ROBUST DECENTRALIZED CONTROLLER DESIGN: STATE SPACE VERSION OF THE EQUIVALENT SUBSYSTEMS METHOD

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Abstract: The paper deals with the robust PID controller design for performance for linear MIMO systems with state decentralized structure using output feedback. The proposed design procedure is a straightforward state space extension of the frequency domain Equivalent Subsystems Method (ESM) guaranteeing feasible performance achieved in equivalent subsystems for the full system.

Keywords: decentralized controller, guaranteed performance, output feedback controller, PID controller

1 INTRODUCTION

The main concept of the Equivalent Subsystems Method originally developed as a Nyquist-based frequency domain decentralized controller (DC) design technique is the so called *equivalent subsystem*; equivalent subsystems are generated by shaping Nyquist plot of each decoupled subsystem using any selected characteristic locus of the matrix of interactions. Local controllers of equivalent subsystems independently tuned for stability and specified feasible performance in terms of degree of stability constitute the resulting decentralized controller guaranteeing the same degree of stability for the full system. The ESM state-space version for continuous-time systems proposed in this paper provides new perspectives to further development of the approach.

The robust decentralized PID controller is designed using the polytopic description of the uncertain system and applying the robust optimal control design procedure with extended cost function (Rosinová and Veselý, 2006) for state-space equivalent subsystems generated in each vertex of the polytopic uncertainty domain.

The paper is organized as follows: Section 2 includes preliminaries and problem statement, Section 3 presents the main results followed by a simple example in Section 4. Conclusions are drawn in the last section.

2 PROBLEM STATEMENT AND PRELIMINARIES

Consider a linear time-invariant affine dynamic system with state decentralized structure

$$\begin{aligned} \dot{x} &= (A + \delta A)x + (B + \delta B)u \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in R^n$ is state, $u \in R^m$ is control input and $y \in R^\ell$ is output of the system; *A*, *B*, and *C* are constant matrices of corresponding dimensions whereby both *B* and *C* have a decentralized structure, $\delta A, \delta B$ are matrices of uncertainties of compatible dimensions. Uncertainties are considered to be affine, described by as follows

$$\delta A = \sum_{j=1}^{p} \varepsilon_j \tilde{A}_j, \quad \delta B = \sum_{j=1}^{p} \varepsilon_j \tilde{B}_j \tag{2}$$

where $\underline{\varepsilon}_j \le \varepsilon_j \le \overline{\varepsilon}_j$ are unknown uncertainty parameters, $\tilde{A}_j, \tilde{B}_j, \ j = 1,2,...,p$ are constant matrices of uncertainties of appropriate dimensions and structure. The uncertain system can equivalently be described by a polytopic model given by the vertices of the uncertainty polytope

$$\{(A_1, B_1, C), (A_2, B_2, C) \ldots (A_N, B_N, C)\}, \ N = 2^p \tag{3}$$

For the nominal system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{4}$$

the decentralized feedback control law will be considered in the form

$$u(t) = FCx(t) \tag{5}$$

where *F* is a block diagonal matrix compatible with the block diagonal structure of *B* and *C*.

Performance is assessed using the quadratic performance index and the degree-of-stability of the closed-loop system. To damp oscillations and limit the response rate, the additional term for the state variable derivative is included in the performance index (Rosinová and Veselý, 2006; 2008).

$$J = \int_0^t [x^T(t)Qx(t) + u^T(t)Ru(t) + \dot{x}^T(t)S\dot{x}(t)]dt \tag{6}$$

where $Q, S \in R^{n \times n}, R \in R^{m \times m}$ are symmetric positive definite matrices.

The PID control law to be designed as proposed in (Rosinová and Veselý, 2006) is considered in the form

$$u(t) = K_p y(t) + K_{in} \int_0^t y(t)dt + F_d C_d \dot{x}(t) \tag{7}$$

If an auxiliary state variable is defined as follows

$$z(t) = \int_0^t y(t)dt, \tag{8}$$

proportional and integral terms can be included into the augmented state vector. Then, for the PI part of the PID controller, the state-space description of the closed-loop system is

$$\dot{x}_d = \begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A + BK_pC & BK_{in} \\ C_{in} & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_d(t) = A_c x_d(t) + B[F_d C_d \ \ 0]\dot{x}_d \tag{9}$$

where

$u_d(t) = F_d C_d \dot{x}(t);\ A_c = \begin{bmatrix} A + BK_p C & BK_{in} \\ C_{in} & 0 \end{bmatrix}$; C_{in} is output matrix for integral feedback

Manipulating (9) as follows

$$0 = A_c x_d(t) - M_d \dot{x}_d(t) \tag{10}$$

where $M_d = [\, I - BF_d C_d \,]$ includes the derivative part of the PID controller, the main results on PID controller design are given in the Theorem 1 (Rosinová and Veselý, 2006).

Theorem 1

Consider a continuous-time linear system (4) with the PID controller (7) and the cost function (6). The closed-loop system is stable if there exist matrices $H, P = P^T > 0, G$ and the gain matrices K_p, K_{in}, K_d such that the following condition holds

$$\begin{bmatrix} A_c^T H^T + HA_c + Q + C^T F^T RFC & P - M_d^T H + G^T A_c \\ P - M_d^T H + G^T A_c & -M_d^T G - G^T M_d + S \end{bmatrix} < 0 \tag{11}$$

where $F = [\, K_p \ \ K_{in} \,]$.

3 MAIN RESULTS

3.1 DC design for guaranteed degree-of-stability of the nominal system

To design a decentralized controller, the nominal system (3) has to be rewritten as follows

$$\dot{x} = (A_d + A_m)x + \sum_{i=1}^{m} B_i u$$
$$y = \sum_{i=1}^{m} C_i x \tag{12}$$

where $A_d = block\ diag(A_i)$ and *u(t)* is considered to have a decentralized control structure, i.e.

$$u(t) = \sum_{i=1}^{m} F_i C_i x(t) \tag{13}$$

Hence, the resulting closed-loop system can be written as

$$\dot{x}(t) = \left(\sum_{i=1}^{m} A_i + B_i F_i C_i \right) x(t) + A_m x(t) \tag{14}$$

Denote *p* the maximum real part out of all eigenvalues of the off-diagonal matrix A_m, i.e. for $\lambda_i(A_m),\ i = 1,\dots,n$

$$p = \max_i [Re\{\lambda_i(A_m)\}],\quad i = 1,\dots,n \tag{15}$$

Definition 1:

The closed-loop system (14) is stable if and only if individual subsystenms $S_i = A_i + B_i F_i C_i,\ i = 1,2,\dots,m$ satisfy the inequality (n_i is the size of the particular submatrix A_i)

$$Re\{\lambda_k[A_i + B_i F_i C_i]\} < -pI_i \quad k = 1,\dots,n_i \tag{16}$$

In the state-space version, equivalent subsystems are defined

$$A_i^{eq} = A_i + pI_i \qquad i = 1,2,...,m \tag{17}$$

where I_i is identity matrix of the same size as A_i.

Lemma 1:
The closed-loop system (14) is stable with a guaranteed minimum degree of stability $\alpha:\ 0 < \alpha < \alpha_{max}$ if and only if individual subsystems S_i, $i = 1,2,...,m$ satisfy the following inequality

$$\max_k Re\{ \lambda_k [A_i + B_i F_i C_i] \} \le -(p + \alpha) I_i, \quad k = 1,2,...,n_i \tag{18}$$

i.e. equivalent subsystems applied in the design of a decentralized PID controller for specified degree of stability are generated as follows

$$A_i^{eq}(\alpha) = A_i + (p + \alpha) I \tag{19}$$

3.2 Robust DC design for specified degree-of-stability and guaranteed cost

If the nominal system (4) belongs to the class of polytopic uncertain systems (1) described by N vertices of the polytope (3), obtained using (1), (2) with

$$\sum_{j=1}^{p} \varepsilon_j = 1,\ 0 \le \varepsilon_j \le 1, \tag{20}$$

hence submatrices A_i, B_i, C_i, $i = 1,\dots,N$ with constant entries are matrices in the vertices of the uncertainty box.

As (11) is linear in the uncertain parameter ε_j, for the sake of robust controller design it can be split in N inequalities as follows

$$\begin{bmatrix} A_{cij}^T H_i^T + H_i A_{cij} + Q + C_i^T F_i^T R F_i C_i & * \\ P_{ij} - M_{dij}^T H_i + G_i^T A_{cij} & - M_{dij}^T G_i - G_i^T M_{dij} + S \end{bmatrix} < 0 \tag{21}$$

$$i = 1,...,m;\ j = 1,...,N$$

If there exists solution to (21) with respect to the matrices $P_{ij} > 0,\ H_i,\ G_i,\ F_i,\ F_{di}$ then each equivalent subsystem is robustly stable with guaranteed cost and the full system is parameter dependent quadratic stable with guaranteed cost (i.e. there exists one Lyapunov function for the whole set of polytope vertices and stability for the whole uncertainty domain is guaranteed).

4 EXAMPLE

Theoretical results are illustrated on a simple academic example.

Consider a uncertain polytopic system (1) with $N = 2^p = 4$ vertices. Due to limited space, numeric values of particular matrices are shown for one vertex only:

$$A_1 = \begin{bmatrix} -0.1 & 0.02 & 0.022 & 0.210 \\ 0 & -0.6 & 0.1 & 0.01 \\ 0.1 & 0.2 & -0.23 & 0.01 \\ 0.032 & 0.21 & -0.22 & -0.6 \end{bmatrix} \quad B_1^T = \begin{bmatrix} 0.55 & 0.21 & 0 & 0 \\ 0 & 0 & 0.12 & 0.65 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

In the cost function (6) following values were selected: $Q = qI,\ q = 0.01$; $R = rI,\ r = 1$; $S = sI,\ s = 0.001$.

The problem has been solved for two different degrees of stability using (21) yielding following results:

1. for $\alpha = 0$

Resulting gain matrices are

$$F = \begin{bmatrix} -0.5788 & 0 & -0.3363 & 0 \\ 0 & -0.2519 & 0 & -0.2220 \end{bmatrix}, \quad F_d = \begin{bmatrix} 1.0965 & 0 \\ 0 & 1.0813 \end{bmatrix}$$

Evaluation of the maximum real part of the closed-loop matrices eigenvalues in each vertex $A_{cij},\ i = 1,2;\ j = 1,2,3,4;\ k = 1,\dots,n_i$ (n_i is the size of the particular submatrix A_i) gives:

1st subsystem (all vertices): $\max_j [Re\{\lambda_k(A_{c1j})\}] = -0.25738$

2nd subsystem (all vertices): $\max_j [Re\{\lambda_k(A_{c2j})\}] = -0.05066$

Full system (all vertices): $\max_j [Re\{\lambda_k(A_{cj})\}] = -0.19128$

2. for $\alpha = 0.05$

Resulting gain matrices are

$$F = \begin{bmatrix} -0.7059 & 0 & -0.4410 & 0 \\ 0 & -0.2540 & 0 & -0.3001 \end{bmatrix}, \quad F_d = \begin{bmatrix} 1.0965 & 0 \\ 0 & 1.0822 \end{bmatrix}$$

Evaluation of the maximum real part of the closed-loop matrices eigenvalues in each vertex $A_{cij},\ i = 1,2;\ j = 1,2,3,4$ gives:

1st subsystem (all vertices): $\max_j [Re\{\lambda_k(A_{c1j})\}] = -0.2396$

2nd subsystem (all vertices): $\max_j [Re\{\lambda_k(A_{c2j})\}] = -0.0500$

Full system (all vertices): $\max_j [Re\{\lambda_k(A_{cj})\}] = -0.1772$

For $\alpha = 0.05$, step responses of the full closed-loop system in all four vertices are shown in Fig. 1.

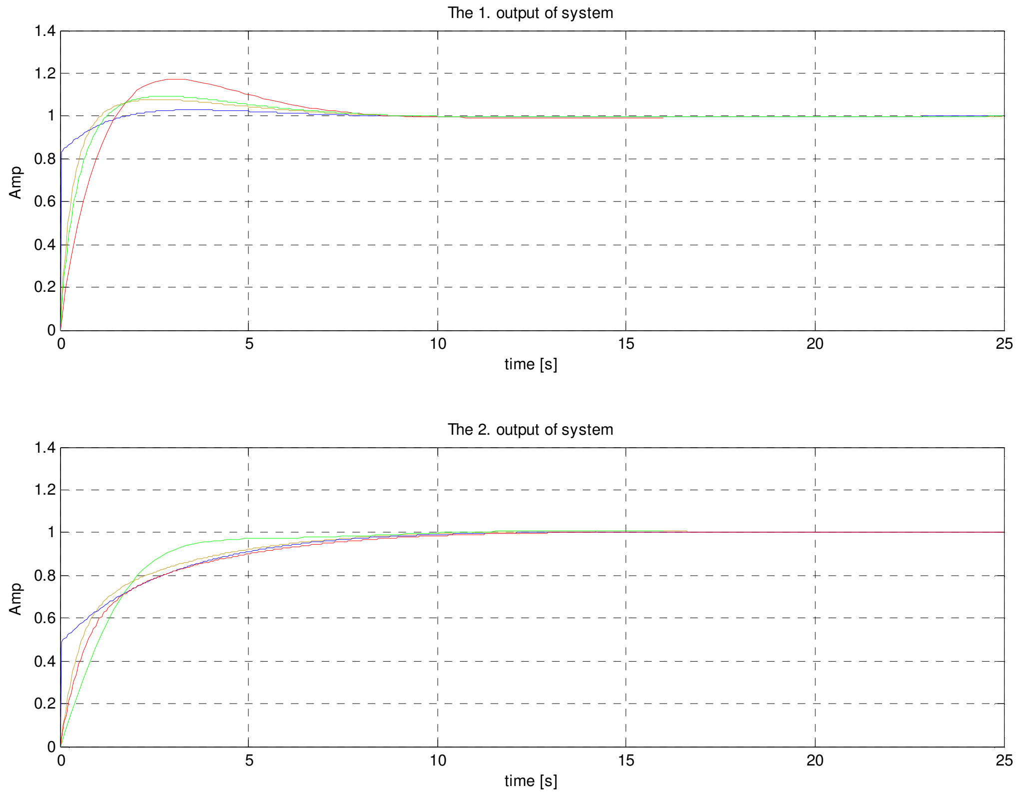

Figure 1: Closed-loop system step responses in all four vertices for α = 0.05

5 CONCLUSION

In this paper is the first attempt to extend the Equivalent Subsystem approach (originally developed in the frequency domain) for application in the state-space. Robust decentralized PID controller has been designed using the polytopic description of the uncertain system and applying the robust optimal control design procedure with extended cost function (Rosinová and Veselý, 2006) to state-space equivalent subsystems generated in each vertex of the polytopic uncertainty domain.

The main advantage of the proposed approach is that the order of the PID design procedure reduces to the order of the particular subsystem. The main limitation for application of the proposed approach is the required block diagonal structure of both the input and output matrices.

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