RELAY IDENTIFICATION BY ANALYZING NONSYMMETRICAL OSCILLATIONS FOR SINGLE INTEGRATOR WITH TIME DELAY

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Abstract: The paper deals with approximation of systems with the dominant first order dynamics and nonsymmetrical input by the Integrator Plus Dead Time (IPDT) model by means of analysis of the nonsymmetrical oscillations arising under relay control. The results achieved by identification of optical plant are then experimentally verified by Disturbance Observer (DOB) based controller.

Keywords: relay, disturbance observer, integrator plant, nonsymmetrical oscillations

1 INTRODUCTION

Relay feedback test is very popular approach used in a commercial autotuners. The current research in this area was closely analyzed in (Tao Liu, Furong Gao 2009). There are two types of relay tests, unbiased and biased. When using the unbiased test the process gain can be highly deflorated by a load disturbance. Many relay feedback methods have been proposed to reject static disturbances (Hang, Åström, & Ho, 1993; Park, Sung, & Lee, 1997, 1998; Shen, Wu, & Yu, 1996). Their approaches bias the reference value of the relay on–off as much as a static disturbance (that must be known in advance), in order to achieve the same accuracy as in the case of no disturbance. Nevertheless none of these approaches can be applied to large static disturbance, of which the magnitude is bigger than that of the relay. By inserting a proportional integral (PI) controller behind the relay for the test, (Sung and Lee, 2006) proposed an identification method for application against large static disturbance, larger than the magnitude of the relay. The drawback of the method is given be necessity to tune an additional controller.

The other important question is related to the model used for approximating the plant dynamics. Almost 70 years ago, Ziegler and Nichols (Ziegler & Nichols, 1942) proposed to use the sustained oscillations for process dynamics characterization giving finally PID controller tuning, whereby the process dynamics approximation was equivalent to the use of the IPDT model. It is, however, well known that the method is appropriate also for dealing with many systems with more complicated and typically static dynamics. Several papers investigate the transition point when the designer should choose to use either First Order Plus Dead Time (FOPDT) based or IPDT based PID control design methods (e.g. Skogestad, 2003; Jones and Tham, 2004). Also Huba (2003) shows that for the relatively low ratio of the dead time and the plant time constant T_d / T_p it is enough to use the to Integrator Plus Dead Time (IPDT) approximations also for dealing with the FOPDT processes used in this paper. However, when using the IPDT approximation for the FOPDT process, the plant feedback that is around an operating point is equivalent to a load disturbance is leading to assymmetrical behavior also in the case with symmetrical relay. So, this oscillation asymmetry with respect to the precision of the whole approximation is playing in the identification an important issue. For a noncompensated disturbance (including also the internal plant feedback around the operating point), the deformation of oscillations leads to increased influence of higher harmonics and to decreased precision of the identification both by using the describing functions method and the Fast Fourier Transform (FFT). The main advantage of constraining the plant approximation to the IPDT model

$$S(s) = K_s e^{-T_d s} / s \tag{1}$$

is that both the experiment setup and the corresponding formulas remain relatively simple. There is no need to tune the PI controller before the identification, or to use the PI controler with the additional anti windup circuitry.

2 PROPOSED METHOD DERIVATION

Let us consider oscillations in the control loop with a relay with the output $u_r = \pm M$ and a piecewise constant input disturbance $v = const$. The actual plant input will be given as a piecewise constant signal $u_A = \pm M + v$. Possible transients are shown in Fig. 2.

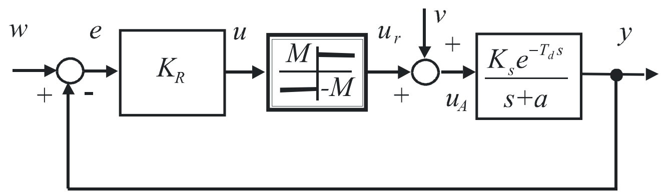

Figure 2: Relay identification with nonsymmetrical plant input

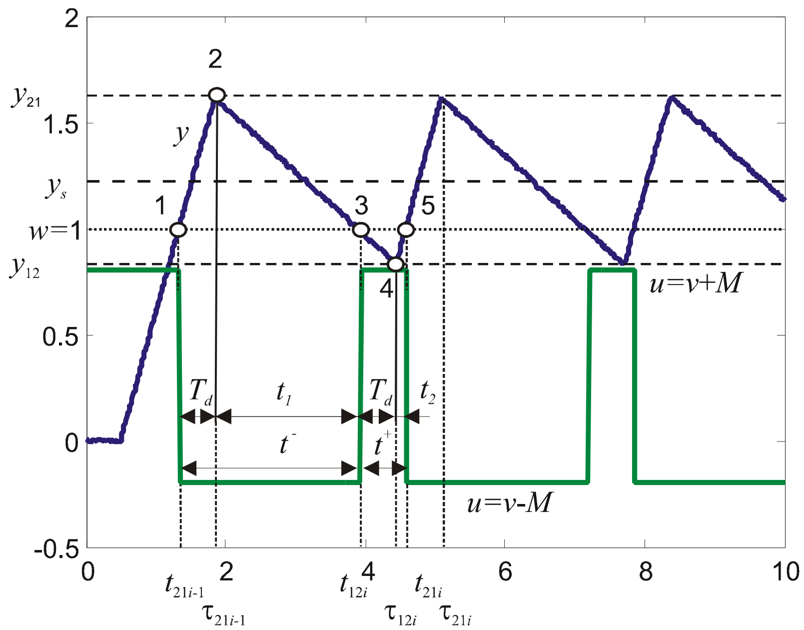

Figure 2: Transients of basic variables of the loop in Fig. 1

By assuming relay switching from the positive relay output $u = M$ to the negative value $u = -M$ (point 1) at the time moment t_{21i-1}, due to the dead time the influence of the positive plant input $U_2 = (v + M)K_s$ will keep over interval with the length equal to the dead time value T_d. Then, after reaching output value y_{21} at the time moment τ_{21i-1} (point 2) due to the effective plant input $U_1 = (v - M)K_s$ the output starts to decrease. After the time interval t_1 it reaches the reference value w (point 3). Even though at this moment the relay switches to the positive value $u = M$ the plant output continues to fall the time T_d longer and reaches the value y_{12} (point 4). The total length of the interval with negative relay output will be denoted as t^-. Under virtue of the positive relay output the plant output starts to rise and reaches the reference value after the time t_2 (point 5). The total duration of the positive relay output may be denoted as

$$t^+ = t_2 + T_d \tag{2}$$

As a result of the time delay, the plant output turnover time instants τ_{21i} are shifted with respect to the relay reversal moments t_{21i} by T_d. Similar time shift exists among time instants τ_{12i} and t_{12i}, i.e.

$$\tau_{21i} = t_{21i} + T_d \quad ; \quad \tau_{12i} = t_{12i} + T_d \tag{3}$$

For the single integrator it is possible to formulate relations

$$\begin{aligned} y_{21} - w &= U_2 T_d \;;\; t_1 = (w - y_{21})/U_1 \\ y_{12} - w &= U_1 T_d \;;\; t_2 = (w - y_{12})/U_2 \end{aligned} \tag{4}$$

Period of one cycle may be denoted as

$$P_u = t^+ + t^- = 2T_d + t_1 + t_2 = \frac{4T_d M^2}{M^2 - v^2} \tag{5}$$

For a known value of the relay amplitude M and a known ratio of the positive and negative relay output duration over one cycle

$$\varepsilon = \frac{t^+}{t^-} = \frac{t_2 + T_d}{t_1 + T_d} = -\frac{v - M}{v + M} \tag{6}$$

it is possible to express the identified disturbance as

$$v = u_0 + v_n \tag{7}$$

This may be composed of the known intentionally set offset at the relay output u_0 and of an unknown external disturbance v_n. Together, it may be identified as

$$v = M\,\frac{1-\varepsilon}{1+\varepsilon} \tag{8}$$

From (5) it then follows

$$T_d = \frac{P_u}{4}\left[1 - \left(\frac{v}{M}\right)^2\right] = P_u\,\frac{\varepsilon}{(1+\varepsilon)^2} \tag{9}$$

The output mean value over one cycle period may be expressed as

$$\begin{aligned} y_s = \frac{1}{P_u}\Bigg[w + \int_0^{T_d} U_2 t\,dt + \int_{T_d}^{2T_d + t_1} \left(y_{21} + U_1(t - T_d)\right)dt + \\ + \int_{2T_d + t_1}^{P_u} \left(y_{12} + U_2(t - 2T_d - t_1)\right)dt \Bigg] \end{aligned} \tag{10}$$

$$y_s = \frac{1}{P_u}\Bigg[w + \int_0^{T_d} K_s (v+M) t dt + \\ + \int_{T_d}^{2T_d+t_1} \big(y_{21} - K_s (v-M)(t-T_d)\big)dt + \\ + \int_{2T_d+t_1}^{P_u} \big(y_{12} + K_s (v+M)(t-2T_d-t_1)\big)dt \Bigg] \tag{11}$$

Finally, one gets formula for the plant gain

$$K_s = \frac{(1+\varepsilon)^2}{\varepsilon P_u}\frac{(y_s - w)}{v} = \frac{(1+\varepsilon)^3}{\varepsilon(1-\varepsilon)P_u}\frac{(y_s - w)}{M} \tag{12a}$$

It is also possible to calculate the plant gain by using the area *A* limited by $y(t)$ around w over one period (5), when

$$K_s = \frac{(1+\varepsilon)^4}{\varepsilon\left(1+\varepsilon^2\right)}\frac{A}{MP_u^2} \tag{12b}$$

In difference to (12a), this may also be used in the symmetrical case with $v = 0$ and $y_s = w$. So, to get the model parameters (1) it is enough to calculate the mean plant output value over one sustained cycle of relay switching (11), or the equivalent area *A*, the period of oscillation (5) and the ratio of time slots with positive and negative relay output (6). The approximation is invariant agains a constant input disturbance $v = const$. This may be considered to be composed of the intentionally introduced disturbance u_0 and of the external disturbance v_n

$$v = u_0 + v_n \tag{13}$$

In this way it is possible to introduce an additional free parameter for tuning enabling to work in any working point. When i tis chosen in the vicinity of the steady=state input=output characteristic, it enables to work with relatively low relay module *M*.

After carrying out the above procedure at least for two different reference signal values w_1 and w_2 and by evaluating changes of the identified disturbance values v_1 and v_2 in dependence on the mean output values y_{s1} and y_{s2} it is then possible to approximate the dependence

$$v = f(y_s) \tag{14}$$

If it has a negligible slope with respect to changes in y_s, the system is sufficiently well approximated by the IPDT model..

3 REAL EXPERIMENT

The thermo-optical plant laboratory model (Fig.3) offers measurement of 8 process variables: controlled temperature, its filtered value, ambient temperature, controlled light intensity, its derivative and filtered value, the fan speed of rotation and current. The temperature and the light intensity control channels are interconnected by 3 manipulated voltage variables influencing the bulb (heat & light source), the light-diode (the light source) and the fan (the system cooling). Besides these, it is possible to adjust two parameters of the light intensity derivator. Within Matlab/Simulink or Scilab/Scicos schemes [10] the plant is represented as a single block and so limiting needs on costly and complicated software packages for real time control. The (supported) external converter cards are necessary just for sampling periods below 50ms. Currently, more than 40 such plants are used in labs of several EU universities.

Figure 3: udaq28/LT

The thermal plant consists of a halogen bulb 12V DC/20W (elements 1-6), of a plastic pipe wall (element 7), of its internal air column (element 8) containing the temperature sensor PT100, and of a fan 12V DC/0,6W (element 9 that can be used for producing disturbances, but also for control).

The filtered optical channel was used for the experiment. The following table summarizes the plant‘s parameters identified in several working points.

Table 1: Parameter values

M	relay bias	setpoint	Ks	Td	load disturbance
2,5	2,5	18	0,517352527	0,41972879	-2,001845018
2,5	2,5	14	0,489313995	0,440021655	-1,603235014
2,5	2,5	10	0,430658533	0,429437785	-1,212268744
2,5	2,5	6	0,30954285	0,412814878	-0,876338851
2	3	18	0,628492828	0,348484848	-2,212121212
2	3	14	0,563738186	0,346666622	-1,904672897
2	3	10	0,440228574	0,286839239	-1,718606783
2	3	6	0,303412414	0,163469911	-1,484375

The following figures compare the mesured data with the simulated model for various experiment setups.

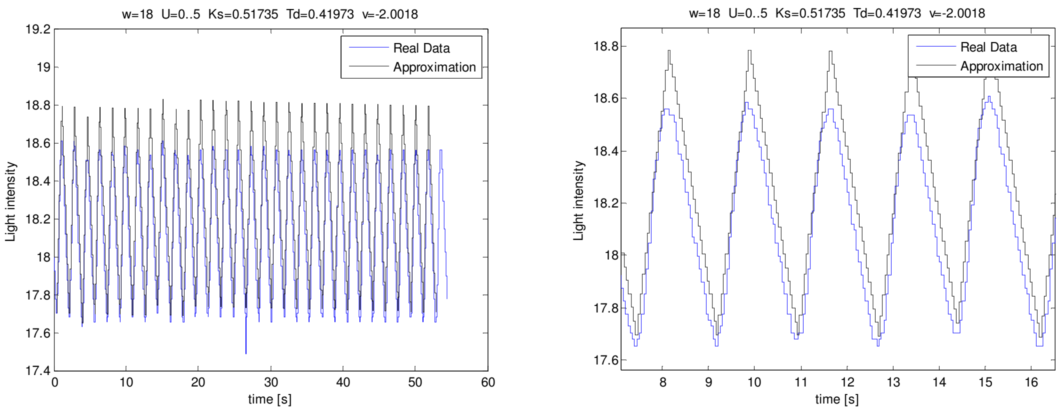

Figure 4: Mesurement and approximation

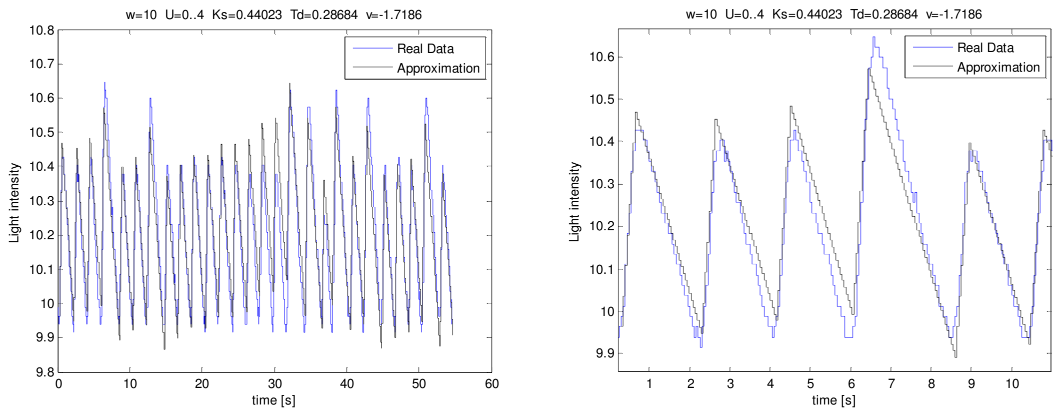

Figure 5: Mesurement and approximation

4 PI_1 controller

The PI_1 controller employs disturbance observer as the I-action. The controller structure consisting of P- action and DOB is presented in fig. 6. Index "1" used in its title has to be related to one saturated pulse of the control variable that can occur in accomplishing large reference signal steps. In this way it should be distinguished from the PI_0 controller reacting to a reference step by monotonic transient of the manipulated variable.

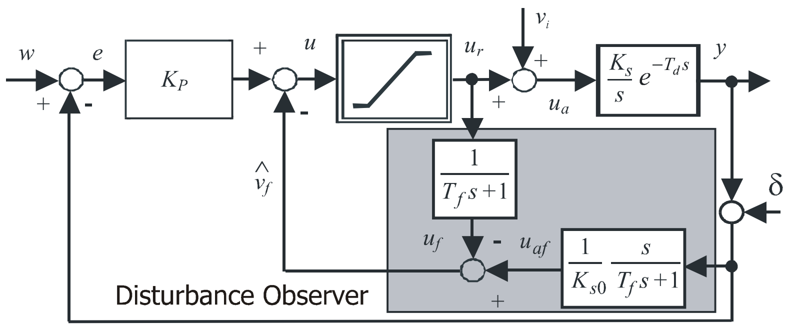

Figure 6: PI1-controller

To achieve fastest possible transients without overshooting the following controller tuning was used (Huba 2003).

The closed loop pole corresponding to the fastest monotonic output transients using simple P-controller

$$\alpha_e = \frac{1}{T_{d0} e} \tag{15}$$

whereby T_{d0} represents estimate of the plant dead time. When using the P-controller together with the DOB based I-action, the "slower" closed loop pole should be used

$$\alpha_{eI} = \alpha_e / c = \frac{1}{T_{d0} e} \frac{1}{c}; c \in \langle 1.3 \quad 1.5 \rangle \tag{16}$$

The gain of P-controller corresponding to the closed loop pole (16) is then

$$K_P = -\alpha_{eI} / K_s \tag{17}$$

For the time constant of the filter used in the DOB one gets

$$T_f = -\frac{1}{\alpha_{eI}} \tag{18}$$

A more detailed analysis of the tuning in the constrained case is to find in the paper Huba (2010).

The following figures show the real experiment results for various setpoint changes. The maximal value of process gain and the maximum value of dead time were used for the controller tuning in the first experiment $K_{s0} = \max\{K_s\}; T_{d0} = \max\{T_d\}$.

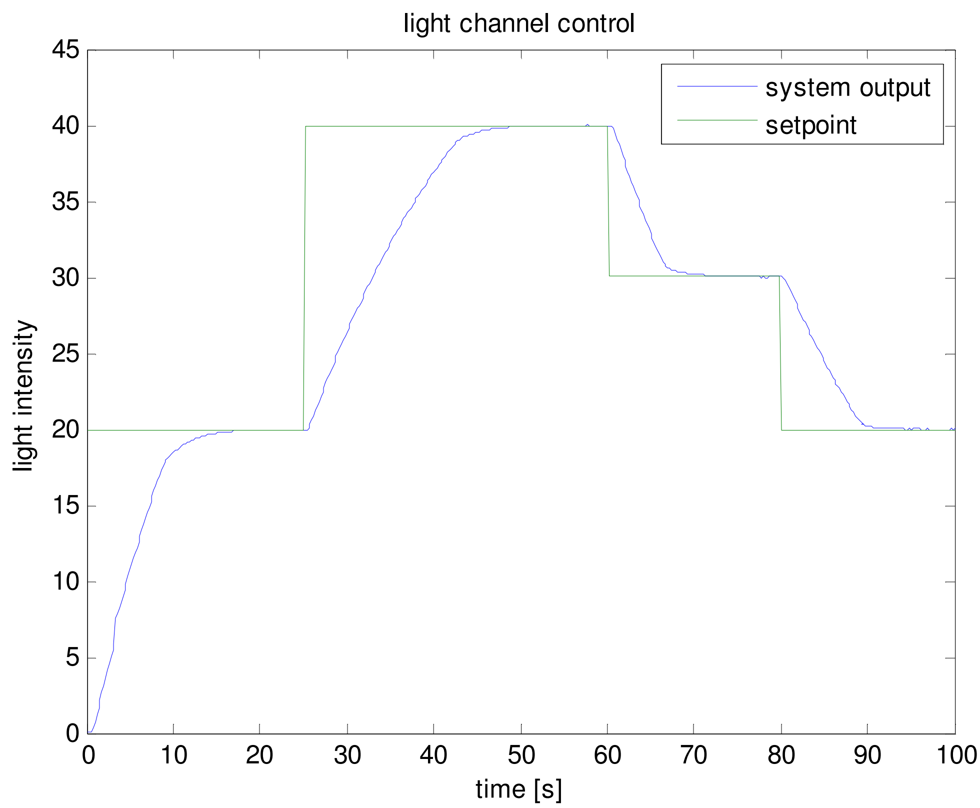

Figure 7a: Closed-loop - several setpoint changes; $K_{s0} = \max\{K_s\}; T_{d0} = \max\{T_d\}$

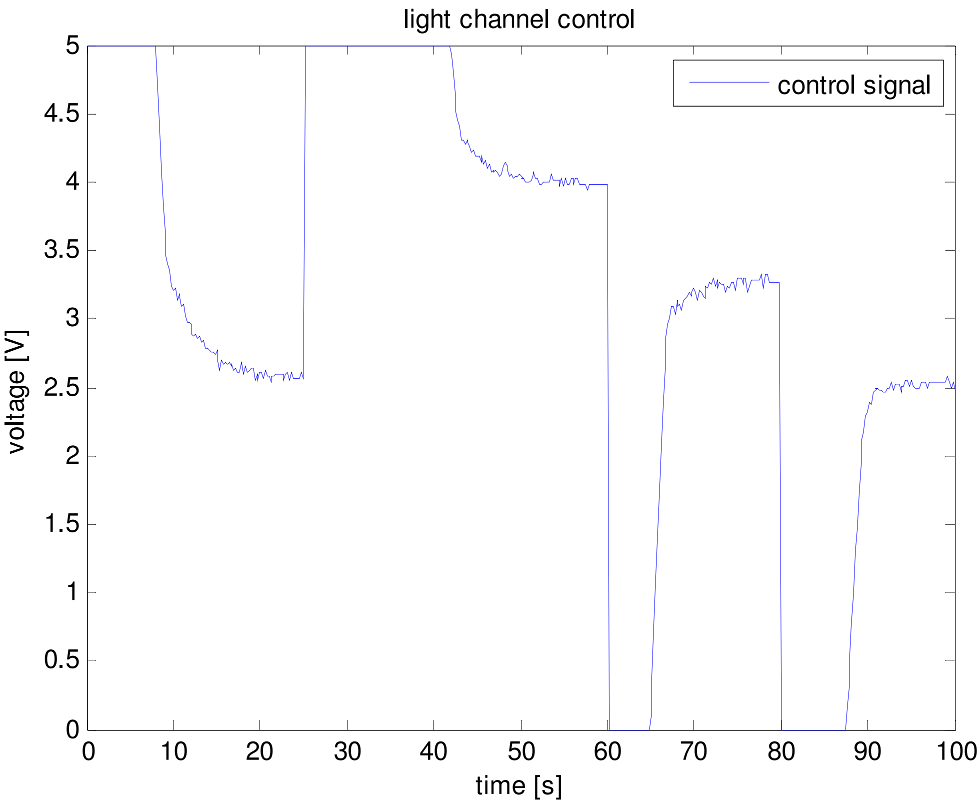

Figure 7b: Closed-loop - several setpoint changes; $K_{s0} = \max\{K_s\}; T_{d0} = \max\{T_d\}$

The control gives fast transients without overshooting. The control signal consists of two control phases: one can observe an interval at the saturation followed by the control signal's monotonic transient to the new steady state value.

There was an average value of the process gain $K_{s0} = mean\{K_s\}$ used in the next experiment for the controler tuning. Although the output of the system does not overshoot, the change in parameters leads to a slightly oscillatory control signal. When the average value of the dead time was used similar control quality was achieved.

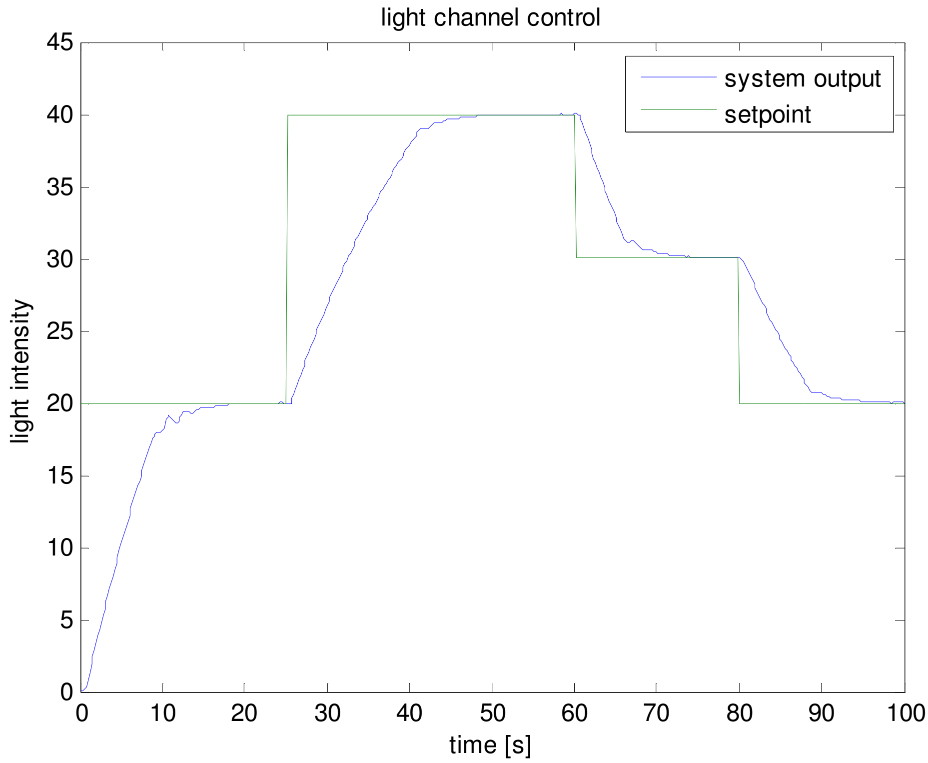

Figure 8a: Closed-loop - several setpoint changes $K_{s0} = mean\{K_s\}$

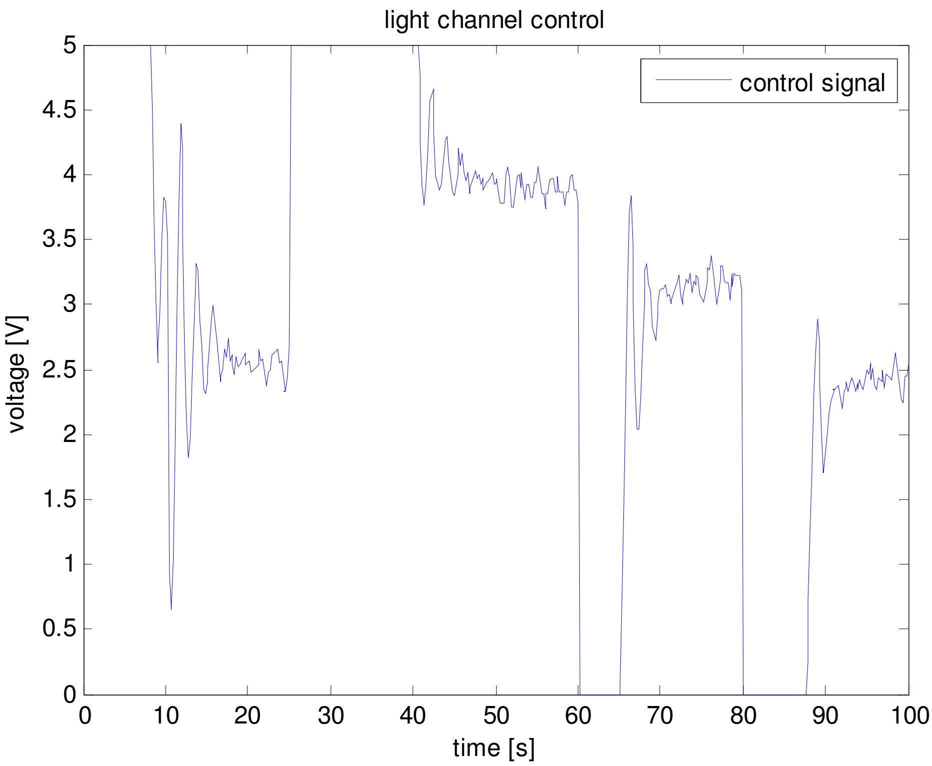

Figure 8b: Closed-loop - several setpoint changes; $K_{s0} = mean\{K_s\}$

5 CONCLUSION

New relay experiment identification method has been proposed for the IPDT plant. Stable optical plant with the first order dominant dynamics was used for the verifying the method by the real experiment. The method benefits from obtaining the load disturbance value without need of tuning a PI controller firstly. The drawback is the sensitivity to the measurement noise that may, however, be partially eliminated by sampled-data relay control using longer sampling periods.

In applying the proposed method to controlling optical plant, the relay test yields results depending on the working point that obviously points out on nonlinear plant behaviour. In this paper, the nonlinear properties were treated by a robust controller. One of the strong advantages of the proposed method, however, is its possible extention to identifying nonlinear systems with dominant first order dynamics + dead time.

6 ACKNOWLEDGEMENTS

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