymbol{B}u\big) + \boldsymbol{C}\dot{\boldsymbol{A}}(\boldsymbol{x})\boldsymbol{x}.\end{aligned} \tag{15}$$

To obtain the second derivative of y, the derivative of $\boldsymbol{A}$ with respect to the time t is needed. Using standard rules from matrix differential calculus [21], the time derivative of $\boldsymbol{A}$ follows as

$$\begin{aligned}\dot{\boldsymbol{A}}(\boldsymbol{x}) &= \frac{\partial \boldsymbol{A}(\boldsymbol{x})}{\partial \boldsymbol{x}^{\mathrm{T}}}\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \frac{\partial \boldsymbol{A}(\boldsymbol{x})}{\partial \boldsymbol{x}^{\mathrm{T}}}\big(\dot{\boldsymbol{x}} \otimes \mathbf{I}_n\big)\\ &= \frac{\partial \boldsymbol{A}(\boldsymbol{x})}{\partial \boldsymbol{x}^{\mathrm{T}}}\big(\boldsymbol{A}(\boldsymbol{x})\boldsymbol{x} \otimes \mathbf{I}_n + \boldsymbol{B}u \otimes \mathbf{I}_n\big).\end{aligned} \tag{16}$$

From the system description (5), the derivative of $\boldsymbol{A}$ with respect to the transposed state vector $\boldsymbol{x}$ follows as

$$\frac{\partial \boldsymbol{A}(\boldsymbol{x})}{\partial \boldsymbol{x}^{\mathrm{T}}} = \frac{\partial \boldsymbol{A}_0}{\partial \boldsymbol{x}^{\mathrm{T}}} + \frac{\partial \bar{\boldsymbol{A}}(\boldsymbol{x})}{\partial \boldsymbol{x}^{\mathrm{T}}} + \frac{\partial \tilde{\boldsymbol{A}}(\boldsymbol{x})}{\partial \boldsymbol{x}^{\mathrm{T}}}\,. \tag{17}$$

Since $\boldsymbol{A}_0$ is constant, it follows that

$$\frac{\partial \boldsymbol{A}_0}{\partial \boldsymbol{x}^{\mathrm{T}}} = \boldsymbol{0}\,. \tag{18}$$

Using the matrix chain rule and the matrix product rule [21], the second and the third term on the right hand side of (13) become

$$\begin{aligned}\frac{\partial \bar{\boldsymbol{A}}(\boldsymbol{x})}{\partial \boldsymbol{x}^{\mathrm{T}}} &= \sum_{p=1}^{N}\frac{\partial \bar{\boldsymbol{A}}(\boldsymbol{x})}{\partial \boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}}\,\frac{\mathrm{d}\,\boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}}{\mathrm{d}\,\boldsymbol{x}^{\mathrm{T}}}\\ &= \sum_{p=1}^{N}\frac{\partial \bar{\boldsymbol{A}}(\boldsymbol{x})}{\partial \boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}}\,(\boldsymbol{v}_p^{\mathrm{T}} \otimes \mathbf{I}_n)\end{aligned} \tag{19}$$

and

$$\begin{aligned}\frac{\partial \tilde{\boldsymbol{A}}(\boldsymbol{x})}{\partial \boldsymbol{x}^{\mathrm{T}}} &= \sum_{p=1}^{N}\frac{\partial \tilde{\boldsymbol{A}}(\boldsymbol{x})}{\partial \boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}}\,\frac{\mathrm{d}\,\boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}}{\mathrm{d}\,\boldsymbol{x}^{\mathrm{T}}}\\ &= \sum_{p=1}^{N}\frac{\partial \tilde{\boldsymbol{A}}(\boldsymbol{x})}{\partial \boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}}\,(\boldsymbol{v}_p^{\mathrm{T}} \otimes \mathbf{I}_n)\,.\end{aligned} \tag{20}$$

To calculate these derivatives, the derivatives of the matrices $\bar{\boldsymbol{A}}(\boldsymbol{x})$ and $\tilde{\boldsymbol{A}}(\boldsymbol{x})$ with respect to $\boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}$ are required. These derivatives can be easily calculated from (6) and (7) and are given as

$$\begin{aligned}\frac{\partial \bar{\boldsymbol{A}}(\boldsymbol{x})}{\partial \boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}} &= \sum_{\mu=-L}^{L}\bar{\boldsymbol{M}}_{p,\mu}\;\mathrm{j}\mu\;\mathrm{e}^{\,\mathrm{j}\mu\,\boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}}\\ &= \begin{bmatrix}\bar{\boldsymbol{M}}_{p,-L} & \cdots & \bar{\boldsymbol{M}}_{p,L}\end{bmatrix}\left(\begin{bmatrix}-L\mathrm{j}\cdot\mathrm{e}^{\left(-L\,\mathrm{j}\boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}\right)}\\ \vdots\\ L\mathrm{j}\cdot\mathrm{e}^{\left(L\,\mathrm{j}\boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}\right)}\end{bmatrix}\otimes \mathbf{I}_n\right)\end{aligned} \tag{21}$$

and

$$\begin{aligned}&\frac{\partial \tilde{\boldsymbol{A}}(\boldsymbol{x})}{\partial \boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}} =\\ &= \sum_{h=1}^{N}\sum_{l=1}^{N}\sum_{\mu=-L}^{L}\tilde{\boldsymbol{M}}_{h,l,\mu}\left(\mathrm{e}^{\,\mathrm{j}\mu\,\boldsymbol{v}_l^{\mathrm{T}}\boldsymbol{x}}\,\frac{\partial \boldsymbol{v}_h^{\mathrm{T}}\boldsymbol{x}}{\partial \boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}} + \frac{\partial\,\mathrm{e}^{\,\mathrm{j}\mu\,\boldsymbol{v}_l^{\mathrm{T}}\boldsymbol{x}}}{\partial \boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}}\,\boldsymbol{v}_h^{\mathrm{T}}\boldsymbol{x}\right)\\ &= \sum_{l=1}^{N}\sum_{\mu=-L}^{L}\tilde{\boldsymbol{M}}_{p,l,\mu}\,\mathrm{e}^{\,\mathrm{j}\mu\,\boldsymbol{v}_l^{\mathrm{T}}\boldsymbol{x}} + \sum_{h=1}^{N}\sum_{\mu=-L}^{L}\tilde{\boldsymbol{M}}_{h,i,\mu}\,\mathrm{j}\mu\,\mathrm{e}^{\,\mathrm{j}\mu\,\boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}}\,\boldsymbol{v}_h^{\mathrm{T}}\boldsymbol{x}\\ &= \sum_{l=1}^{N}\begin{bmatrix}\tilde{\boldsymbol{M}}_{p,l,-L} & \cdots & \tilde{\boldsymbol{M}}_{p,l,L}\end{bmatrix}\left(\begin{bmatrix}\mathrm{e}^{-\mathrm{j}\,L\boldsymbol{v}_l^{\mathrm{T}}\boldsymbol{x}}\\ \vdots\\ \mathrm{e}^{\,\mathrm{j}\,L\boldsymbol{v}_l^{\mathrm{T}}\boldsymbol{x}}\end{bmatrix}\otimes \mathbf{I}_n\right) +\\ &+ \sum_{h=1}^{N}\begin{bmatrix}\tilde{\boldsymbol{M}}_{h,p,-L} & \cdots & \tilde{\boldsymbol{M}}_{h,p,L}\end{bmatrix}\cdot\\ &\cdot\left(\begin{bmatrix}-\mathrm{j}L\,\mathrm{e}^{-\mathrm{j}\,L\boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}}\\ \vdots\\ \mathrm{j}L\,\mathrm{e}^{\,\mathrm{j}\,L\boldsymbol{v}_p^{\mathrm{T}}\boldsymbol{x}}\end{bmatrix}\otimes\left(\boldsymbol{v}_h^{\mathrm{T}}\boldsymbol{x}\,\mathbf{I}_n\right)\right).\end{aligned} \tag{22}$$

From (21) and (22) the advantage of the description in the introduced matrix notation becomes obvious because the matrices remain unchanged since their elements are constant. Only the vectors including the exponential functions of the Fourier series and the second factor of the Kronecker product change due to the differentiation.

With (16) to (22) all necessary expressions are given to compute the control law

$$\begin{aligned}u = &\frac{-\boldsymbol{C}\left(\dfrac{\partial \boldsymbol{A}(\boldsymbol{x})}{\partial \boldsymbol{x}^{\mathrm{T}}}\big(\boldsymbol{A}(\boldsymbol{x})\boldsymbol{x}\otimes\boldsymbol{I}_n\big) + \boldsymbol{A}^2(\boldsymbol{x})\right)\boldsymbol{x}}{\boldsymbol{C}\left(\dfrac{\partial \boldsymbol{A}(\boldsymbol{x})}{\partial \boldsymbol{x}^{\mathrm{T}}}\big(\boldsymbol{B}\otimes\boldsymbol{I}_n\big)\boldsymbol{x} + \boldsymbol{A}(\boldsymbol{x})\boldsymbol{B}\right)} +\\ &+ \frac{-\gamma_1\boldsymbol{C}\boldsymbol{A}(\boldsymbol{x})\boldsymbol{x} - \gamma_0\boldsymbol{C}\boldsymbol{x} + \beta w}{\boldsymbol{C}\left(\dfrac{\partial \boldsymbol{A}(\boldsymbol{x})}{\partial \boldsymbol{x}^{\mathrm{T}}}\big(\boldsymbol{B}\otimes\boldsymbol{I}_n\big)\boldsymbol{x} + \boldsymbol{A}(\boldsymbol{x})\boldsymbol{B}\right)},\end{aligned} \tag{23}$$

which can be calculated from (15). Applying the control law (23) leads to the resulting closed loop dynamics described by

$$\ddot{y} = -\gamma_1\dot{y} - \gamma_0 y + \beta w \tag{24}$$

As mentioned above, the derivation shown here is for systems with a relative degree of two. An extension to systems with a higher relative degree is fairly straightforward.

IV. Simulation Studies

The control design described in the previous section is applied to an industrial machine in a simulation study. A schematic diagram of the model is shown in Fig. 3.

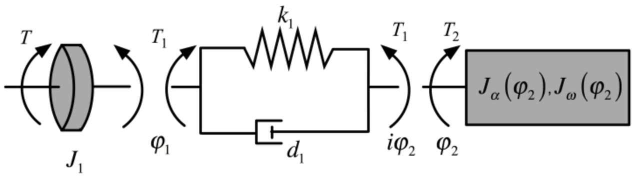

Fig. 3 Schematic diagram of the industrial machine (belt not shown)

The dynamics of this model are described by

$$J_1 \ddot{\varphi}_1 = T - T_1 \,, \tag{25}$$

$$J_\alpha(\varphi_2)\ddot{\varphi}_2 + J_\omega(\varphi_2)\dot{\varphi}_2^2 = T_2 \tag{26}$$

with

$$T_1 = k_1(\varphi_1 - i\varphi_2) + d_1(\dot{\varphi}_1 - i\dot{\varphi}_2), \tag{27}$$

and

$$T_2 = iT_1 \ . \tag{28}$$

In this model, T is the driving torque generated from an electrical machine (not included in the model) and T_1 and T_2 are inner torques that are transmitted through a belt drive (not shown in the diagram) with gear ratio i; φ_1 and φ_2 are the rotation angles of the drive side and the load side; J_1 is the moment of inertia at the driving end, k_1 is the stiffness and d_1 the damping of the system; J_α and J_ω are angle-dependent parameters that can be calculated from the construction data of the machine and were provided by an industrial partner that manufactures this type of machine. The variations of the parameters J_α over the angle φ_2 and J_ω over the angle φ_2 are shown in Fig. 2 and Fig. 4, respectively.

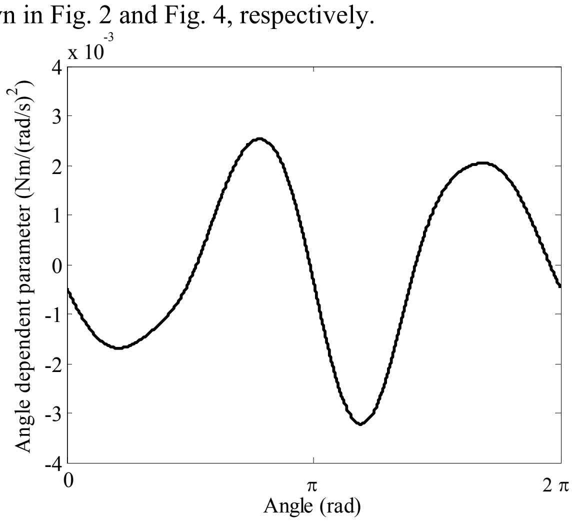

Fig. 4 Parameter J_ω over the rotation angle φ_2

If the system is driven by a constant input torque, the resulting angular speed fluctuates around a mean value. This undesirable behavior is shown in Fig. 5. The potential of standard cascaded PID/PI speed control for reducing these oscillations is limited. The tuning of PID/PI control for such systems is difficult since certain settings of the controller parameters can even amplify the oscillations in certain operating conditions.

The system equations can be written as the state space model given by (11) and (12) with

$$\boldsymbol{x} = \begin{bmatrix} \varphi_1 & \dot{\varphi}_1 & \varphi_2 & \dot{\varphi}_2 \end{bmatrix}^{\mathrm{T}}, \tag{29}$$

$$\boldsymbol{A}(\boldsymbol{x}) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_1}{J_1} & \frac{-d_1}{J_1} & \frac{ik_1}{J_1} & \frac{id_1}{J_1} \\ 0 & 0 & 0 & 1 \\ \frac{ik_1}{J_\alpha(x_3)} & \frac{id_1}{J_\alpha(x_3)} & -\frac{i^2 k_1}{J_\alpha(x_3)} & -\frac{i^2 d_1 + J_\omega(x_3)x_4}{J_\alpha(x_3)} \end{bmatrix}, \tag{30}$$

$$\boldsymbol{B} = \begin{bmatrix} 0 & 1/J_1 & 0 & 0 \end{bmatrix}^{\mathrm{T}}, \tag{31}$$

and

$$\boldsymbol{C} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}. \tag{32}$$

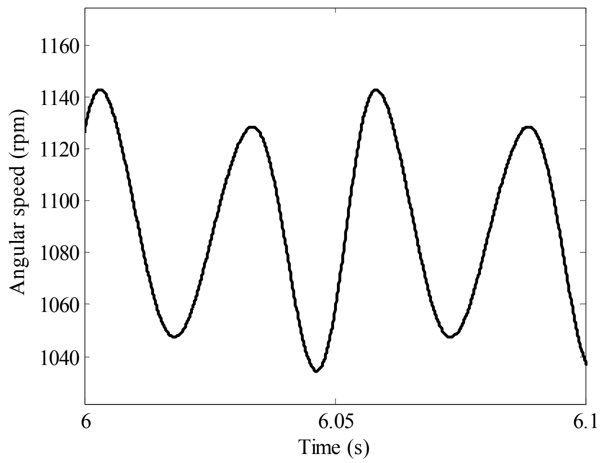

Fig. 5 Angular speed $\dot{\varphi}_2$ if the system is driven by constant torque

From (30) it can be seen that the behavior of the considered system depends on the rotation angle φ_2 which corresponds to the state x_3 and its derivative $\dot{\varphi}_2$ which corresponds to the state x_4.

For this system the state space representation of the form introduced in Sec. II becomes

$$\dot{\boldsymbol{x}} = \left(\boldsymbol{A}_0 + \bar{\boldsymbol{A}}(\boldsymbol{v}_1^{\mathrm{T}}\boldsymbol{x}, \boldsymbol{v}_2^{\mathrm{T}}\boldsymbol{x}) + \tilde{\boldsymbol{A}}(\boldsymbol{v}_1^{\mathrm{T}}\boldsymbol{x}, \boldsymbol{v}_2^{\mathrm{T}}\boldsymbol{x})\right)\boldsymbol{x} + \boldsymbol{B}u \tag{33}$$

$$y = \boldsymbol{C}\boldsymbol{x}\,, \tag{34}$$

with the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^{\mathrm{T}} \tag{35}$$

and

$$\mathbf{v}_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}} \tag{36}$$

the relevant states x_3 and x_4 can be selected. Thus, $N = 2$.

From the system matrix in (30) the linear part of the system matrix can be read off directly and gives

$$\boldsymbol{A}_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_1/J_1 & -d_1/J_1 & ik_1/J_1 & id_1/J_1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \quad (37)$$

As introduced in (5), the nonlinear parts of (30) are divided into a part $\bar{\boldsymbol{A}}$ with a harmonic dependence on the states and $\tilde{\boldsymbol{A}}$ with a harmonic and a linear dependence on the states. These nonlinear parts can be represented by a Fourier series.

The matrix coefficients $\bar{\boldsymbol{M}}_{l,\mu}$ for the matrix $\bar{\boldsymbol{A}}$ in (6) are obtained as

$$\bar{\boldsymbol{M}}_{1,\mu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ ik_1\bar{m}_\mu & id_1\bar{m}_\mu & -i^2k_1\bar{m}_\mu & -d_1i^2\bar{m}_\mu \end{bmatrix} \quad (38)$$

and

$$\bar{\boldsymbol{M}}_{2,\mu} = \boldsymbol{0}, \quad (39)$$

where the scalars $\bar{m}_\mu$ are the coefficients of the Fourier expansion of J_α^{-1}. Similarly, the matrix coefficients for $\tilde{\boldsymbol{A}}$ in (7) are obtained as

$$\tilde{\boldsymbol{M}}_{2,1,\mu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{m}_\mu \end{bmatrix}, \quad (40)$$

$$\tilde{\boldsymbol{M}}_{1,1,\mu} = \tilde{\boldsymbol{M}}_{1,2,\mu} = \tilde{\boldsymbol{M}}_{2,2,\mu} = \boldsymbol{0}, \quad (41)$$

where the scalars $\tilde{m}_\mu$ are the coefficients of the Fourier expansion of J_ω / J_α. Fourier series of eight order are used, that is, $L = 8$.

The simulated control result for the model of the industrial machine is shown in Fig. 6. The effects of the nonlinearities are attenuated by the calculated control law with feedback linearization. Also the desired linear behavior as a second order system is achieved.

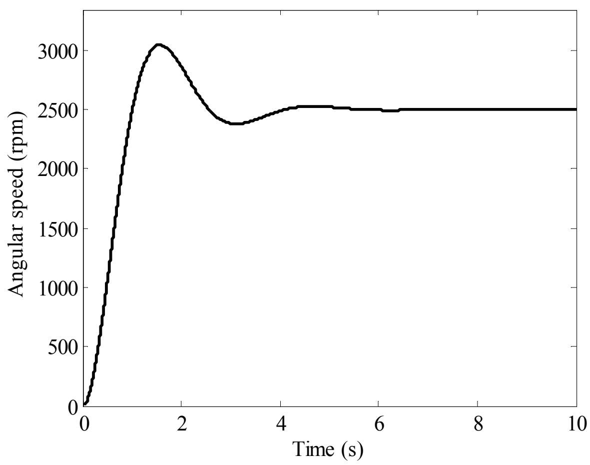

Fig. 6 Step response of the controlled nonlinear system

In Fig. 7 it can be seen that the angular speed at the drive side is still fluctuating while the angular speed at the load side is constant.

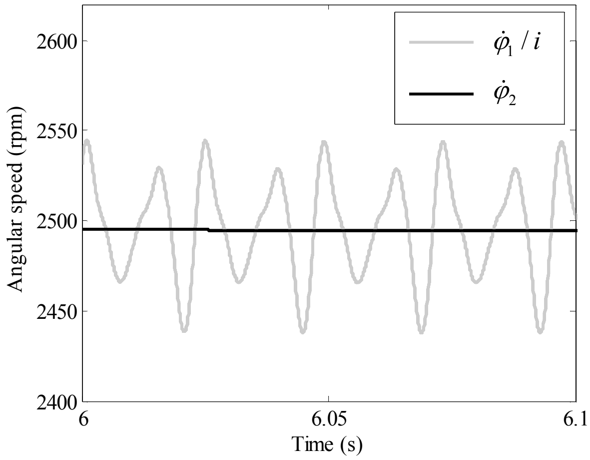

Fig. 7 Fluctuating angular speed $\dot{\varphi}_1$ at the drive side (related to the load side by including the gear ratio i) and constant angular speed $\dot{\varphi}_2$ at the load side

The control signal for the considered system to achieve the desired behavior is shown in Fig. 8. No calculations are needed to obtain the derivatives and the control law since the matrix coefficients can be simply plugged into the corresponding equations and the control law is readily obtained.

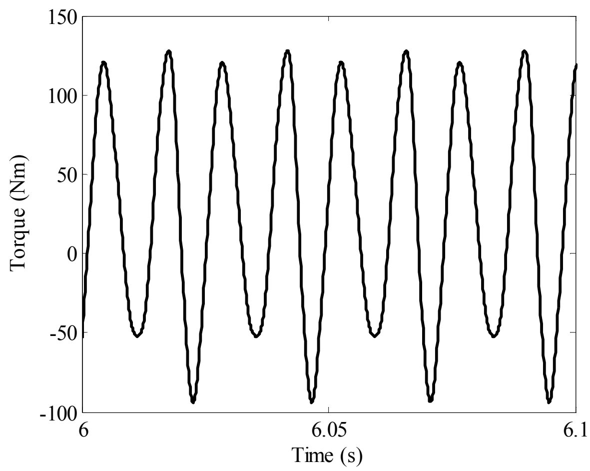

Fig. 8 Control signal to achieve the desired behavior on the output

V. Summary and Conclusion

In this paper, the active control of speed fluctuations in rotating machines has been considered. After a simple motivating example, a very general state-space description of the considered systems was introduced. With this system description, it is very easy to derive a control law that eliminates the speed fluctuations and results in a second-order linear dynamics for the controlled system. The proposed control law has been applied to a model that represents the behavior of an industrial machine that shows undesired speed fluctuations. To the best of the authors knowledge, this is the first time that feedback linearization has been used for active vibration control for such systems.

The steps that need to be carried out in the control design process consist of transforming the model equations into the general form. This involves reading off entries from the model equations and expanding periodic model parameters into Fourier series. This can be easily done using the standard discrete Fourier transform.

Although the simulation results show that the approach works well (as predicted by theory), it is stressed that this is a preliminary study. The setup considered here corresponds to an ideal case (noise free measurements, no actuator and sensor dynamics included, all system parameters exactly known, all states available for feedback, continuous-time control possible). The applicability of this approach in more realistic cases will be investigated in future studies, where robustness issues (sensitivity to noise and model inaccuracies), implementation issues (discretization of the control algorithm) and extensions (including actuator and sensor dynamics, using an observer to reconstruct unmeasured state variables) will be investigated. The main contribution of this paper is the idea of using feedback linearization for the active control of torsional vibrations.

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