CYBERNETICA
Model based control systems for the process industry

Validation and assessment of reduced models on an industrial distillation column

Stefan de Graaf, Cybernetica AS (Trondheim)
Leyla Özkan, Technische Universiteit Eindhoven
Andreas Linhart, Norwegian University of Science and Technology
Reinout Romijn, RWTH Aachen University

Contents

- Introduction to my working environment: Cybernetica
- Description of distillation column model
- Purpose of model reduction
- Model reduction methods and their resulting models
- Open-loop validation and results
- Summary

Introduction to Cybernetica AS

Model based control systems for the process industry

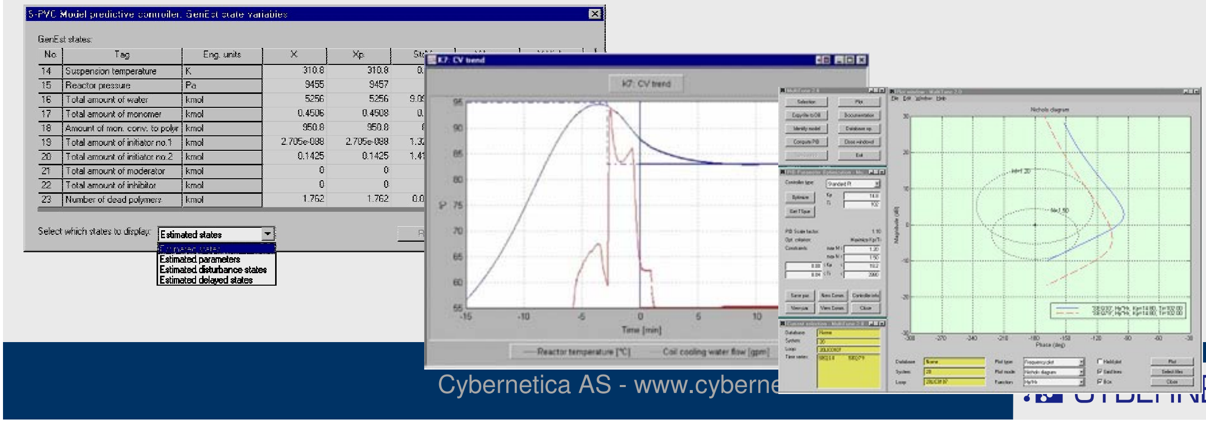
Background

- Founded in 2000
- At present 15 employees
- A spin-off company from the research groups in process control and engineering cybernetics at SINTEF, NTNU and Statoil

- Office in Trondheim, Norway

Products: Overview

- Model based control - Cybernetica CENIT
- Optimal grade transitions - Cybernetica CENIT
- Plant optimization - Cybernetica PlantOptimize
- Batch process optimization - Cybernetica BatchOptimize
- Controller tuning - Cybernetica MultiTune
- Dynamic simulation - Cybernetica SIMON


5-PVC Model predictive controller. GenEst state variables

GenEst states:

No.	Tag	Eng. units	X	Xp	St
14	Suspension temperature	K	310.8	310.8	0.
15	Reactor pressure	Pa	9455	9457	
16	Total amount of water	kmol	5256	5256	9.0
17	Total amount of monomer	kmol	0.4506	0.4508	0.
18	Amount of mon. conv. to polyr	kmol	950.8	950.8	
19	Total amount of initiator no.1	kmol	2.705e-088	2.705e-088	1.3
20	Total amount of initiator no.2	kmol	0.1425	0.1425	1.4
21	Total amount of moderator	kmol	0	0	
22	Total amount of inhibitor	kmol	0	0	
23	Number of dead polymers	kmol	1.762	1.762	0.0

Select which states to display: Estimated states

Applications: Overview

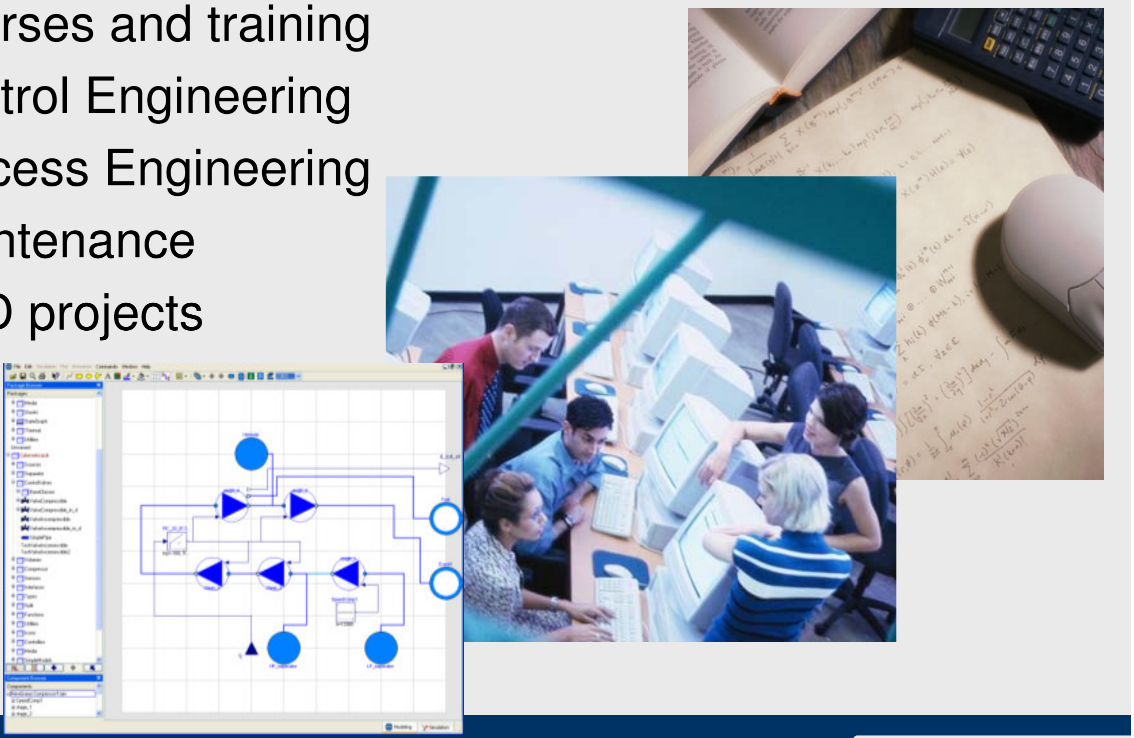
Polymer industry Light metals Oil and gas






Services

- Courses and training
- Control Engineering
- Process Engineering
- Maintenance
- R&D projects



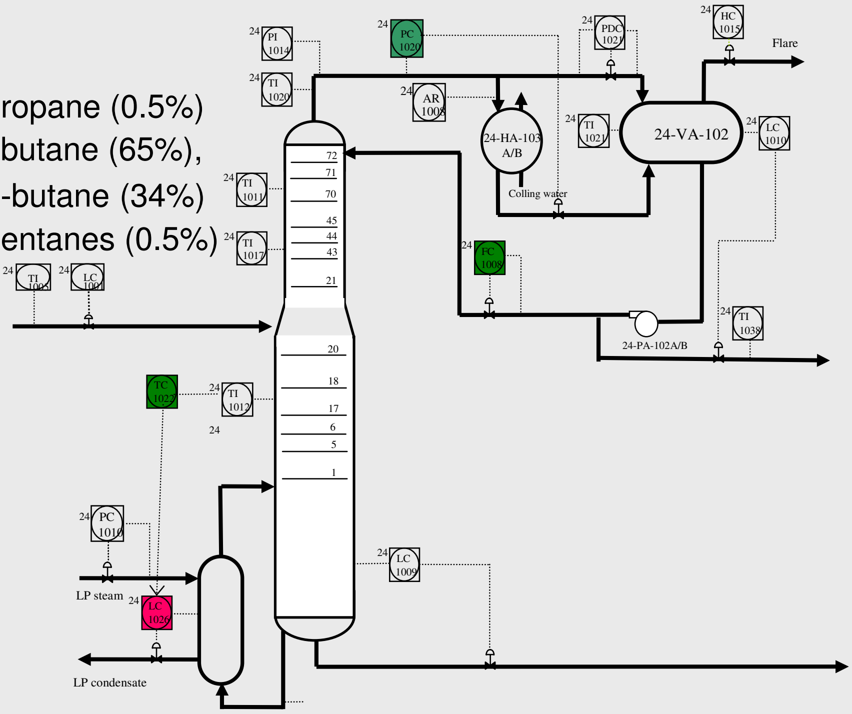
Contact us!

Cybernetica AS
Leirfossveien 27
NO-7038 Trondheim
Norway

Phone: +47 73822870
Fax: +47 73822871
E-mail: email@cybernetica.no
Web: www.cybernetica.no

Description of distillation column model

propane (0.5%)
i-butane (65%),
n-butane (34%)
pentanes (0.5%)



Inputs for MPC:

- reflux rate
- temperature setpoint at 19th tray

Outputs for MPC:

- n-butane fraction at top
- i-butane fraction at bottom

Model description

- mass balance per component for each tray, sump and reflux drum
- Energy balance for each tray, sump and reflux drum
- Complete flash at all trays, sump and in refluxdrum.
- Vapour-liquid equilibrium is based on Soave-Redlich-Kwong equations

Model description

- Vapour flow rates between trays depend on pressure differences between trays.
- Liquid flow rates between trays depend on liquid level on trays (weir equation)
- 4 PI controllers included in the model

Summary of equations

474 differential equations, of the type :

$$\frac{dM_{k,i}}{dt} = L_{k-1}x_{k-1,i} + V_{k+1}y_{k+1,i} - L_k x_{k,i} - V_k y_{k,i} + \text{specific feeds for feed tray, sump and reflux drum}$$

$$\frac{dU_k}{dt} = L_{k-1}h^L_{k-1} + V_{k+1}h^V_{k+1} - L_k h^L_k - V_k h^V_k + \text{specific feeds for feed tray, sump and reflux drum}$$

k = tray number {1...94}

i = component number {1,2,3,4}

Summary of equations

Outputs of the model: x, y, T, P, levels in sump and refluxdrum

$[L, V, x, y, h, T, P, \text{levels}]^T = g(M,U)$

g(M,U) contains 188 implicit (non)linear algebraic equations ,
e.g. SRK-equations

Conclusion: Many equations have to be integrated and solved using DAE-solvers, which leads to high computational loads

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Purpose of model reduction

Equations used in a distillation model and its application:

474 differential equations: $\frac{dx}{dt} = f(x,z,u,d)$

188 algebraic equations: $0 = g(x,z,u,d)$

2 output equations: $y = h(x,z,d)$

Application: compute optimal input trajectories *u* with MPC

Problem: Computation of *u* is too slow for fluctuations in *d*.

This leads to sub-optimal values for *u*.

Cause: Computation and use of large Jacobian: $\begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial z} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial z} \end{bmatrix}$

Reduction methods and reduced models

Two used methods:

1. Tray aggregation (Andreas Linhart)
2. POD-grey box modeling (Reinout Romijn)

Tray aggregation

Reduction of number of state and algebraic equations:

Step 1: Reducing number of state equations (f), which leads to new state equations (f_1) and new implicit algebraic equations (f_2).

Step 2: Reducing number of implicit algebraic equations (f_2 and g), including those created in step 1 by calling for their solutions that are stored in tables.

Step 3: Efficient computation and storage of non-zero Jacobian ellements for system that is obtained with steps 1 and 2.

Result (Linhart): reduced model is 6 times faster than full model at accuracies that are dominated by simulation errors.

Tray aggregation

original model

$$\frac{dx}{dt} = f(x,z,u,d)$$

$$0 = g(x,z,u,d)$$

$$y = h(x,z,d)$$

$$J = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial z} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial z} \end{bmatrix}$$

after step 1

$$\frac{dx'_1}{dt} = f'_1(x',z,u,d)$$

$$0 = f'_2(x',z,u,d)$$

$$0 = g(x',z,u,d)$$

$$J = \begin{bmatrix} \frac{\partial f'_1}{\partial x'_1} & \frac{\partial f'_1}{\partial x'_2} & \frac{\partial f'_1}{\partial z} \\ \frac{\partial f'_2}{\partial x'_1} & \frac{\partial f'_2}{\partial x'_2} & \frac{\partial f'_2}{\partial z} \\ \frac{\partial g}{\partial x'_1} & \frac{\partial g}{\partial x'_2} & \frac{\partial g}{\partial z} \end{bmatrix}$$

after step 2

$$\frac{dx'_1}{dt} = f'_1(x',z,u,d)$$

$$\begin{bmatrix} x'_2 & z \end{bmatrix} = g'(x'_1,u,d)$$

$$y = h(x',z,d)$$

$$J = \begin{bmatrix} \frac{\partial f'_1}{\partial x_1} \end{bmatrix}$$

POD and grey-box modeling

original model

$$\frac{dx}{dt} = f(x,z,u,d)$$

$$0 = g(x,z,u,d)$$

$$y = h(x,z,d)$$

$$J = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial z} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial z} \end{bmatrix}$$

reduced model

$$\frac{dx'}{dt} = Ax' + Bu$$

$$y = (Cx' + Du)^T \varphi$$

POD and grey-box modeling

$$J = [A]$$

Open-loop validation

Follwing characteristics for full model and reduced models are compared:

- relative gain array (RGA)
- condition number
- simulations with input step changes that show directionality

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Explanation of RGA

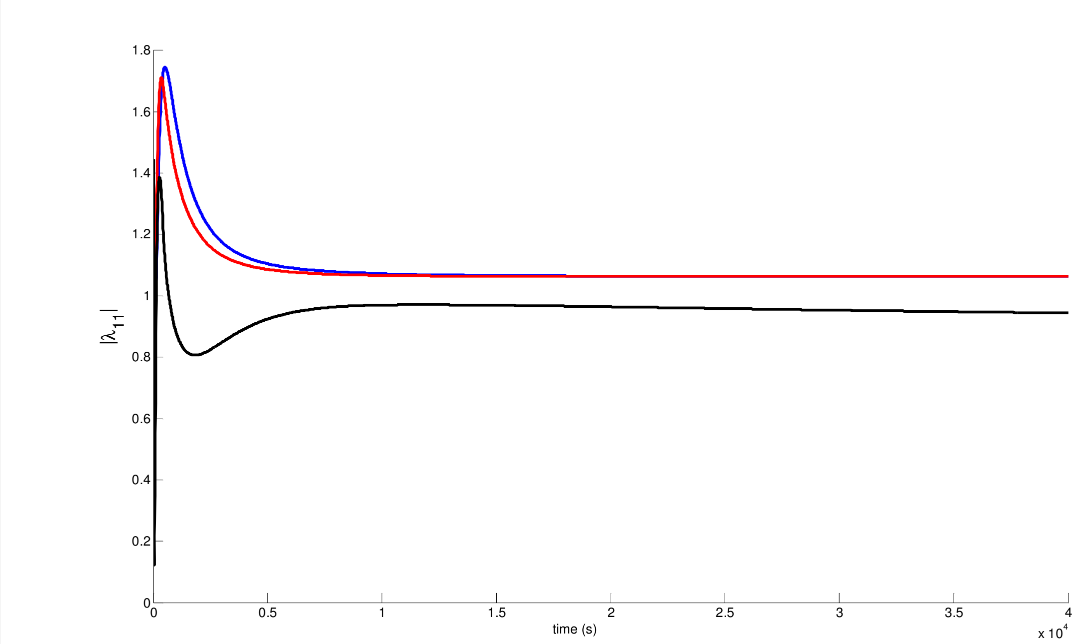
Numbers in RGA matrix for full model and reduced models should be the same:

$$\Lambda = \begin{array}{cc} \begin{array}{cc} \mathrm{u}_1 & \mathrm{u}_2 \end{array} & \\ \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} & \begin{array}{c} \mathrm{y}_1 \\ y_2 \end{array} \end{array} \qquad \text{where} \qquad \lambda_{ij} = \frac{\left(\frac{\partial y_i}{\partial u_j}\right)_u}{\left(\frac{\partial y_i}{\partial u_j}\right)_y}$$

$$\sum_{i=1}^{N} \lambda_{ij} = 1 \qquad \text{for} \qquad \text{j=1,2} \qquad\qquad \sum_{i=1}^{N} \lambda_{ij} = 1 \qquad \text{for} \qquad \text{i=1,2}$$

Easy computation: $\Lambda(s) = G(s) \otimes \left(G(s)^{-1}\right)^T$ where $s = iw$

RGA results



blue line: full model red line: aggregated model black line: POD+grey-box

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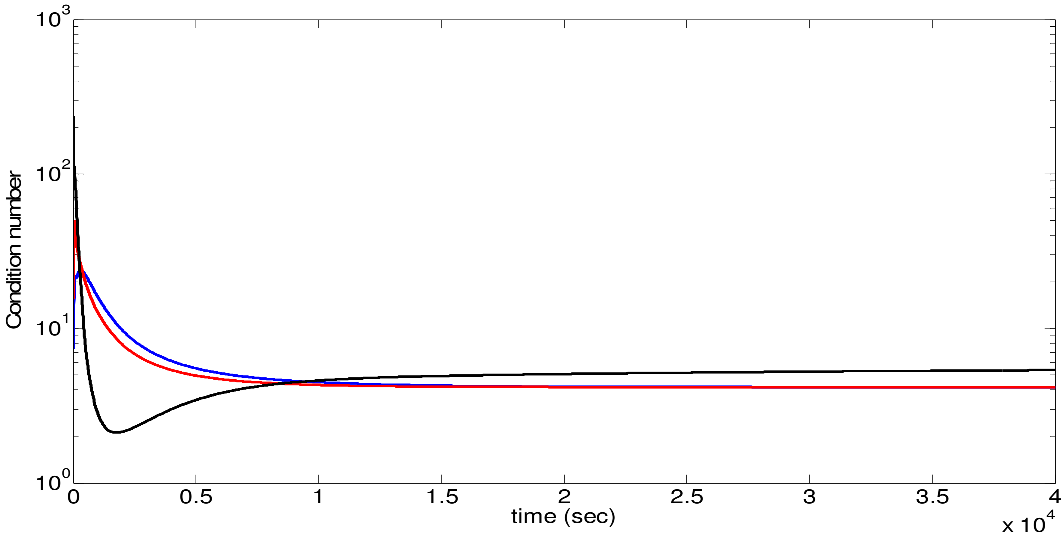
Condition number

Condition number of linearized full model and linearized reduced models should be the same: $\gamma = \frac{\sigma_{\text{max}}}{\sigma_{\text{min}}}$

where $\Sigma = diag\left(\sigma_1, \sigma_2, \sigma_3, \cdots, \sigma_m\right)$ is computed by carrying out a singular value decomposition on $G(0)_t$: $G\left(0\right)_{\text{t}} = W \Sigma V^T$

Condition number



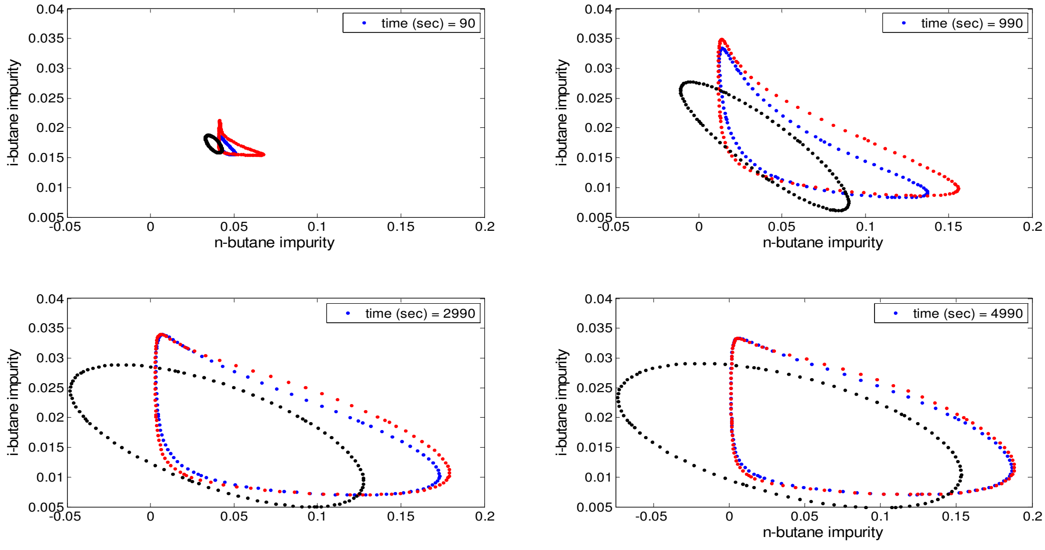
blue line: full model

red line: aggregated model

black line: POD+grey-box

Simulations with input step changes

100 simulations with initial conditions set at at steady state values and step changes in u_1 and u_2 such that: $\Delta u_1^2 + \Delta u_2^2 = 1$



Relation simulation results ↔ directionality

Axis directions of steady state output ellipses are the colums of W-matrices obtained by carrying out a singular value decomposition on *G(0)*: $G(0) = W\Sigma V^T$

The σ-s in $\Sigma = diag(\sigma_1, \sigma_2, \sigma_3, \cdots, \sigma_m)$ determine the length of the axes.

This reasoning can be extended to non-linear systems although they do not have transfer function matrices and perfect ellipses of output values.

Summary

- Presentation of a large DAE-model for a distillation process.
- Relation between large Jacobian of DAE-models and large computational loads.
- Presentation of two reduced models, their reduced Jacobians and reduced computational loads.
- Explanation of open-loop validation tools for reduced models: RGA, condition number, simulations with input step changes that show directionality.
- Presentation of validation results for the reduced models.