UNIFIED APPROACH TO INPUT-TO-STATE LINEARIZATION OF NONLINEAR CONTROL SYSTEMS

Miroslav Halás

Institute of Control and Ingustrial Informatics
Slovak University of Technology, Faculty of Electrical Engineering and Information Technology
Ilkovičova 3, 812 19 Bratislava, Slovak Republic
Tel.: +421 2 60291458
e-mail: miroslav.halas@stuba.sk

Abstract: The input-to-state linearization of a system is usually necessary to consider whenever the input-to-output linearization of the system yields a possibly unstable cloosed-loop system. In this work a unified approach to the input-to-state linearization of nonlinear systems, both continuous- and discrete-time, is suggested. The problem is studied from a uniform standpoint. In that respect methods and tools of the so-called pseudo-linear algebra, applied to the algebraic formalism of differential forms, play a key role.

Keywords: nonlinear control systems, input-to-state linearization, unification, algebraic approach, pseudo-linear algebra, differential forms

1 INTRODUCTION

Many solutions to control problems of continuous- and discrete-time nonlinear systems show significant similarities. Usually, behind the similarities lies a mathematical abstraction that accommodates both cases. In this paper such an abstraction, called pseudo-linear algebra (Bronstein and Petkovšek, 1996; Abramov, Le and Li, 2005), is introduced to unify the algebraic formalism of differential one-forms of Conte, Moog and Perdon (2007) with an application to input-to-state linearization of nonlinear control systems, both continuous- and discrete-time. Though the differential and shift operators have remarkably different properties and calculations with differential and shift operators are based on different rules, they both accommodate into pseudo-linear algebra as special cases. So, the main contribution of the paper is unification. However, besides the conventional shift operator description of the discrete-time control systems, this approach is also valid for discrete-time systems described via the difference operator that provides a smooth transition from the sampled data algorithms to their continuous-time counterparts. Another contribution of the paper is extension. Pseudo-linear algebra covers not just classical continuous and discrete-time cases, but also q-shift and q-difference operators, where the state at time t determines the state at time qt with $q \in \mathbf{R}$. Applications employing other types of operators in the control theory can be found in (Guo, 2005; Anderson and Kadirkamanathan, 2007). For instance, the q-shift operator can be in some cases used to model discrete-time systems with the varying sampling period. The sampling period may be, for instance, changed based on the availabilty of resources (Albertos, 2007). Note that q-shift operators can be also found in quantum mechanics.

A preliminary discussion concerning the unification of the study of nonlinear control systems was given in (Halás and Kotta, 2007).

2 ALGEBRAIC SETTING

In this section an algebraic setting for dealing with properties of different nonlinear control systems is constructed.

2.1 Pseudo-linear algebra

Pseudo-linear algebra, also known as Ore algebra, is the study of common properties of linear ordinary differential, difference, q-difference and other types of operators (Bronstein and Petkovšek, 1996; Abramov, Le and Li, 2005). Such operators are expressed in terms of so-called skew polynomials. We start with introducing the notion of a pseudo-derivation.

Let K be a field and $\sigma : K \to K$ an automorphism of K. A map $\delta : K \to K$ which satisfies

$$\begin{aligned} \delta(a+b) &= \delta(a) + \delta(b) \\ \delta(ab) &= \sigma(a)\delta(b) + \delta(a)b \end{aligned} \qquad (1)$$

is called a pseudo-derivation (or a σ-derivation).

Obviously, $\sigma(a/b) = \sigma(a)/\sigma(b)$ and $\delta(a/b) = (b\delta(a) - a\delta(b))/(\sigma(b)b)$ for $a, b \in K$ with $b \neq 0$.

The notion of a pseudo-derivation unifies the notions of a derivation and various types of difference operators (Bronstein and Petkovšek, 1996; Abramov, Le and Li, 2005). The following three examples exhaust all the possible pseudo-derivations over a field K:

- if $\sigma = 1_K$ then (1) is just a rule for a derivation on K and the pair (K, δ) is called a differential field,
- for any automorphism σ and any $\alpha \in K$, the map $\delta_\alpha = \alpha(\sigma - 1_K)$ is a pseudo-derivation. By choosing appropriate $\alpha \in K$ and σ we can specify various types of difference operators. For instance, if $\alpha = 1$ and σ is the automorphism over K which takes time t to $t+1$ then $\Delta = \delta_1 = \sigma - 1_K$ represents the usual difference operator over K. One can easily check that

$$\Delta(ab) = \sigma(a)\Delta(b) + \Delta(a)b = \sigma(a)(\sigma(b) - b) + (\sigma(a) - a)b = \sigma(ab) - ab$$

- for any automorphism σ the zero map $\delta = 0$ is a pseudo-derivation. The pair (K, σ) is called a difference field in that case. If necessary, one can equivalently reduce it to the previous case, i.e. a nontrivial pseudo-derivation, by associating the difference operator $\Delta = \delta_1 = \sigma - 1_K$.

Table 1: Basic types of operators

Case	σ	δ	*f(t)*
differential	1_K	$\frac{\mathrm{d}}{\mathrm{d}t}$	$\frac{\mathrm{d}f(t)}{\mathrm{d}t}$
shift	$t \to t+1$	0	$f(t+1)$
difference	$t \to t+1$	$\sigma - 1_K$	$f(t+1) - f(t)$
q-shift	$t \to qt$	0	$f(qt)$
q-difference	$t \to qt$	$\sigma - 1_K$	$f(qt) - f(t)$

Different types of operators can be now simply specified by choosing appropriate σ and δ. Some basic types of operators are listed in Table 1. Of course, if needed, one can define more exotic operators, as for instance Mahlerian which takes t to t^p.

Definition 1. A σ-differential field is a triple (K,σ,δ) where K is a field, σ is an automorphism of K and δ is a pseudo-derivation. ■

Thus, the σ-differential field unifies the notions of a differential field and a difference field. It will be the starting point for constructions used in charactering theoretic properties of various nonlinear control systems.

Definition 2. Let V be a vector space over a field K. A map $\theta : V \to V$ is called pseudo-linear if

$$\begin{aligned} \theta(u+v) &= \theta(u) + \theta(v) \\ \theta(au) &= \sigma(a)\theta(u) + \delta(a)u \end{aligned} \tag{2}$$

for any $a \in K$ and $u, v \in V$. ■

In comparison with a pseudo-derivation which unifies the notions of a derivation and various types of difference operators, a pseudo-linear map unifies in addition the notion of a shift operator. Note that any field K is a vector space itself. Hence, we can consider pseudo-linear maps over K assuming that (2) holds for any $a, u, v \in K$:

- if $\sigma = 1_K$ then (2) is a derivation. The pseudo-linear operator being $\theta = \delta$,

$$\delta(au) = a\delta(u) + \delta(a)u$$

- for any automorphism σ and any $\alpha \in K$, the map $\Delta = \delta_\alpha = \alpha(\sigma - 1_K)$ is a pseudo-derivation and the pseudo-linear operator being $\theta = \Delta$,

$$\Delta(au) = \sigma(a)\Delta(u) + \Delta(a)u$$

- if $\delta = 0$ then (2) is a shift operator. Now, the pseudo-linear operator being $\theta = \sigma$,

$$\sigma(au) = \sigma(a)\sigma(u)$$

Thus, pseudo-linear maps allow us to handle differential, difference and shift structures from a uniform standpoint. However, difference and shift operators represent two alternative ways of describing discrete-time dynamics. The shift-operator based model can always be transformed into the difference-operator domain and vice versa. In particular, this can be done in general and the problem can thus always be reduced either to a purely differential case or to a purely shift case.

Remark also that the shift operators are, in general, easier to handle than the difference operators. On the other hand, the difference-operator based models are closely linked to the continuous-time models in terms of both parameters and structure and improve the numerical properties of structure detection algorithms (Anderson and Kadirkamanathan, 2007).

3 CONTROL SYSTEMS

In this section we define a wide class of nonlinear control systems. For the sake of simplicity, for $x(t)$ we write just x. In what follows, the symbol $x^{\langle 1 \rangle}$ stands for a pseudo-linear operator: $x^{\langle 1 \rangle} = \theta(x)$. It can be a derivation, $x^{\langle 1 \rangle} = \dot{x}$, that corresponds to the continuous-time case (see also Table 1), a shift, $x^{\langle 1 \rangle} = \sigma(x)$, or a difference, $x^{\langle 1 \rangle} = \alpha(\sigma(x) - x)$ with $\alpha \in \mathbf{R}$, that correspond to two alternative discrete-time cases.

The nonlinear control systems considered in this paper are objects of the form

$$\begin{aligned} x^{\langle 1\rangle} &= f(x,u) \\ y &= g(x) \end{aligned} \qquad (3)$$

where the entries of f and g are meromorphic functions, which we think of as elements of the quotient field of the ring of analytic functions, and $x \in \mathbf{R}^n$, $u \in \mathbf{R}^m$ and $y \in \mathbf{R}^p$ denote respectively state, input and output to the system.

In constructing an appropriate algebraic setting we follow the lines given in (Conte, Moog and Perdon, 2007), or (Aranda-Bricaire, Kotta and Moog, 1996) for the discrete-time counterpart. Let K denote now the field of meromorphic functions of variables $\{x, u^{\langle k\rangle}; k \geq 0\}$. We assume that system (3) is generically submersive, i.e.

$$\operatorname{rank}_K \frac{\partial \sigma(x)}{\partial (x,u)} = n$$

Under this assumption, σ is an automorphism of K and there exists, up to an isomorphism, a unique difference field K^* called the inversive closure of K (Cohn, 1965). Here we assume that the inversive closure is given and by abuse of notation we use the same symbol K for both. An explicit construction follows the same lines as in (Aranda-Bricaire, Kotta and Moog, 1996) and (Bartosiewicz, Kotta, Pawluszewicz and Wyrwas, 2007) for the shift operator and the difference operator based cases, respectively. Note that if $\sigma = 1_K$, $K^* = K$.

Let δ be a pseudo-derivation defined on K. The field K can be endowed with a σ-differential structure, determined by system equations (3). In this case the triple (K, σ, δ) forms a σ-differential field. We define a pseudo-linear operator θ, acting on K, separately for derivation, shift and difference operators.

First, if $\sigma = 1_K$ and $\delta = \mathrm{d}/\mathrm{d}t$, a pseudo-linear operator $\theta = \delta$ and

$$\delta\varphi(\{x, u^{\langle k\rangle}\}) = \sum_{i=1}^{n} \frac{\partial \varphi}{\partial x_i}\delta x_i + \sum_{\substack{j=1 \\ k\geq 0}}^{m} \frac{\partial \varphi}{\partial u_j^{\langle k\rangle}} \delta u_j^{\langle k\rangle}$$

where $\delta x_i = f_i(x,u)$ and $\delta u_j^{\langle k\rangle} = u_j^{\langle k+1\rangle}$.

Second, if $\delta = 0$, a pseudo-linear operator $\theta = \sigma$ and

$$\sigma\varphi(\{x, u^{\langle k\rangle}\}) = \varphi(\{\sigma x, \sigma u^{\langle k\rangle}\})$$

where $\sigma x = f(x,u)$ and $\sigma u^{\langle k\rangle} = u^{\langle k+1\rangle}$.

Finally, if $\delta = \alpha(\sigma - 1_K) := \Delta$ with $\alpha \in \mathbf{R}$, then a pseudo-linear operator $\theta = \Delta$ and

$$\Delta\varphi(\{x, u^{\langle k\rangle}\}) = \alpha\left[\varphi(\{\sigma x, \sigma u^{\langle k\rangle}\}) - \varphi(\{x, u^{\langle k\rangle}\})\right]$$

where $\sigma = \dfrac{1}{\alpha}\Delta + 1_K$, $\Delta x = f(x,u)$ and $\Delta u^{\langle k\rangle} = u^{\langle k+1\rangle}$.

Next, define the vector space E of one-forms spanned over K by differentials of elements of K; that is

$$E = \operatorname{span}_K \{\mathrm{d}\xi; \xi \in K\}$$

Any element $v \in E$ is a vector of the form $v = \sum_i c_i \xi_i$ where only a finite number of c_i's are nonzero elements in K. We say that $v \in E$ is exact if $v = \varphi$ for some $\varphi \in K$ and φ is then usually referred to as a differential of φ.

The vector space E can also be endowed with the σ-differential structure determined by the system equations (3). However, this time there is no need to define actions separately. Each pseudo-linear operator $\theta : K \to K$ induces a pseudo-linear operator $\theta : E \to E$ as follows

$$\theta(v) = v^{\langle 1 \rangle} = \sum_i \left[\sigma(c_i)(\theta(\xi_i)) + \delta(c_i)\xi_i\right] \tag{4}$$

The operator θ commutes with the operator d, $\theta(\mathrm{d}\varphi) = \mathrm{d}(\theta(\varphi))$, and reduces to the well-known rules

$$\delta v = \sum_i \left[c_i(\delta\xi_i) + \delta(c_i)\xi_i\right]$$

and

$$\sigma v = \sum_i \sigma(c_i)(\sigma\xi_i)$$

for the special cases of continuous-time systems ($\sigma = 1_K$, $\theta = \delta = \mathrm{d}/\mathrm{d}t$) and distrecte-time systems ($\delta = 0$, $\theta = \sigma$), respectively.

We briefly demonstrate some basic computations in (K, σ, δ) and E by the following example.

Example 1. Consider the nonlinear q-difference system with q = 2.

$$x(2t) - x(t) = x(t)u(t) \qquad x^{\langle 1 \rangle} = \Delta x = xu$$
$$y(t) = x(t) \qquad y = x$$

The corresponding σ-differential field is (K, σ, δ) where σ takes t to $2t$ and $\Delta = \sigma - 1_K$, the pseudo-linear operator being $\theta = \Delta$. (K, σ, δ) has the σ-differential structure given by the system equations. If, for instance, $\varphi = x^2$, then

$$\varphi^{\langle 1 \rangle} = \Delta\varphi = (\sigma x)^2 - x^2 = (\Delta x + x)^2 - x^2 = (xu + x)^2 - x^2$$

Also E has σ-differential structure given by the system equations. If, for instance, $v = 2ux$, then $v^{\langle 1 \rangle} = \Delta v = 2\sigma(u)\sigma(x) - 2ux = 2(\Delta u + u)(\Delta x + x) - 2ux = 2(u^{\langle 1 \rangle} + u)(xu + x) - 2ux$. Or directly by (4), $v^{\langle 1 \rangle} = 2\sigma(u)(\Delta x) + 2\Delta(u)x = 2(u^{\langle 1 \rangle} + u)(xu) + 2u^{\langle 1 \rangle}x$ which yields the same result. ■

4 MODELLING, ANALYSIS AND SYNTHESIS PROBLEMS

Within the algebraic formalism based on differential one-forms the necessary and sufficient solvability conditions as well as the solutions have been obtained for different fundamental modelling, analysis and synthesis problems (Aranda-Bricaire, Kotta and Moog, 1996; Conte, Moog and Perdon, 2007; Kotta, Zinober and Liu, 2001). These solutions are intrinsic and coordinate-free. A significant example is offered by the way in which the notion of accessibility, static state feedback linearization and state space realizability are dealt with. In all cases, a single tool, based on the notion of relative degree, gives the key for carrying on a deep analysis and for characterizing relevant dynamical properties. The formalism is based on classification of differential forms, related to a control system. Within this algebraic formalism the sequence of subspaces is associated to a control system which contains a lot of information on structural properties of the system.

Let $X = \mathrm{span}_K\{\mathrm{d}x\}$ and θ be a pseudo-linear map over E. For the sake of simplicity, let $\theta^k(v) = v^{\langle k \rangle}$ for any $v \in E$.

Definition 3. The relative degree of a one-form $\omega \in X$ is given by

$$r = \min\{k \in \mathbf{N}; \operatorname{span}_K \{\omega, \cdots, \omega^{\langle k \rangle}\} \not\subset X\}$$ ■

Now the sequence of subspaces is defined by induction as follows:

$$\begin{aligned} H_1 &= \operatorname{span}_K \{\mathrm{d}x\} \\ H_j &= \operatorname{span}_K \{\omega \in H_{j-1}; \omega^{\langle 1 \rangle} \in H_{j-1}\} \end{aligned}$$

Clearly, H_k is the subspace of one-forms which have relative degree greater than or equal to k. Furthermore, there exists an integer $k^* > 0$ such that

$$H_1 \supset \ldots \supset H_{k^*} \supset H_{k^*+1} = H_{k^*+2} = \ldots = H_\infty$$

In what follows we mainly provide a unification of the existing results for continuous- and discrete-time cases plus an extension of these results to the difference operator domain using the notion of a pseudo-linear operator. Moreover, this approach covers also the less frequent cases of q-shift and q-difference based system descriptions.

4.1 Input-to-state linearization

Static state feedback linearization is a basic control prolbem that allows to linearize a nonlinear system and thereby a use of linear methods. The input-to-state linearization of a system is usually necessary to consider whenever the input-to-output linearization of the system yields a cloosed-loop system that contains a possibly unstable unobservable subsystem.

Using the idea of H_k filtration one can easily carry over the results valid for the static state feedback linearization problem (Conte, Moog and Perdon, 2007; Aranda-Bricaire, Kotta and Moog, 1996). The aim is to find, if possible, a regular static state feedback $u = \varphi(x, v)$ and a state transformation $\xi = \phi(x)$, where φ and ϕ are meromorphic and $\operatorname{rank}_K \frac{\partial \varphi}{\partial v} = m$, such that, in the new coordinates, the closed-loop system reads

$$\xi^{\langle 1 \rangle} = A\xi + Bv$$

where the pair (A, B) is controllable (in Brunovsky canonical form).

Theorem 1. System (3) is linearizable by regular static state feedback if and olny if

- $H_\infty = 0$,
- H_i are completely integrable for any $i \geq 1$. ■

The proofs follow the same line as in (Conte, Moog and Perdon, 2007; Aranda-Bricaire, Kotta and Moog, 1996) using the notion of a pseudo-linear map.

Example 2. Consider the nonlinear system

$$\begin{aligned} x_1(t+1) - x_1(t) &= -x_1(t) + x_2^2(t)u(t) \\ x_2(t+1) - x_2(t) &= -x_2(t) + x_2(t)u(t) \end{aligned}$$

which can be modeled either over the σ-differential field (K, σ, Δ) with $\sigma : t \to t+1$, $\Delta = \sigma - 1_K$ and the pseudo-linear operator being $\theta = \Delta$, i.e. a difference operator, or over the σ-differential field $(K, \sigma, 0)$ with $\sigma : t \to t+1$ and the pseudo-linear operator being $\theta = \sigma$, i.e. a shift operator.

If the system is modeled over (K, σ, Δ), that is

$$\begin{aligned} \Delta x_1 &= -x_1 + x_2^2 u \\ \Delta x_2 &= -x_2 + x_2 u \end{aligned} \tag{5}$$

then the filtration can be computed as

$$
\begin{aligned}
H_1 &= \operatorname{span}_K\{\mathrm{d}x_1, \mathrm{d}x_2\} \\
H_2 &= \operatorname{span}_K\{\omega \in H_1; \Delta\omega \in H_1\}
\end{aligned}
$$

If we consider $\omega = \alpha_1 x_1 + \alpha_2 x_2$, where $\alpha_1, \alpha_2 \in K$, then

$$
\begin{aligned}
\Delta\omega &= \Delta(\alpha_1 x_1) + \Delta(\alpha_2 x_2) \\
&= \sigma(\alpha_1)(\Delta x_1) + \Delta\alpha_1 x_1 + \sigma(\alpha_2)(\Delta x_2) + \Delta\alpha_2 x_2 \\
&= \sigma(\alpha_1)(-x_1 + 2x_2 u x_2 + x_2^2 u) + \Delta\alpha_1 x_1 + \sigma(\alpha_2)(-x_2 + u x_2 + x_2 u) + \Delta\alpha_2 x_2
\end{aligned}
$$

Now $\Delta\omega \in H_1$ implies $\sigma(\alpha_1)x_2^2 + \sigma(\alpha_2)x_2 = 0$ from which $\alpha_2 = -\alpha_1\sigma^{-1}(x_2)$. Note that the system is generically submersive and from the system equations $\sigma^{-1}(x_2) = x_1/x_2$. Thus, we get

$$
\begin{aligned}
H_2 &= \operatorname{span}_K\{\mathrm{d}x_1 - \frac{x_1}{x_2}\mathrm{d}x_2\} \\
H_3 &= 0
\end{aligned}
$$

However, notice that even if the system is modeled over $(K, \sigma, 0)$, that is

$$
\begin{aligned}
\sigma x_1 &= x_2^2 u \\
\sigma x_2 &= x_2 u
\end{aligned} \tag{6}
$$

one gets the identical accessibility filtration

$$
\begin{aligned}
H_1 &= \operatorname{span}_K\{\mathrm{d}x_1, \mathrm{d}x_2\} \\
H_2 &= \operatorname{span}_K\{\omega \in H_1; \sigma\omega \in H_1\}
\end{aligned}
$$

This time

$$
\begin{aligned}
\sigma\omega &= \sigma(\alpha_1)(\sigma x_1) + \sigma(\alpha_2)(\sigma x_2) \\
\sigma\omega &= \sigma(\alpha_1)(2x_2 u x_2 + x_2^2 u) + \sigma(\alpha_2)(u x_2 + x_2 u)
\end{aligned}
$$

which implies $\sigma(\alpha_1)x_2^2 + \sigma(\alpha_2)x_2 = 0$ and we again get $\alpha_2 = -\alpha_1\sigma^{-1}(x_2)$.

$$
\begin{aligned}
H_2 &= \operatorname{span}_K\{\mathrm{d}x_1 - \frac{x_1}{x_2}\mathrm{d}x_2\} \\
H_3 &= 0
\end{aligned}
$$

Since all the subspaces are completely integrable

$$
\begin{aligned}
H_1 &= \operatorname{span}_K\{\mathrm{d}x_1, \mathrm{d}x_2\} \\
H_2 &= \operatorname{span}_K\{\mathrm{d}x_1 - \frac{x_1}{x_2}\mathrm{d}x_2\} = \operatorname{span}_K\{\frac{1}{x_2}\mathrm{d}x_1 - \frac{x_1}{x_2^2}\mathrm{d}x_2\} = \operatorname{span}_K\{\mathrm{d}\frac{x_1}{x_2}\} \\
H_3 &= 0
\end{aligned}
$$

the linearizing output, with relative degree 2, can be chosen as $y_f = x_1/x_2$. The corresponding state transformation can be found as $\xi_1 = y_f$, $\xi_2 = y_f^{\langle 1\rangle}$ and the linearizing feedback from $v = y_f^{\langle 2\rangle}$.

However, note that if the system is modeled over (K, σ, Δ), (5), then

$$
\begin{aligned}
y_f &= \frac{x_1}{x_2} \\
\Delta y_f &= x_2 - \frac{x_1}{x_2} \\
\Delta^2 y_f &= x_2 u - 2x_2 + \frac{x_1}{x_2} = v
\end{aligned}
$$

from which

$$u = \frac{v}{x_2} + 2 - \frac{x_1}{x_2^2}$$

and $(\xi_1, \xi_2) = (x_1/x_2, x_2 - x_1/x_2)$, while if it is modeled over $(K, \sigma, 0)$, (6), then

$$
\begin{aligned}
y_f &= \frac{x_1}{x_2} \\
\sigma y_f &= x_2 \\
\sigma^2 y_f &= x_2 u = v
\end{aligned}
$$

from which

$$u = \frac{v}{x_2}$$

and $(\xi_1, \xi_2) = (x_1/x_2, x_2)$. ■

5 CONCLUSION

In this paper, a unification and an extension of the algebraic formalism of one-forms were discussed for a wide class of nonlinear control systems. In both the unification and the extension the pseudo-linear algebra played a key role. Though differential, shift and diffrence operators have remarkably different properties, they all accommodate into this mathematical abstraction as special cases. Moreover, the suggested framework includes also less conventional operators like *q*-shift and *q*-difference operators. Pseudo-linear algebra was applied to demonstrate that the earlier solutions to input-to-state linearization of continuous- and discrete-time cases can be presented in a unified manner and include the earlier solutions as the special cases.

ACKNOWLEDGEMENT

This work was supported by Slovak scientific grant agency VEGA grants No. VG-1/0656/09 and VG-1/0369/10.

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