PREDICTIVE PID CONTROL

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Abstract: Model predictive control has proved its effectiveness in many industrial applications. Predictive algorithms are available in various commercial control packages but their implementation costs could be considerable. Control of industrial processes is often realized using small industrial controllers that offer only simple control structures, like PID control loop. The aim of the paper is to derive the conditions for the equivalence of the generalized predictive control (GPC) and the PID control implemented on simple programmable logical controller. The performances of the GPC and PID control schemes are shown to be identical by simulation and an application of the predictive PID to a laboratory plant is presented.

Keywords: predictive control, PID controller, PLC

1 INTRODUCTION

Model predictive control (MPC) refers to a family of advanced control methods which make explicit use of a model of the process to predict the future process behavior and to calculate a future control sequence minimizing an objective function (Camacho and Bordons, 2004). The objective function is formulated as a combination of the set-point tracking performance and control effort. As predictive control belongs to the category of the open-loop optimization techniques, its implementation is based on the receding horizon strategy, i.e. only the first control signal of the future sequence is used at each sampling instant and the calculation is repeated in the next sampling time. This allows to incorporate a feedback into the control loop and to improve the control performances in the presence of disturbances and unmodelled dynamics.

First predictive control algorithms have been proposed at the end of the 1970s and quickly became popular and developed considerably over the last three decades both within the research control community and in industry. Their popularity is mainly due to the fact, that they can be used to control a great variety of processes including time-delayed systems or nonminimum phase or the unstable ones. The multivariable case can easily be dealt with as well. Another important feature is that the constraints can be systematically incorporated into the design procedure, which can influence the resulting control system performances and the process operation safety. MPC technology can now be found in a wide variety of application areas including petrochemical, chemical, food processing, automotive and aerospace industries.

Despite the wide development of advanced control methods, the PID controllers are still commonly used in industry for its structural simplicity and design rules of thumb. The PID control function can be found in many medium-size programmable logical controllers (PLC) and all large PLC. Such functions can be used directly by entering the parameters for given PID controller structure.

The aim of this paper is to employ the classical PID algorithm on industrial computers to do advanced control without the necessity of the specialized software. More specifically, the PID control structure implemented on SIMATIC S7-200 programmable logical controller has been considered. The PID controller parameters are obtained by equating the discrete PID control law with the classical pole-placement control structure of generalized predictive control (GPC) given some conditions on the orders of the polynomials involved in the GPC control structure (Miller *et al*, 1999). As these orders depend on the process model, the process model order is restricted to a maximum of two, the first order model results in a PI controller while a second order plant yields a PID structure. On the other hand, there is no restriction on the choice of GPC tuning parameters so that the advantages of model predictive control can fully be exploited.

Performance of the predictive PID scheme is shown to be equivalent to GPC by means of simulation. An application of the predictive PID algorithm for the control of a laboratory plant is also presented.

2 GENERALIZED PREDICTIVE CONTROL

Generalized predictive control (GPC) developed in (Clarke *et al*, 1987) belongs to the most popular predictive algorithms based on the parametric plant model. It can handle various control problems for a wide range of plants, its implementation is relatively simple and due to several design parameters it can be tuned to specific applications.

Consider that the operation of the single-input single-output (SISO) plant around the particular setpoint can be described by the following CARIMA model

$$A(z^{-1})y(t) = B(z^{-1})u(t-d-1) + v(t) \tag{1}$$

$$D(z^{-1})v(t) = C(z^{-1})\xi(t) \tag{2}$$

with

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + \ldots + a_{na} z^{-na} \qquad & B(z^{-1}) &= b_0 + b_1 z^{-1} + \ldots + b_{nb} z^{-nb} \\ C(z^{-1}) &= 1 + c_1 z^{-1} + \ldots + c_{nc} z^{-nc} \qquad & D(z^{-1}) &= 1 - z^{-1} \end{aligned} \tag{3}$$

where $u(t)$ is the control variable, $y(t)$ the measured plant output, d denotes the minimum plant model time-delay in sampling periods, $v(t)$ represents the external disturbances and $\xi(t)$ is the random variable with zero mean value and finite variance. For simplicity in the following the $C(z^{-1})$ polynomial is chosen to be 1.

The GPC control objective is to compute the future control sequence in such a way that the future plant output is driven close to the prescribed reference trajectory; this is accomplished by minimizing the following cost function

$$J(t, ph, ch, sh, \rho) = E\left\{ \sum_{j=sh}^{ph} \left(\hat{y}(t+j/t) - y^*(t+j) \right)^2 + \rho \left(D(z^{-1}) u(t+j-sh) \right)^2 \right\} \tag{4}$$

subject to:

$$D(z^{-1})u(t+i) = 0 \ \text{ for } \ ch \le i \le ph \tag{5}$$

where sh, ph and ch are positive scalars defining the starting horizon, prediction horizon and control horizon, ρ is a nonnegative control weighting scalar. $\hat{y}(t+j/t)$ denotes the j-step ahead prediction of $y(t)$ based on the data available up to time t and $y^*(t+j)$ is the future reference trajectory, that can be generated as an output of the reference model of the form

$$A_m\left(z^{-1}\right)y^*\left(t+d+1\right)=B_m\left(z^{-1}\right)u^*\left(t\right) \tag{6}$$

The j-step ahead predictor is expressed as follows

$$\begin{aligned}\hat{y}\left(t+j/t\right)&=G_{j-d}\left(z^{-1}\right)D(z^{-1})u\left(t+j-d-1\right)+y_0\left(t+j/t\right)\\ y_0\left(t+j/t\right)&=H_{j-d}(z^{-1})D(z^{-1})u\left(t-1\right)+F_j(z^{-1})y\left(t\right)\end{aligned} \tag{3.11}$$

where the polynomials $F_j\left(z^{-1}\right), G_{j-d}\left(z^{-1}\right), H_{j-d}\left(z^{-1}\right)$ are solutions of the following Diophantine equations

$$1=A(z^{-1})D(z^{-1})E_j(z^{-1})+z^{-j}F_j(z^{-1}) \tag{7}$$

$$E_j(z^{-1})B(z^{-1})=G_{j-d}(z^{-1})+z^{-j+d}H_{j-d}(z^{-1}) \tag{8}$$

The cost function (4)-(5) may be rewritten in the suitable vector form

$$\begin{aligned}J\left(t,ph,ch,sh,\rho\right)=&\left(G_1U\left(t+ch-1\right)+Y_0\left(t\right)-Y^*\left(t+ph\right)\right)^T\left(G_1U\left(t+ch-1\right)+Y_0\left(t\right)-Y^*\left(t+ph\right)\right)\\&+\rho U\left(t+ch-1\right)^T U\left(t+ch-1\right)\end{aligned} \tag{9}$$

where $Y^*\left(t+ph\right)=\left[y^*\left(t+d+1\right),\ldots,y^*\left(t+ph\right)\right]^T$

$$Y_0\left(t\right)=\left[y_0\left(t+d+1/t\right),\ldots,y_0\left(t+ph/t\right)\right]^T$$

$$U\left(t+ch-1\right)=\left[D\left(z^{-1}\right)u\left(t\right),\ldots,D\left(z^{-1}\right)u\left(t+ch-1\right)\right]^T$$

$$G_1=\begin{bmatrix} g_{sh-d-1} & \cdots & g_0 & 0 & 0 & 0\\ g_{sh-d} & \cdots & \cdots & g_0 & 0 & 0\\ \vdots & \cdots & \cdots & \cdots & \ddots & \vdots\\ g_{ch-1} & \cdots & \cdots & \cdots & \cdots & g_0\\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots\\ g_{ph-d-1} & \cdots & \cdots & \cdots & \cdots & g_{ph-ch-d}\end{bmatrix} \tag{10}$$

The vector that minimizes the cost function (9) is given by

$$U\left(t+ch-1\right)=-\left[G_1^TG_1+\rho I_{ch}\right]^{-1}G_1^T\left(Y_0\left(t\right)-Y^*\left(t+ph\right)\right) \tag{11}$$

The GPC control law is implemented in a receding horizon sense, which means that only the first component of the optimal vector $U\left(t+ch-1\right)$ is taken into account and the optimization process is repeated in the next sampling period

$$D\left(z^{-1}\right)u\left(t\right)=-\sum_{j=sh}^{ph}\gamma_j\left(y_0\left(t+j/t\right)-y^*\left(t+j\right)\right) \tag{12}$$

where the coefficients γ_j for $j=sh,\ldots,ph$ are the first line components of the matrix $\left[G_1^TG_1+\rho I_{ch}\right]^{-1}G_1^T$.

The control law (12) may also be implemented using a standard pole-placement control structure (shown in Fig. 1)

$$S\left(z^{-1}\right)D\left(z^{-1}\right)u\left(t\right)+R\left(z^{-1}\right)y\left(t\right)=T\left(z^{-1}\right)y^*\left(t\right) \tag{13}$$

where

$$R(z^{-1}) = \sum_{j=sh}^{ph} \gamma_j F_j(z^{-1}) = r_0 + r_1 z^{-1} + \ldots + r_{nr} z^{-nr} \tag{14}$$

$$S(z^{-1}) = 1 + \sum_{j=sh}^{ph} \gamma_j z^{-1} H_{j-d}(z^{-1}) = 1 + s_1 z^{-1} + \ldots + s_{ns} z^{-ns} \tag{15}$$

$$T(z^{-1}) = \sum_{j=sh}^{ph} \gamma_j z^{-ph+j} = t_0 + t_1 z^{-1} + \ldots + t_{nt} z^{-nt} \tag{16}$$

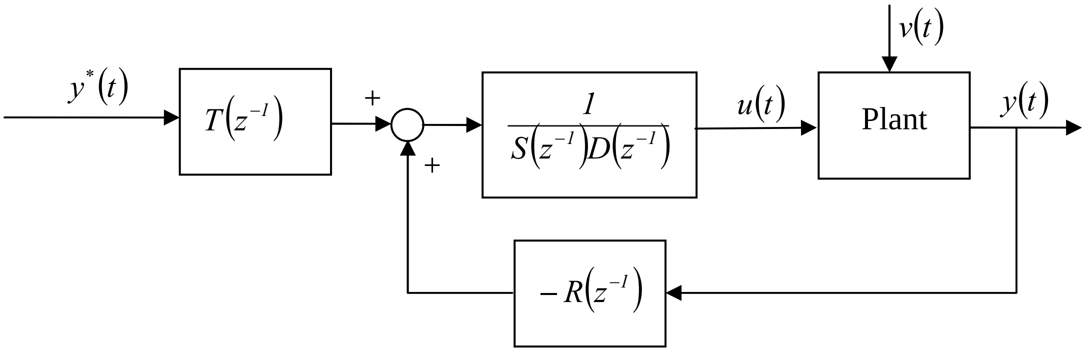


Figure 1 Standard pole-placement control structure

3 PID ALGORITHM IMPLEMENTED ON PLC

Advanced predictive algorithms take part of several commercial control packages (Quin and Badgwell, 2003). However, the automation of many industrial processes is done through the use of small computers called the programmable logic controllers (PLCs), where hardware and software are specifically adapted to industrial environment. This type of controllers usually offers only simple control structures, such as on-off control or PID control loops.

The Siemens SIMATIC S7-200 series is a line of micro-programmable logic controllers that can control a variety of small applications. The PID control law implemented on this PLC is of the form (S7-200 PLC System manual, 2005)

$$u(t) = M_P(t) + M_I(t) + M_D(t) \tag{17}$$

with

$M_P(t) = K_c e(t)$ is the proportional term (18)

$M_I(t) = K_c \dfrac{T_s}{T_i} e(t) + M_X$ is the integral term (19)

$M_D(t) = K_c \dfrac{T_d}{T_s}\left(e(t) - e(t-1)\right)$ is the differential term

where $e(t) = w(t) - y(t)$ is the control error and $w(t)$ denotes the value of setpoint. The bias M_X is the running sum of all previous values of the integral term. T_s represents the sample time. K_c, T_i and T_d are the loop gain, derivative time constant and integral time constant, respectively, that have to be specified.

To avoid step changes or bumps in the output due to derivative action on setpoint changes, this equation is modified to assume that the setpoint is a constant ($w(t) = w(t-1)$). This results in the calculation of the change in the process variable instead of the change in the error

$$M_D(t) = K_c \frac{T_d}{T_s}\left(w(t) - y(t) - w(t) + y(t-1)\right) = K_c \frac{T_d}{T_s}\left(y(t-1) - y(t)\right) \qquad (20)$$

The resulting PID control structure is depicted in Figure 2.

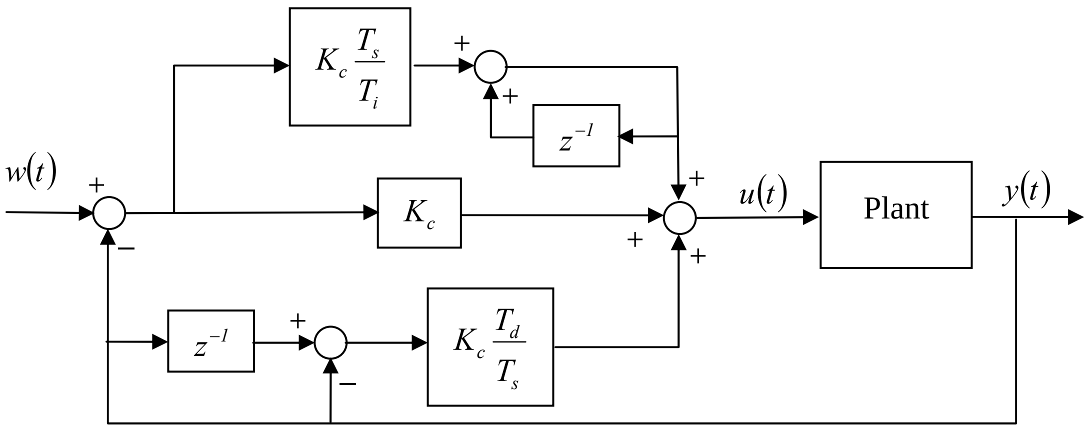


Figure 2 PID control structure

4 PREDICTIVE PID CONTROL

The aim is to implement the predictive control algorithm by means of the above described PID control structure. Let us compare the pole placement control law

$$S(z^{-1})D(z^{-1})u(t) = T(z^{-1})y^*(t) - R(z^{-1})y(t) \qquad (21)$$

to the incremental form of the PID control law (17) – (20)

$$\begin{aligned} D(z^{1})u(t) &= u(t) - u(t-1) = \\ &= \left(K_c + K_c\frac{T_s}{T_i}\right)w(t) - K_c w(t-1) + \left(-K_c - K_c\frac{T_s}{T_i} - K_c\frac{T_d}{T_s}\right)y(t) + \\ &\quad + \left(K_c + 2K_c\frac{T_d}{T_s}\right)y(t-1) - K_c\frac{T_d}{T_s}y(t-2) = \\ &= P_w(z^{-1})w(t) - P_y(z^{-1})y(t) \end{aligned} \qquad (22)$$

where

$$P_w(z^{-1}) = \left(K_c + K_c\frac{T_s}{T_i}\right) - K_c z^{-1} \qquad (23)$$

$$P_y(z^{-1}) = \left(K_c + K_c\frac{T_s}{T_i} + K_c\frac{T_d}{T_s}\right) + \left(-K_c - 2K_c\frac{T_d}{T_s}\right)z^{-1} + K_c\frac{T_d}{T_s}z^{-2} \qquad (24)$$

and in case of PI control law

$$P_y\left(z^{-1}\right)=\left(K_c+K_c\frac{T_s}{T_i}\right)-K_c z^{-1} \tag{25}$$

Assuming that $w(t)=y^*(t)$ it yields that

$$S\left(z^{-1}\right)=1 \tag{26}$$

$$R\left(z^{-1}\right)=P_y\left(z^{-1}\right) \tag{27}$$

$$T\left(z^{-1}\right)=P_w\left(z^{-1}\right) \tag{28}$$

i.e. the GPC control law is equivalent to the PID control law if the $T\left(z^{-1}\right)$ polynomial is of the first order and the $R\left(z^{-1}\right)$ polynomial is of the second order. In case of PI control law $R\left(z^{-1}\right)$ has to be the first order polynomial. These conditions can be satisfied by the proper choice of the plant model structure (1), namely the second order plant model ($na=2, nb=0$) for PID control and the first order plant model ($na=1, nb=0$) for PI control, respectively.

The PID tuning constants K_c, T_i and T_d can be derived from equation (27)

$$K_c=-r_1-2r_2 \tag{29}$$

$$T_i=-\frac{r_1+2r_2}{r_0+r_1+r_2}T_s \tag{30}$$

$$T_d=\frac{-r_2}{r_1+2r_2}T_s \tag{31}$$

As it can be seen from the equations (29) – (31), the PID tuning constants depend only on the parameters of the $R\left(z^{-1}\right)$ polynomial. According to (14), the calculation of these parameters necessitates the knowledge of the G_1 matrix and $F_j\left(z^{-1}\right)$ $j=sh,\dots ph$ polynomials.

The coefficients of G_1 matrix (10) can be obtained from the samples of plant step response, i.e. for the unit input step

$$\{u(0),u(1),u(2),\dots\}=\{1,1,1,\dots\} \tag{32}$$

the plant output signal is of the form

$$\{y(0),y(1),y(2),y(3),\dots\}=\{0,g_0,g_1,g_2,\dots\} \tag{33}$$

The $F_j\left(z^{-1}\right)$ $j=sh,\dots ph$ polynomials are solutions of the Diophantine equations (7) with

$$\begin{aligned}E_j(z^{-1})&=e_0+e_1z^{-1}+\dots+e_{j-1}z^{-j+1}\\F_j(z^{-1})&=f_0^j+f_1^jz^{-1}+\dots+f_{na}^jz^{-na}\end{aligned} \tag{34}$$

and they can be calculated recursively as follows

$$E_1\left(z^{-1}\right)=1 \tag{35}$$

$$F_1\left(z^{-1}\right)=z\left(1-A\left(z^{-1}\right)D\left(z^{-1}\right)\right) \tag{36}$$

$$\begin{aligned}E_j\left(z^{-1}\right)&=E_{j-1}\left(z^{-1}\right)+f_0^{j-1}z^{-j+1}\\F_j\left(z^{-1}\right)&=z^j\left(1-E_j\left(z^{-1}\right)A\left(z^{-1}\right)D\left(z^{-1}\right)\right)\end{aligned}\quad \text{for}\quad j=2,\dots,ph \tag{37}$$

In order to compare the performances of the proposed PID control loop to those of the GPC control scheme a simulation of the second order plant

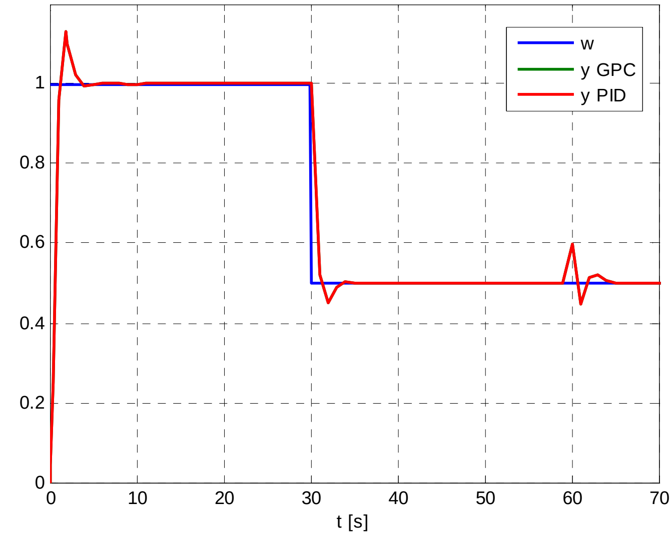
$$G(s)=\frac{1.308s+2.702}{s^2+0.5108s+0.6756}$$

has been performed with the sample time $T_s=1s$. At the time 60 s a step disturbance of 0.1 has been added to the plant output. The control design parameters are summarized in Table 1.

Table 1: Parameter values

GPC	ch=2	ph=15	ρ=1
PID	K_c=0,2102	T_i=0,7777	T_d=1,3241

As it can be seen from Fig. 3 the plant output and the control signal plots for both control schemes are identical.



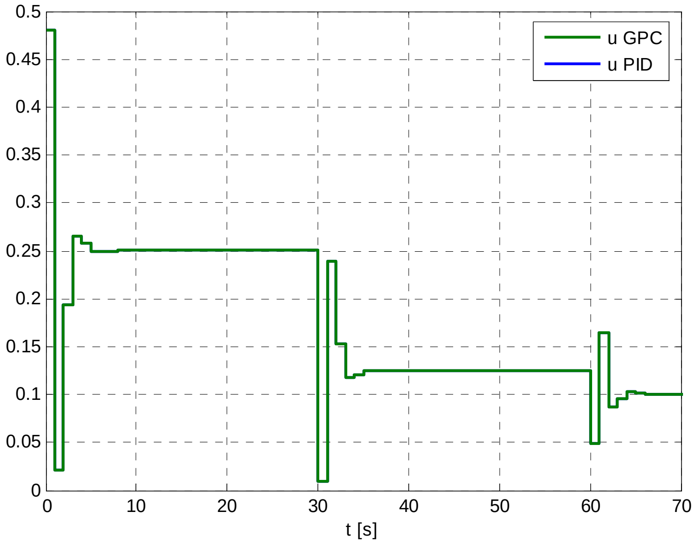


Figure 3 Comparison of GPC and PID control performances

5 EXPERIMENTAL EVALUATION

In order to verify the proposed predictive PID control design the control of simple cylindrical laboratory tank using the Simatic S7-200 has been realized. Because the GPC synthesis is based on the plant model transfer function, it is necessary to identify the ARX model of this nonlinear system around the operating point using the least-squares method. The system input is the inflow valve opening and the system output is tank level. The tank has also the outflow valve which has been used to generate a disturbance. In Figure 4 the time response of the real system is compared to the identified model time response round the operating point. The identified model transfer function is as follows:

$$G\left(z^{-1}\right)=\frac{0.01122z^{-1}}{1-1.207z^{-1}+0.2146z^{-2}}, \quad T_{VZ}=1s \tag{38}$$

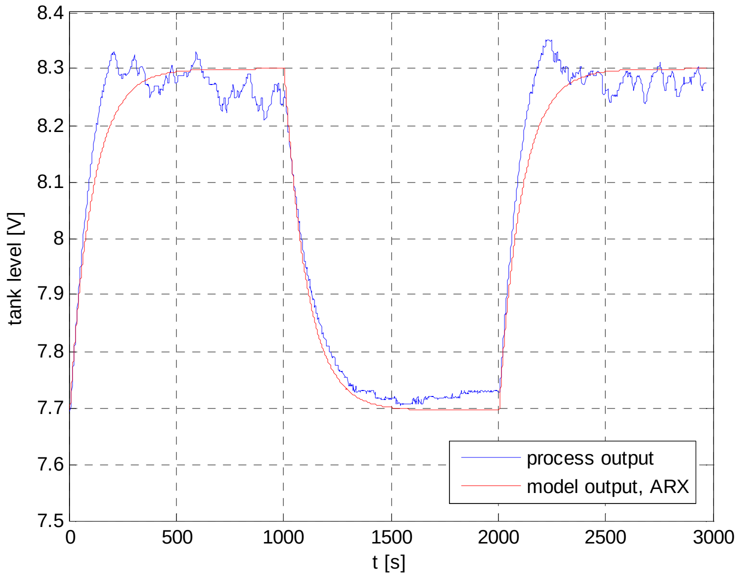


Figure 4 System identification

Table 2: Parameter values

GPC	ch=1	ph=10	ρ=1
PID	K_c=4,6121	T_i=6,7646	T_d=0,2654

Based on the plant model (38) the predictive PID controller has been designed according to the section 4. Table 2 summarizes the control design parameters and the resulting PID constants.

Time response of tank level and desired value are in Figure 5. The time response of the manipulated variable and disturbance has also been monitored (Figure 6).

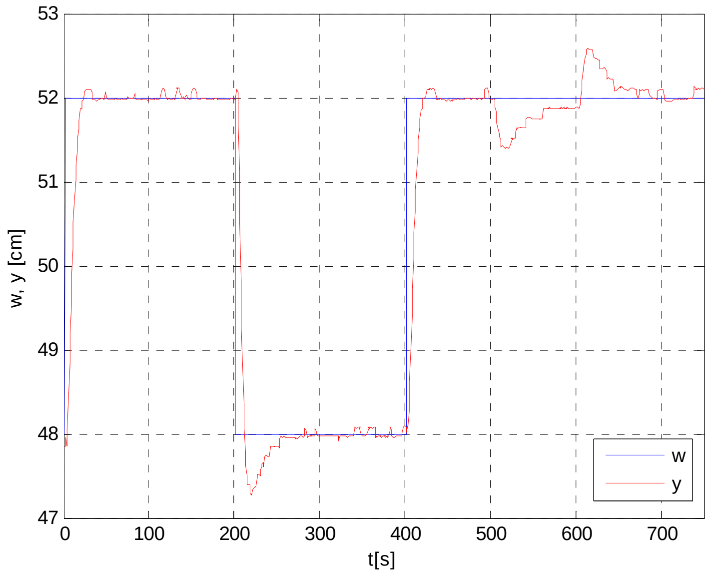


Figure 5 Time response of tank level

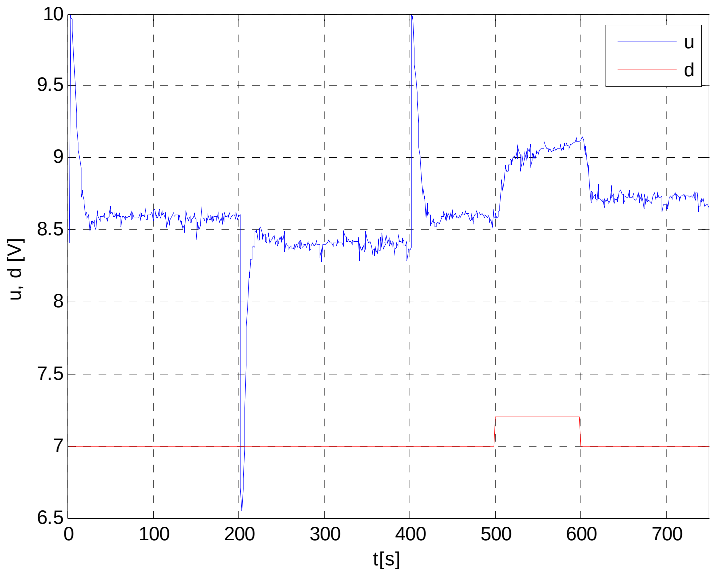


Figure 6 Time response of manipulated variable and disturbance

6 CONCLUSION

In this paper the PID control design based on predictive control approach has been proposed. More specifically, the PID control structure implemented on programmable logical controller SIMATIC S7-200 has been considered. The PID controller parameters have been derived by comparison of the discrete PID control law to the pole-placement control structure of GPC provided that the plant model structure is properly chosen. The equivalence of both control schemes has been tested by simulation. An implementation of the predictive PID controller for the control of simple laboratory tank has also been presented.

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