COOPERATION OF SUBSYSTEMS IN DISCRETE EVENT SYSTEMS

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Abstract: A modular approach to synthesis and control of DES (discrete-event systems) is proposed and tested. The subsystems of DES are understood to be modules of a more complex structure. They are modelled by place/transition Petri nets (P/T PN) and assembled in order to cooperate. In such a way more complex subsystems of the global system or the global system itself can be synthesized. Supervisor synthesis methods are utilized in order to ensure the prescribed cooperation of the modules.

Keywords: Discrete event systems (DES), control, cooperation, multi agent systems (MAS), place/transition Petri nets (P/T PN), supervising

1 INTRODUCTION AND PRELIMINARIES

DES represent a wide class of systems used in human practice. They are discrete in nature, i.e. driven by discrete events. Petri nets (PN) are widely used at DES modelling and control, at the DES supervisor synthesis, even at synthesis of the DES structure. Here, P/T PN will be used. The PN place invariants and the Parikh's vector will be utilized in order to synthesize the cooperation of autonomous subsystems with the aim to satisfy demands posed on the global system. Both the author's previous results (Čapkovič, 2002,2005,2007,2009) and existing knowledge in the area of the supervisory control (Iordache & Antsaklis, 2006) are utilized on this way. The applicability of the approach to the cooperation of DES subsystems (agents) by means of the supervisor synthesis as well as to the DES structure synthesis is illustrated in examples. The origin of PN dates back to 60th of the last century. Then, they were defined by C. A. Petri. But more serious mathematical background for PN was given later. The mostly cited paper related to the PN background (Murata, 1989) yields the serious base of knowledge about PN and their properties. Consider here the general P/T PN-based mathematical model of the DES in the following form

$$\begin{aligned}
&\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{B}.\mathbf{u}_k, \quad k = 0,\cdots,K, \quad \mathbf{B} = \mathbf{G}^T - \mathbf{F}, \quad \mathbf{F}.\mathbf{u}_k \le \mathbf{x}_k \\
&\mathbf{x}_k = (\sigma_{p_1}^k,\cdots,\sigma_{p_n}^k)^T, \quad \sigma_{p_i}^k \in \{0,1,\cdots,c_{p_i}\}, \quad 0 \le \sigma_{p_i}^k \le c_{p_i}, i = 1,\cdots,n \\
&\mathbf{u}_k = (\gamma_{t_1}^k,\cdots,\gamma_{t_m}^k)^T, \quad \gamma_{t_j}^k \in \{0,1\}, j = 1,\cdots,m \\
&\mathbf{F} = \{f_{ij}\}_{n\times m}, \quad f_{ij} \in \{0, M_{f_{ij}}\}, \quad i = 1,\cdots,n; \ j = 1,\cdots,m \\
&\mathbf{G} = \{g_{ij}\}_{m\times n}, \quad g_{ij} \in \{0, M_{g_{ij}}\}, \quad i = 1,\cdots,m; \ j = 1,\cdots,n
\end{aligned} \tag{1}$$

where, k is the discrete step; $\mathbf{x}_k$ is the state vector of the system; $\sigma_{p_i}^k$ is an integer expressing the state of the place p_i and represents its activities (marking of p_i) in the step k; $0 < c_{p_i} \le \infty$ is the capacity of the place p_i (as to number of tokens); $\mathbf{u}_k$ is the control vector (quasi the state vector of transitions representing discrete events); $\gamma_{t_j}^k$ represents occurring a discrete event (when 1), i.e. enabling the PN transition t_j in the step k; **B**, **F**, **G** are the incidence matrices representing the PN structure, i.e. the causal relations between the DES activities and discrete events; **F** expresses the causal relations between the states of the places (marking) and the discrete events (enabling the transitions), while **G** expresses the causal relations between the discrete events and the states of places; $(.)^T$ symbolizes the transpose of matrices and vectors.

Besides the state vector $\mathbf{x}_k$ and the control vector $\mathbf{u}_k$ the Parikh's vector **v** is very important too. It follows from the step-by-step dynamics development of the P/T PN-based model of DES:

$$\mathbf{x}_1 = \mathbf{x}_0 + \mathbf{B}.\mathbf{u}_0, \quad \mathbf{x}_2 = \mathbf{x}_1 + \mathbf{B}.\mathbf{u}_1 = \mathbf{x}_0 + \mathbf{B}.(\mathbf{u}_0 + \mathbf{u}_1), \quad \cdots$$
$$\mathbf{x}_k = \mathbf{x}_0 + \mathbf{B}.(\mathbf{u}_0 + \mathbf{u}_1 + \cdots + \mathbf{u}_{k-1}) = \mathbf{x}_0 + \mathbf{B}.\mathbf{v}; \text{ i.e. } \mathbf{v} = (\mathbf{u}_0 + \mathbf{u}_1 + \cdots + \mathbf{u}_{k-1})$$

It yields (by means of its integer entries) information about how many times the particular transitions are fired during the development of the system dynamics from a given initial state $\mathbf{x}_0$ to the prescribed terminal state $\mathbf{x}_k$. The other important vectors in P/T PN are the P-invariants (place invariants) and T-invariants (transition invariants). The vector **w** is understood to be the P-invariant when $\mathbf{w}^T.\mathbf{B} = \mathbf{0}$, while it is named T-invariant when $\mathbf{B}.\mathbf{w} = \mathbf{0}$.

2 PROBLEM STATEMENT

The modular approach to DES modelling will be used here. It means that the structure of the system is understood here to be consisting of mutually cooperating modules – subsystems (agents). Thus, the matrices **F**, **G** in the system (1) can be expressed in the form as follows

$$\mathbf{F} = \begin{pmatrix} \mathbf{F}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & | & \mathbf{F}_{c_1} \\ \mathbf{0} & \mathbf{F}_2 & \cdots & \mathbf{0} & \mathbf{0} & | & \mathbf{F}_{c_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & | & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{F}_{N_A-1} & \mathbf{0} & | & \mathbf{F}_{c_{N_A-1}} \\ \mathbf{0} & \mathbf{0} & \cdots & 0 & \mathbf{F}_{N_A} & | & \mathbf{F}_{c_{N_A}} \\ \mathbf{F}_{d_1} & \mathbf{F}_{d_2} & \cdots & \mathbf{F}_{d_{N_A-1}} & \mathbf{F}_{d_{N_A}} & | & \mathbf{F}_{d\leftrightarrow c} \end{pmatrix} = \begin{pmatrix} blockdiag(\mathbf{F}_i)_{i=1,\dots,N_A} & | & \mathbf{F}_c \\ \mathbf{F}_d & | & \mathbf{F}_{d\leftrightarrow c} \end{pmatrix}$$

$$\mathbf{G} = \begin{pmatrix} \mathbf{G}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & | & \mathbf{G}_{d_1} \\ \mathbf{0} & \mathbf{G}_2 & \cdots & \mathbf{0} & \mathbf{0} & | & \mathbf{G}_{d_2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & | & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{G}_{N_A-1} & \mathbf{0} & | & \mathbf{G}_{d_{N_A-1}} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{G}_{N_A} & | & \mathbf{G}_{d_{N_A}} \\ \mathbf{G}_{c_1} & \mathbf{G}_{c_2} & \cdots & \mathbf{G}_{c_{N_A-1}} & \mathbf{G}_{c_{N_A}} & | & \mathbf{G}_{c\leftrightarrow d} \end{pmatrix} = \begin{pmatrix} blockdiag(\mathbf{G}_i)_{i=1,\dots,N_A} & | & \mathbf{G}_d \\ \mathbf{G}_c & | & \mathbf{G}_{c\leftrightarrow d} \end{pmatrix}$$

where $\mathbf{B}_i = \mathbf{G}_i^T - \mathbf{F}_i$; $\mathbf{B}_{d_i} = \mathbf{G}_{d_i}^T - \mathbf{F}_{d_i}$; $\mathbf{B}_{c_i} = \mathbf{G}_{c_i}^T - \mathbf{F}_{c_i}$; $i = 1, ..., N_A$; $\mathbf{B}_{d\leftrightarrow c} = \mathbf{G}_{c\leftrightarrow d}^T - \mathbf{F}_{d\leftrightarrow c}$, i.e. the complex DES consisting of many cooperating subsystems can be modelled by means of (1) too. The submatrices in the diagonals of the matrices **F**, **G** pertain to the N_A autonomouos subsystems (agents) $A_1, \cdots, A_{N_A}$, and the blocks indexed by $c \leftrightarrow d$ are concerning the PN subnet expressing the kernel of the interface structure among agents (e.g. a supervisor). Other submatrices represent the interactions among the agents and the kernel. The index *c* denotes the interactions realized by the PN transitions while the index *d* represent the interactions realized by the PN places. The interactions among the modules can have a different origin. They can arise as an arbitrary man-created structure or they can be synthesized in analytical terms by means of an exact method which guarantees a desired behaviour of the global system. Consequently, the supervisor can also be understood to be one of the subsystems of the global system.

3 SYNTHESIS OF THE SUPERVISOR BASED ON P-INVARIANTS

At the supervisor synthesis the P-invariants can be utilized. In general several P-invariants $\mathbf{w}_1$, ..., $\mathbf{w}_q$ can exist in a given P/T PN-based model of DES. Consider that they create the columns of the $(n \times q)$-dimensional matrix $\mathbf{W}$. Consequently, the following equation

$$\mathbf{W}^T.\mathbf{B} = \mathbf{0}$$

can be utilized in the method of the supervisor synthesis as follows. Suppose that the original system, being characterized by the structural matrix **B,** is extended for s additional places (slacks). Then, it can be written (Iordache & Antsaklis, 2006; Čapkovič, 2009)

$$[\mathbf{L} \quad \mathbf{I}_s].\begin{bmatrix} \mathbf{B} \\ \mathbf{B}_s \end{bmatrix} = \mathbf{0}; \quad \mathbf{L}.\mathbf{B} + \mathbf{B}_s = \mathbf{0}; \quad \mathbf{B}_s = -\mathbf{L}.\mathbf{B}; \quad \mathbf{B}_s = \mathbf{G}_s^T - \mathbf{F}_s$$

where $(s \times n)$-dimensional matrix $\mathbf{L}$ prescribes a set of s conditions imposed on the marking of the P/T PN-based model of DES – e.g. $\mathbf{L}.\mathbf{x} \leq \mathbf{b}$, where $\mathbf{b}$ is the s-dimensional constant vector of integers; $\mathbf{I}_s$ is $(s \times s)$-dimensional identity matrix and the $(s \times m)$-dimensional matrix $\mathbf{B}_s$ is the structure of the supervisor to be synthesized. Because the state of the augmented system (i.e. the original system plus the supervisor) is given as $(\mathbf{x}^T \mathbf{x}_s^T)^T$, it is still necessary to find the initial state $\mathbf{x}_s^0$ of the supervisor. This can be performed as follows

$$[\mathbf{L} \quad \mathbf{I}_s].\begin{bmatrix} \mathbf{x} \\ \mathbf{x}_s \end{bmatrix} = \mathbf{b}; \quad \mathbf{L}.\mathbf{x} + \mathbf{x}_s = \mathbf{b}; \quad \mathbf{x}_s = \mathbf{b} - \mathbf{L}.\mathbf{x}; \quad \mathbf{x}_s^0 = \mathbf{b} - \mathbf{L}.\mathbf{x}_0$$

4 EXAMPLE 1

Consider the set of five subsystems (agents) {A_1, ..., A_5} having the PN models given in Fig. 1.

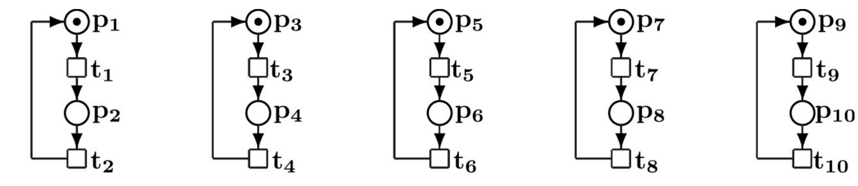

Figure 1: The group of five subsystems (agents)

Regard the places with the odd indices to be representants of the state ‚ready' while the places with the even indices to be representants of the state ‚working'. Hence, the initial state of the autonomous agents, being $\mathbf{x}_0 = (1\ 0\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 0)^T$, means that all of the agents are ready. Prescribe the following conditions for the agents cooperation: it is necessary to ensure that only one agent from the subgroup {A_1, A_4, A_5} and only one agent from the subgroup {A_2, A_4, A_5} and only one agent from the subgroup {A_3, A_4, A_5} can simultaneously cooperate with other agents from the group. In other words, the agents inside the designated subgroups must not work simultaneously. Even, the agents A_4 and A_5 can work only individually (any cooperation with other agents is excluded). However, the agents A_1, A_2, A_3 can work simultaneously. The conditions can be mathematically described as follows

$$\sigma_{p_2} + \sigma_{p_8} + \sigma_{p_{10}} \le 1$$
$$\sigma_{p_4} + \sigma_{p_8} + \sigma_{p_{10}} \le 1$$
$$\sigma_{p_6} + \sigma_{p_8} + \sigma_{p_{10}} \le 1$$

Hence, $s = 3$. Introducing three additional places – i.e. the slacks p_{11}, p_{12}, p_{13} - we can remove the inequalities. Hence,

$$\sigma_{p_2} + \sigma_{p_8} + \sigma_{p_{10}} + \sigma_{p_{11}} = 1$$
$$\sigma_{p_4} + \sigma_{p_8} + \sigma_{p_{10}} + \sigma_{p_{12}} = 1$$
$$\sigma_{p_6} + \sigma_{p_8} + \sigma_{p_{10}} + \sigma_{p_{13}} = 1$$

$$\mathbf{L} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}; b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}; \; F_{A_i} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; G_{A_i} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; i = 1, \cdots, 5$$

Using the above described procedure of the supervisor synthesis we obtain the parameters and the initial state of the supervisor as follows

$$\mathbf{G}_s^T = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}; \quad \mathbf{F}_s = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\mathbf{x}_s^0 = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^T$$

The supervised agents are displayed in Fig. 2. The obtained structure and initial state vector (the initial marking of the slacks) ensure the prescribed behaviour of the cooperating agents.

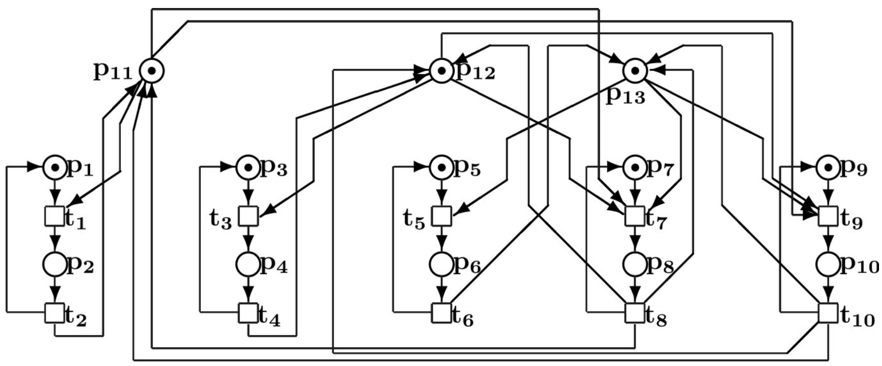

Figure 2: The supervised agents A_1, ..., A_5

5 MORE GENERAL SYNTHESIS OF THE SUPERVISOR

Although the described approach is very useful (because it yields the supervisor synthesis in analytical terms), it does not cover all needs of the supervisor synthesis. It seems to be rather a generalized mutex (mutual exclusion) than a general approach. However, e.g. in case of limited sources (like energy, working space, raw material, etc.), which are commonly utilized by all of the agents, such an approach is irreplaceable. In order to find more general approach applicable in more complicated kinds of the agent cooperation, the control vector $\mathbf{u}_k$ and the Parikh's vector $\mathbf{v}$ are utilized besides the state vector $\mathbf{x}_k$. In (Iordache & Antsaklis, 2006) the more general approach was described. It utilizes not only places but also transitions of the PN model of DES. Its principle can be concisely described as follows. The conditions for prescribing the behaviour of the supervised system are given as the inequality

$$\mathbf{L}_p.\mathbf{x}+\mathbf{L}_t.\mathbf{u}+\mathbf{L}_v.\mathbf{v}\le \mathbf{b}$$

where **b** is n_s-dimensional nonnegative integer vector expressing constraints; $\mathbf{L}_p$, $\mathbf{L}_t$, $\mathbf{L}_v$ are, respectively, $(n_s \times n)$-, $(n_s \times m)$-, $(n_s \times m)$-dimensional matrices of integers. When $\mathbf{b} - \mathbf{L}_p.\mathbf{x} \ge \mathbf{0}$ holds, the supervisor with the following structural partameters and the initial state

$$\mathbf{F}_s = \max\{\mathbf{0}, \mathbf{L}_p.\mathbf{B}+\mathbf{L}_v, \mathbf{L}_t\}$$

$$\mathbf{G}_s^T = \max\{\mathbf{0}, \mathbf{L}_t - \max\{\mathbf{0}, \mathbf{L}_p.\mathbf{B}+\mathbf{L}_v\}\} - \min\{\mathbf{0}, \mathbf{L}_p.\mathbf{B}+\mathbf{L}_v\}$$

$$\mathbf{x}_s^0 = \mathbf{b} - \mathbf{L}_p.\mathbf{x}_0 - \mathbf{L}_v.\mathbf{v}_0$$

guarantees that the constraints are verified by the supervisor for the states resulting from the initial state $\mathbf{x}_0$. It is necessary to say that the max{.} is the maximum operator for matrices. However, the maximum is taken element by element. Namely, in general, **Z** = max{**X**, **Y**} means that $z_{ij} = \max\{x_{ij}, y_{ij}\}$. The minimum can be found analogically.

6 EXAMPLE 2

Let us illustrate the generalized approach on a simple cooperation of two agents A_1, A_2 with the same structure given in Fig.3. The structure of the agents is expressed by the incidence matrices

$$\mathbf{F}_{A_i} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix}; \quad \mathbf{G}_{A_i}^T = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix}; \quad i = 1,2$$

The agent A_1 has the initial state $\mathbf{x}_0^{A1} = (2 \quad 0 \quad 1 \quad 0)^T$ displayed in Fig. 3 on the left, while the agent A_2 has the initial state $\mathbf{x}_0^{A2} = (0 \quad 1 \quad 1 \quad 0)^T$ displayed in Fig. 3 on the right.

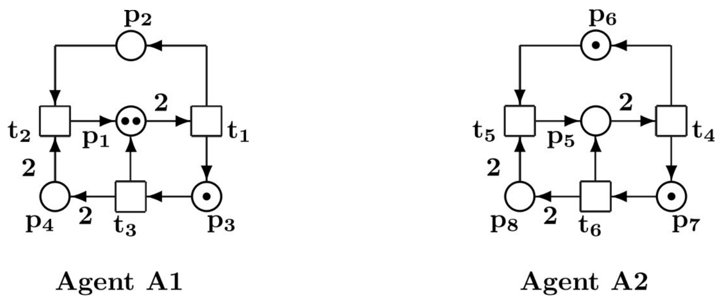

Figure 3: The structure of the agents A_1, A_2 with the designated initial states of the agents.

Let us fulfil the following prescribed conditions: let t_1 affecting the behaviour of A_1 can be fired, even two times consecutively, but in the dependency on the marking of the place p_5 belonging to A_2, and conversely, let t_4 can be fired, even two times consecutively, but in the dependency on the marking of p_1. Thus, the agents are able to affect their behaviour each other in the prescribe manner. This represents an important germ of the agents cooperation. Even, the form of the cooperation can be prescribed in analytical terms. Namely, the conditions for the desired cooperation in question can be expressed mathematically as follows

$$2.\gamma_{t_1} + \sigma_{p_5} \le 4; \quad 2.\gamma_{t_4} + \sigma_{p_1} \le 4; \quad \text{i. e. } \mathbf{b} = (4 \quad 4)^T; \quad \mathbf{L}_v = \mathbf{0}; \quad \mathbf{v}_0 = \mathbf{0}$$

$$\mathbf{L}_p = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}; \quad \mathbf{L}_t = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \end{pmatrix}$$

Considering $\mathbf{u}_0 = (1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1)^T$ we obtain the following structure and initial state of the supervised system. The supervised agents are displayed in the Fig.4.

$$\mathbf{F}_s = \begin{pmatrix} 2 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 & 0 & 0 \end{pmatrix}; \quad \mathbf{G}_s^T = \begin{pmatrix} 2 & 0 & 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 2 & 0 & 0 \end{pmatrix}; \quad \mathbf{x}_s^0 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

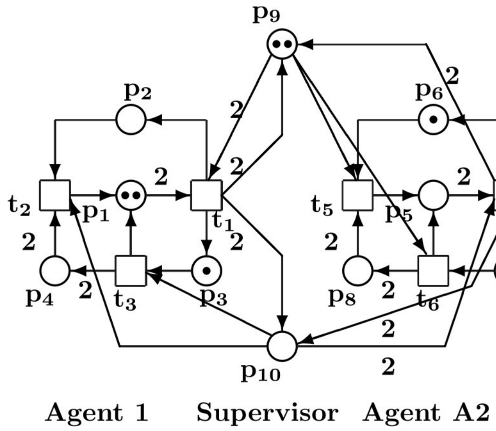

Figure 4: The supervised agents A_1, A_2

7 MODULAR SYNTHESIS OF MANUFACTURING SYSTEMS

The extended approach to the supervisor synthesis can be applied also at the synthesis of manufacturing systems, namely, as to their structure and goal-directed cooperation of their subsystems. The conditions for the cooperation synthesis result (to a certain extent) from the technology used for the production. Namely, there are many different modules (like machines, automatically guided vehicles (AGV), robots, etc.) in the manufacturing systems. They have to be somehow mutually configured (connected) to produce the desired components. Consider three modules of a simple manufacturing system: the unreliable Machine (M) producing parts (sometimes it fails and produce a defected part), Transport 1 – it contains AGV1 transporting the correct parts to a prescribed destination, and Transport 2 - it contains AGV2 transporting the damaged parts to a different space. The initial structure (autonomous subsystems) is displayed in Fig. 5. The final structure will be the product of the cooperation synthesis. The interpretation of the PN places (p_{10} - p_{13} arises during the synthesis) and PN transitions is the following. The places: p_1 - part is being carried to completed parts queue by AGV1; p_2 - AGV1 is free; p_3 - AGV1 is at pick-up position at M; p_4 - part is being carried to damaged-parts queue; p_5 - AGV2 is free; p_6 - AGV2 is at pick-up position at M; p_7 - M is up and busy (part is being processed); p_8 - M is free; p_9 - M is being repaired; p_{10} - completed part is waiting for transfer; p_{11} - damaged part is waiting for transfer; p_{12} - capacity place; p_{13} - machine capacity place. The transitions: t_1 - part is picked up by AGV1; t_2 - part is deposited in completed-parts queue by AGV1; t_3 - AGV1 moves at pick-up position at machine; t_4 - part is picked up by AGV2; t_5 - part is deposited in damaged-parts queue by AGV2; t_6 - AGV2 moves at pick-up position at machine; t_7 - uncontrollable: part processing is complete; t_8 - part is charged in M; t_9 - uncontrollable: machine fails, part is damaged; t_{10} - M is repaired.

The transit from the initial structure represented by the autonomouos agents to the final structure satisfying all prescribed conditions for the agents cooperation can be realized in several steps. The step-by-step procedure of the structure synthesis is more suitable than the procedure realized in one step. It has two main reasons: (i) it is practically impossible to form the whole set of conditions simultaneously or to expect that a chosen (apparently complete) set will be absolutely satisfied without any problem; (ii) forming a simpler set of conditions in any step, finding the corresponding structure, evaluating its quality and forming a new set of conditions seems to be more convenient and more credible procedure. Namely, in any step unexpected problems can appear. They have to be considered and solved by means of additional conditions. Let us illustrate such a procedure.

8 EXAMPLE 3

In the first step let us start from the set of the autonomous agents {Transport 1, Transport 2, Machine} displayed in Fig.5. Denote Transport 1 as A_1, Transport 2 as A_2 and M as A_3. The structural parameters of these agents and their initial states are the following

$$\mathbf{F}_{A_1} = \mathbf{F}_{A_2} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}; \mathbf{G}_{A_1} = \mathbf{G}_{A_2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \mathbf{F}_{A_3} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \mathbf{G}_{A_3} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\mathbf{x}^0_{A_1} = \mathbf{x}^0_{A_2} = (0 \quad 1 \quad 0)^T; \quad \mathbf{x}^0_{A_3} = (0 \quad 1 \quad 0)^T$$

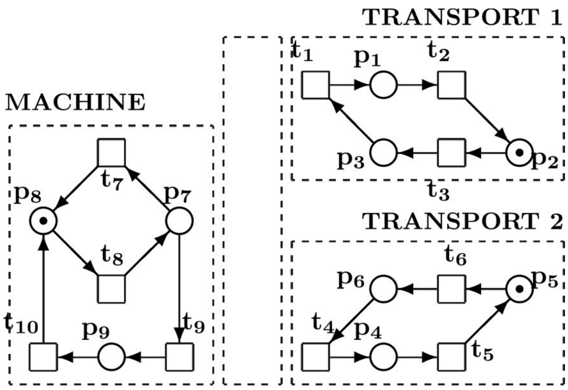

Figure 5: The autonomous subsystems.

As we will see, the synthesis in this case consists of three steps. In each of them the properties of the synthesized structure are tested by virtue of methods (Čapkovič, 2002, 2005, 2007) based on the PN reachability graph (RG), in order to evaluate how the structure in question satisfies our conception. The methods allow to compute the RG parameters (the adjacency matrix and the set of the feasible states being the RG nodes) and to find the state trajectories of the cooperaring subsystems. Let us start from the situation when only the structural parameters and initial states of the PN models of the autonomous devices are at disposal as follows

$$\mathbf{F} = \begin{pmatrix} \mathbf{F}_{A_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_{A_2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{F}_{A_3} \end{pmatrix}; \quad \mathbf{G} = \begin{pmatrix} \mathbf{G}_{A_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_{A_2} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{G}_{A_3} \end{pmatrix}; \quad \mathbf{x}_0 = \begin{pmatrix} \mathbf{x}^0_{A_1} \\ \mathbf{x}^0_{A_{21}} \\ \mathbf{x}^0_{A_3} \end{pmatrix}$$

The first step: To satisfy the technology of the manufacturing system – i.e. the transport of the good parts produced in M by means of AGV1 and the transport of the bad parts produced in M (when this device fails) by means of AGV2 and ensuring the reparation of M - first of all it is necessary to automate the procedure utilizing the contact of t_7 and t_1 as well as that of t_9 and t_4. Consequently, we have the conditions on the Parikh's vector **v** as follows

$$\gamma_{t_1} - \gamma_{t_7} \le 1; \quad \gamma_{t_4} - \gamma_{t_9} \le 1; \quad \text{i.e. } \mathbf{L}_v = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}; \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Considering $\mathbf{v}_0 = \mathbf{0}, \mathbf{L}_p = \mathbf{0}, \mathbf{L}_t = \mathbf{0}$ we obtain the structural parameters and the initial state of the supervisor after the first step of the synthesis in the following form

$$^{1st}\mathbf{F}_s = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}; \; ^{1st}\mathbf{G}_s^T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$^{1st}\mathbf{x}_s^0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \quad ^{1st}\mathbf{F} = \begin{pmatrix} \mathbf{F} \\ ^{1st}\mathbf{F}_s \end{pmatrix}; \quad ^{1st}\mathbf{G}_s^T = \begin{pmatrix} \mathbf{G}^T \\ ^{1st}\mathbf{G}_s^T \end{pmatrix}; \quad ^{1st}\mathbf{x}_0 = \begin{pmatrix} \mathbf{x}_0 \\ ^{1st}\mathbf{x}_s^0 \end{pmatrix}$$

However, because in such a configuration (it is displayed in Fig.6), which apparently seems to be quite natural, the number of the reachable states is too big, namely 256, and moreover, the automatic return to the initial state is impossible (what is very bad in case of such a cyclic processes). Therefore, this structure is insufficient for practical usage and an additional step of the synthesis is necessary.

The second step: The aim of this step of the synthesis is to ensure the reachability of the initial state (to realize the working cycle) and to find the satisfying throughput (to reduce the number of states and feasible trajectories). Try to do this by eliminating the needless and really impossible states. Impose the conditions

$$\begin{aligned} \sigma_{p_3} + \sigma_{p_6} & \leq 1 \\ \sigma_{p_7} + \sigma_{p_{10}} + \sigma_{p_{11}} & \leq 1 \end{aligned}$$

The first condition means that only one of the places needs to be active – namely, when the correct part was produced p_3 is active while p_6 is active when the defective part was produced. The second condition means that only one of the activities represented by p_7, p_{10}, p_{11} is really possible. Namely, the part is either produced at the moment, or the correct part was produced, or the defective part was produced. These activities cannot run simultaneously. Hence,

$$\mathbf{L}_p = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{pmatrix}; \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \quad \mathbf{L}_v = 0; \quad \mathbf{L}_t = 0$$

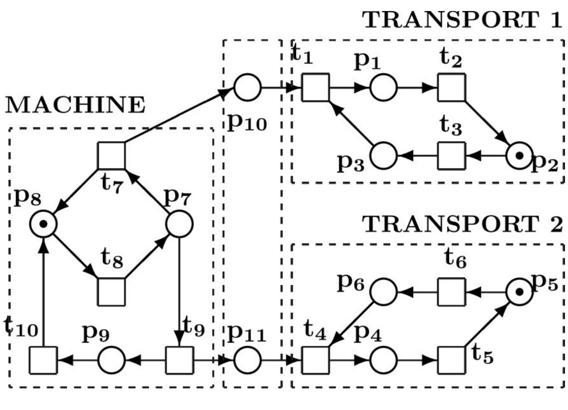

Figure 6: The sythesized structure in the 1st step

In this step of the supervisor synthesis we obtain the following structural parameters and the initial state

$$^{2nd}\mathbf{F}_s = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}; \; ^{2nd}\mathbf{G}_s^T = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$^{2nd}\mathbf{x}_s^0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \quad ^{2nd}\mathbf{F} = \begin{pmatrix} \mathbf{F} \\ ^{2nd}\mathbf{F}_s \end{pmatrix}; \quad ^{2nd}\mathbf{G}_s^T = \begin{pmatrix} \mathbf{G}^T \\ ^{2nd}\mathbf{G}_s^T \end{pmatrix}; \quad ^{2nd}\mathbf{x}_0 = \begin{pmatrix} \mathbf{x}_0 \\ ^{2nd}\mathbf{x}_s^0 \end{pmatrix}$$

The situation after the second step is displayed in Fig.7. Thus, the number of states was strongly reduced to 48 and the initial state is reachable. However, we found that there are two deadlocks here, namely in the states

$$\mathbf{x}_{d1} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}^T$$
$$\mathbf{x}_{d2} = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}^T$$

No transition is enabled in these states. These two states are reachable, respectively, by the sequences $\{t_6, t_8, t_7\}$ and $\{t_3, t_8, t_9, t_{10}\}$. To remove the deadlocks we have to use the additional (already the third) step of the supervisor synthesis.

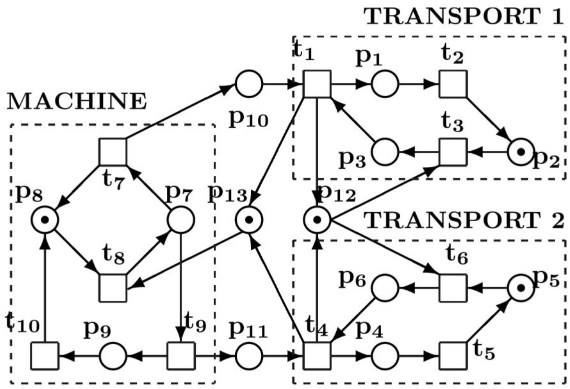

Figure 7: The synthesized structure in the 2nd step

The third step: Here, removing the deadlocks is the main aim. Impose the conditions

$$\gamma_{t_3} - \gamma_{t_7} \le 1; \quad \gamma_{t_6} - \gamma_{t_9} \le 1; \quad \text{i. e.} \quad \mathbf{L}_v = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{pmatrix}; \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

The first condition ties the correctly produced part with its transport to the prescribed space (the movement of the AGV1 into the pick-up space), while the second one analogically represents the same but for the defectively produced part and AGV2. Considering $\mathbf{v}_0 = 0, \mathbf{L}_p = 0, \mathbf{L}_t = 0$ we obtain the structural parameters and the initial state of the supervisor after the third step of the synthesis in the following form

$$^{3rd}\mathbf{F}_s = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}; \; ^{3rd}\mathbf{G}_s^T = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$^{3rd}\mathbf{x}_s^0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \quad ^{3rd}\mathbf{F} = \begin{pmatrix} \mathbf{F} \\ ^{3rd}\mathbf{F}_s \end{pmatrix}; \quad ^{3rd}\mathbf{G}_s^T = \begin{pmatrix} \mathbf{G}^T \\ ^{3rd}\mathbf{G}_s^T \end{pmatrix}; \quad ^{3rd}\mathbf{x}_0 = \begin{pmatrix} \mathbf{x}_0 \\ ^{3rd}\mathbf{x}_s^0 \end{pmatrix}$$

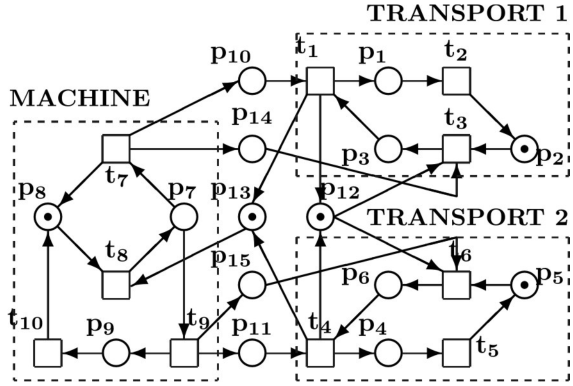

Figure 8: The synthesized structure in the 3rd step

In this manner the deadlocks were removed and simultaneously the number of states was reduced to the number 30. Such a deadlock-free structure with the acceptable number of states seems to be sufficient for us. The final structure of the supervised agents is given in Fig. 8.

9 CONCLUSION

The general approach to the cooperation of the DES subsystems (agents) was presented in this paper. It is based on both the modular approach to expressing the P/T PN-based model of DES and the theory of supervising based on P/T PN. The modules represent the autonomous DES subsystems representing e.g. some devices of manufacturing systems, while the supervisor synthesis is utilized for the interactive cooperation of the modules. Two kinds of the supervisor synthesis were presented in this paper. The first of them is based on the P-invariants of P/T PN, while the second one utilizes in addition also the control vector and the Parikh's vector. Both approaches are storiated by examples. The procedure of the cooperation synthesis is given in analytical terms. It utilizes linear algebra. Consequently, it is lightly applicable in commercial modelling and simulation tools like Matlab, Dymola, Mathematica, etc., as well as in the free-downloadable tools like Scilab.

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