MODIFICATION OF NEIMARK D-PARTITION METHOD FOR DESIRED PHASE MARGIN

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Abstract: In this paper controller design method for SISO systems with performance specification in term of phase margin is presented. Mentioned method is a modification of Neimark D-partition method which ensure desired phase margin of open loop system instead of stability degree. The developed frequency domain design technique is graphical, interactive and insightful. Theoretical results are demonstrated on examples.

Keywords: D-partition, phase margin, PID controller, frequency domain, unstable plant

1 INTRODUCTION

Frequency domain techniques for analysis and controller design dominate SISO control system theory. Bode, Nyquist, Nichols, and root locus are the usual tools for SISO system analysis.
Frequency methods are often used for controller design because it is easy to ensure performance of closed loop system through phase margin in open loop (Nagurka and Yaniv, 2003). To achieve the desired phase margin, controllers are usually designed using Bode characteristics (Fung, *et al* 1998),(Ho, *et al* 1995).
In this paper Neimarks method of D-partition is used (Neimark, 1992). This method is usually used for controller design which ensures closed loop stability with desired stability degree. In this paper is presented modification of this method aimed on phase margin instead of stability degree.

2 PRELIMINARIES AND PROBLEM FORMULATION

Consider classical feedback system depicted in Fig. 1.

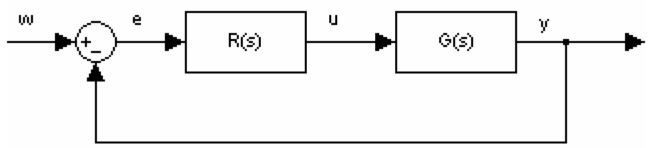

Figure 1: Classical feedback system

The aim is to design PID controller for plant $G(s)$ using Neimark D-partition method so, that not only stability will be ensured but performance in term of phase margin too.

3 THEORETICAL RESULTS

Consider closed loop feedback system with controller $R(s)$ and SISO plant $G(s)$.

$$R(s) = k + \frac{k_i}{s} + k_d s \qquad\qquad G(s) = \frac{B(s)}{A(s)} \tag{1}$$

Substituting (1) into characteristic function (2)

$$1 + R(s)G(s) = 0 \tag{2}$$

and after small manipulations is possible to obtain:

$$k + \frac{k_i}{s} + k_d s = -\frac{A(s)}{B(s)} \tag{3}$$

After other substitution $s = j\omega$ and division into real and imaginary part:

$$k = real\left\{-\frac{A(j\omega)}{B(j\omega)}\right\} \qquad\qquad -\frac{k_i}{\omega}j + k_d j\omega = imag\left\{-\frac{A(j\omega)}{B(j\omega)}\right\} \tag{4}$$

If ω is changing step by step in interval $\omega \in (0,\infty)$ from real part of (4) is possible to calculate frequency dependent vector of complex numbers which plotted in complex plane create D-curve for parameter k. Similar is it with imaginary part of (4) from which is possible obtain k_i or k_d but not both at once. In one step is possible to plot D-curve for parameters k and k_i (PI controller) or k and k_d (PD controller).

With small modification of characteristic function,

$$1 + R(s)G(s)e^{-j\varphi} = 0 \tag{5}$$

it is possible to rotate frequency characteristic of system, where φ is angle of desired rotation in radians (Fig. 2).

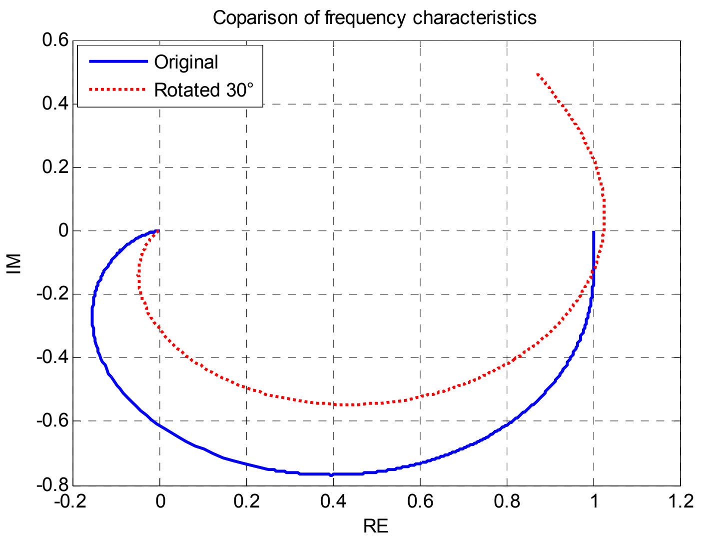

Figure 2: Comparison of frequency characteristic of plant without and with 30° rotation

If D-curves are calculated using characteristic function with rotation (5)

$$k = real\left\{-\frac{A(j\omega)}{B(j\omega)e^{-j\varphi}}\right\} \qquad -\frac{k_i j}{\omega} + k_d j\omega = imag\left\{-\frac{A(j\omega)}{B(j\omega)e^{-j\varphi}}\right\} \tag{6}$$

and controller's parameters are chosen directly from D-curves, designed controller will ensure phase margin equal to angle φ.

If the aim is to design PID controller it is necessary to design the controller in two steps. In first step PD controller can be designed and in second step, PI controller design can by applied on plant with PD controller. Final PID controller is than calculated as

$$PID = (k_1 + k_d s)(k_2 + \frac{k_i}{s}) = k_1 k_2 + k_d k_i + \frac{k_i k_1}{s} + k_d k_2 s \tag{7}$$

In this way controller for unstable plant can be designed, if this plant is possible to stabilize with PD controller. Than in first step PD controller is used for stabilization and PI controller ensure desired phase margin and eliminate steady state offset.

4 CASE STUDY

Example 1: Consider stable 3-th order plant $G(s) = \frac{2s+1}{s^3 + 3s^2 + 3s + 1}$

For this plant PI controller ensuring phase margin $\varphi = 50°$ will be designed. D-curves for parameters k and k_i can be calculated in one step and plotted in one figure, because characteristic function is divided into real and imaginary part and each parameter is calculated from another part.

$$k = real\left\{-\frac{A(j\omega)}{B(j\omega)e^{-j\varphi}}\right\} \qquad k_i = imag\left\{\frac{A(j\omega)}{B(j\omega)e^{-j\varphi}} j\omega\right\},\ \omega \in (0.001;30) \tag{8}$$

In fig. 3 are plotted D-curves for parameters k and k_i with different angle of rotation φ which represent desired phase margin too.

Note: D-curves for k and k_i (or k and k_d) are frequency dependent vectors but the frequency is not important so we plot in figure dependency k and k_i.

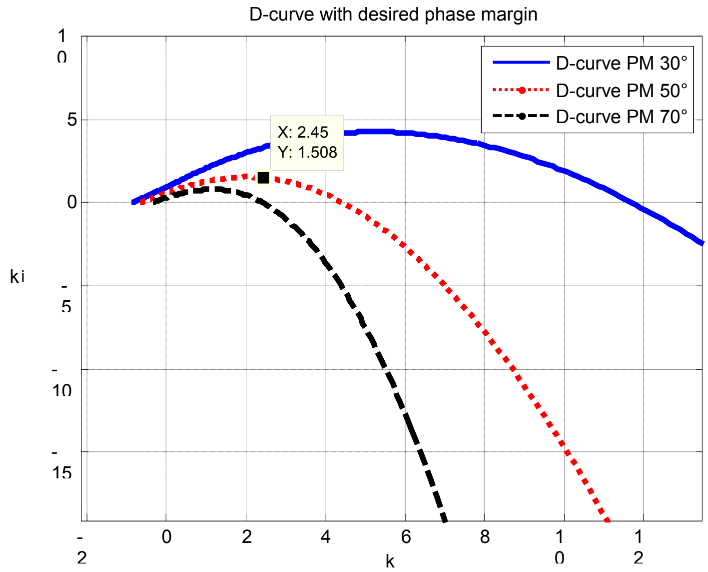

Figure 3: D-curves for parameter k and k_i with different phase margin

In this case we choose parameters from red line (fig. 3). We have to select parameters from the part of line where both parameters have positive values because plant $G(s)$ has positive gain.

The designed controller according to numbers from fig. 3 is:

$$R(s) = 2.45 + \frac{1.508}{s} \tag{9}$$

Bode characteristics in Fig. 4 confirm that the desired phase margin was achieved.

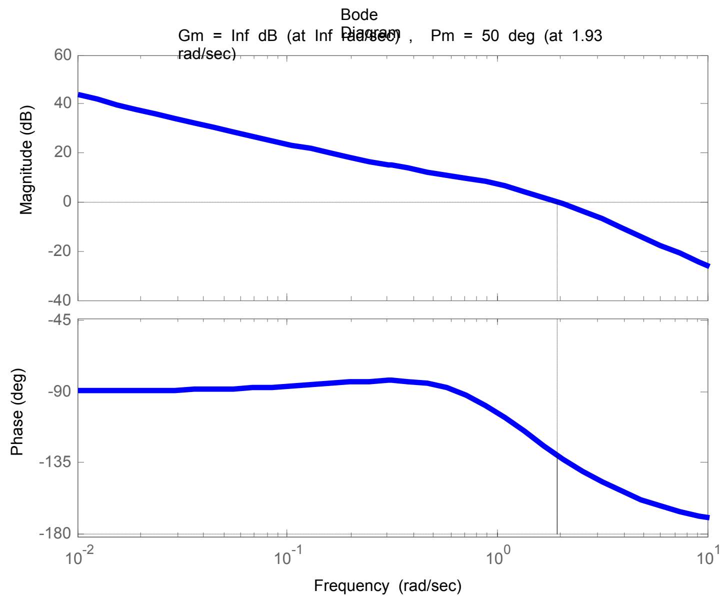

Figure 4: Bode characteristics for plant with desired controller

Example 2: Unstable plant with stable real pole and pair of complex unstable poles $G(s) = \frac{s+3}{s^3 + 4s^2 - 5s + 2}$, $\Lambda = \{-5.061; 0.5326 \pm 0.3335i\}$.

This plant it is necessary to stabilize with PD controller and than PI controller ensuring phase margin can be designed. In this case our aim is to reach phase margin $\varphi = 40°$.

In first step D-curves for k and k_d are calculated and plotted in fig. 5.

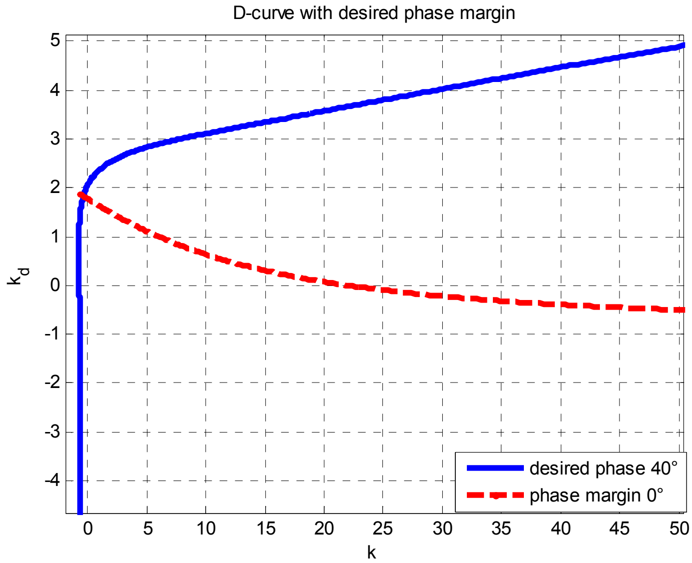

Figure 5: D-curve for parameters k and k_d

In first step it is not necessary to choose parameters from blue line because we just want to stabilize the system. System will be stable if the controller parameters will be chosen from region above red line which represents stability bound. From figure 5 we choosed PD controller parameters $k = 5$ and $k_d = 2$. Closed loop poles of plant with this PD controller are

$$\Lambda = \{-5.4713; -0.2644 \pm 1.7428i\} \tag{10}$$

In second step PI controller is designed for system consisting from plant and PD controller. D-curves for k and k_i were calculated and plotted in fig. 6.

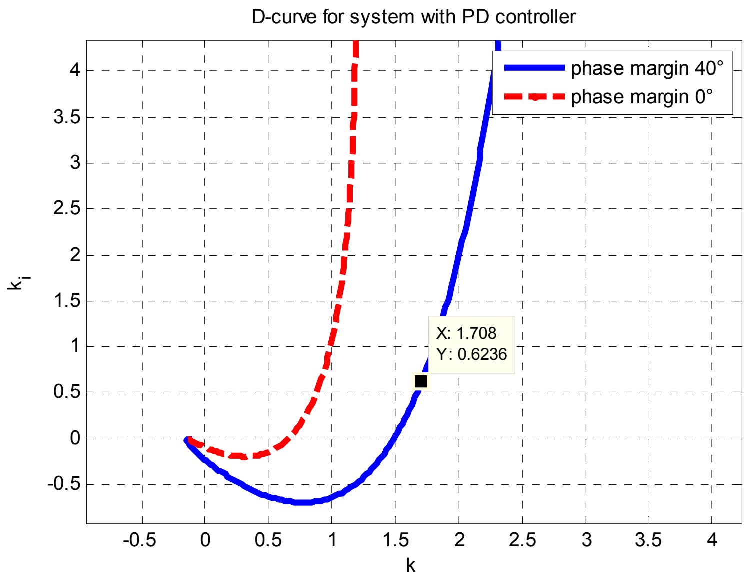

Figure 6: D-curve for parameters k and k_i

From fig. 6 we select parameters from blue line which ensure desired phase margin $\varphi = 40°$. Designed PI controller has following parameters: $k = 1.706$ and $k_i = 0.6236$. Final controller calculated according (7) is:

$$R(s) = 9.787 + \frac{3.118}{s} + 3.416s \quad (11)$$

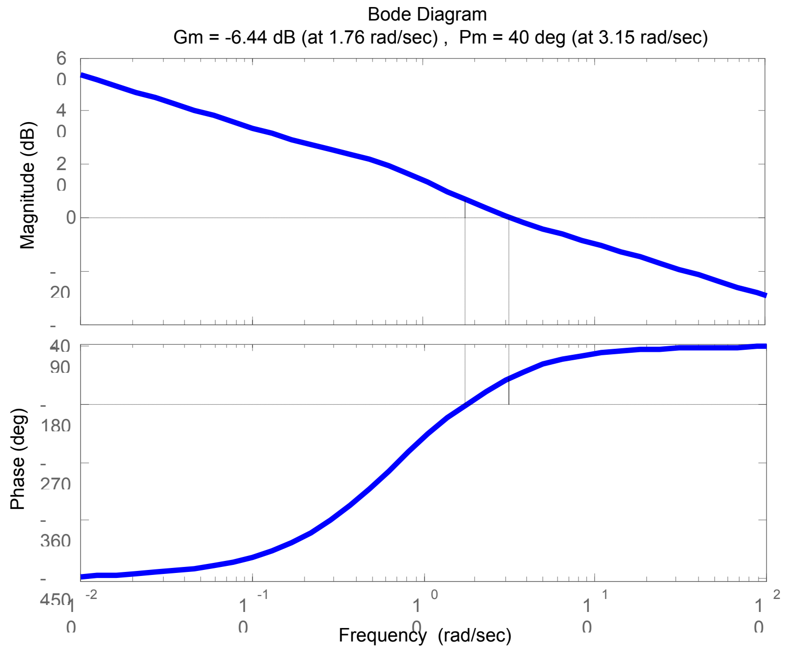

Figure 7: Bode characteristics for plant with desired controller

Fig. 7 confirms that the desired phase margin was achieved. Stability proofs closed loop poles and step response simulation with output disturbation in time 10s and magnitude $d = 0.2$ fig 8.

$$\Lambda = \{-5.8008; -0.6544 \pm 2.1993i; -0.3063\} \quad (12)$$

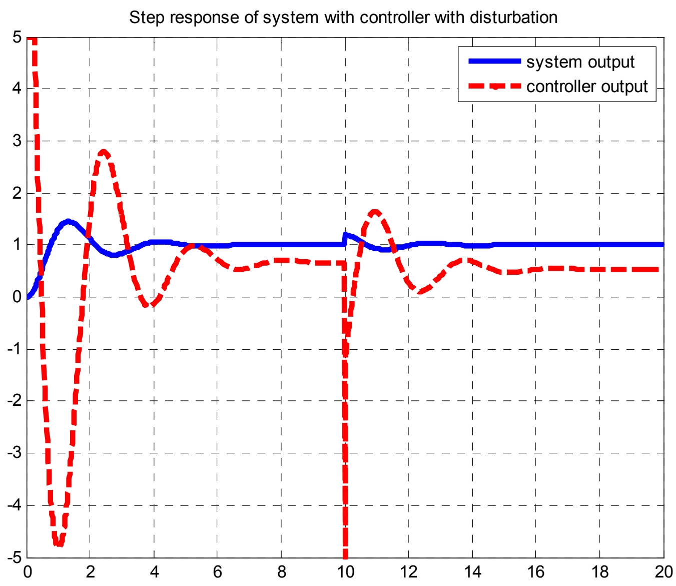

Figure 8: Step response of closed loop system with output disturbation

For several plants controllers for different values of desired phase margin were designed. Dependency of phase margin and overshoot are depicted in fig. 9. For both stable and unstable examples it is possible to see that overshoot is smaller for systems with higher phase margin. In fig. 9 is also depicted curve derived for second order plants (Hudzovic, 1980).

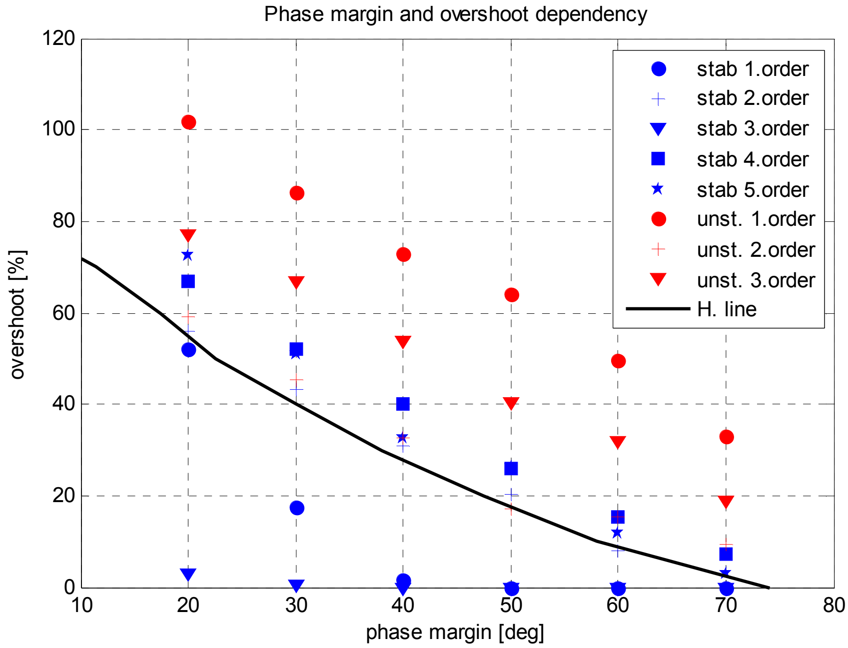

Figure 9: Phase margin and overshoot dependency

5 CONCLUSIONS

In this paper modification of Neimark D-partition method was presented. This controller design approach ensures not only stability but performance in term of phase margin too. The developed frequency domain design technique is graphical, interactive and insightful and it is useful for stable and unstable systems which is possible to stabilize with PD controller. Theoretical results have been verified on case studies.

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