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Temperature control of multidimensional system using decoupled MPC controllers

Jozef Kurilla
Institute of Robotics and Cybernetics
Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava
Bratislava, Slovak Republic
jozef.kurilla@gmail.com

***Abstract*—This paper presents the connection between predictive control and simplification of MIMO system. A model of a part of administrative building is used to observe mutual thermal interactions between individual offices. Multivariable sixth-order system is converted into six linear mutually individual systems of first-order by input-output linearization (decoupling). These SISO systems are controlled by predictive a controller focusing on the accuracy of the room temperature respecting office occupancy profile. Included constraints make from control task the problem of quadratic programing. The final control structure takes advantage of the low computational burden of simple predictive controller and network communication, which ensures the inclusion of constraints. The mutual effect of the output variables and time response of control action is compared in a simulation study.**

Keywords—Mutual interaction, decoupling, predictive control, building automation

I. Introduction

Management of the internal environment is an essential part of building use, whether they are used as industrial, office or residential buildings. Efforts to get efficient use of energy is now encouraging producers to optimize the performance of building materials, used technology, the method of obtaining energy, but it is very important not to forget the fact that even the best technology in the area with outstanding thermal insulation properties, effective management is still necessary.

To ensure a feeling of thermal comfort for people and optimum environmental conditions of various technological processes at first sight may seem an easy task, as the controlled system is not unstable, or system is not too dynamic. The challenge of optimizing control of these systems is not in overcoming the complexities non-linear or system instability, but its impact on overall energy consumption and energy saving options. Energy efficient management of the internal environment of the building has a significant impact on the environment and, certainly, on economic indicators.

Nowadays there are many methods of temperature control which can be divided into groups which may affect the whole system or method which use a simple feedback of every single part of control building (local PID regulation). To provide the suitable quality level of regulation approaches like fuzzy logic [1], genetic algorithms [2], neural networks [3], auto-tuning methods of PID parameters [4] have been proposed in the literature. One of the most dynamic developing methods is model predictive control (MPC). MPC is many times chosen as the most suitable method for buildings, because it can predict the appearance of disturbances such as impact of weather, occupancy or outside temperature [5]. On the other side, MPC requires a model of the building with sufficiently high accuracy and accurate predictions of the disturbances to ensure a suitable control performance. Principle of MPC regulator is computing of control input by solving an optimal problem over selected horizont. To the controlled system is applied only the first element of the command sequence [6]. In the next iteration, a new optimization is executed based on actual measurements. Model predictive control has been applied in various types of application [7] and for control of the indoor climate environment has been created several modifications [8].

This paper is dedicated to control structure consisting of independent MPC regulators of room temperature of the neighbouring offices, which are able to react to change in the neighbouring system respecting constrains. For this purpose the input-output linearization and quadratic programming are used.

Theoretical basis and a model of studied part of the building are presented in section 2. Next section discusses the effect of bonds between neighbouring offices to quality of regulation and also the importance of an input-output linearization. The fourth section is dedicated to principle of MPC regulator, to usage of occupancy profile and to the incorporation of constraints to control system. End of this section is dedicated to the impact of eliminating couplings between systems on quality of control. Section 5 concludes the paper.

II. Thermal spread and model of thermal system

The Heat is an amount of energy, which warmer element transfers to colder element in heat exchange. Heat spreads by conduction, convection, radiation and by latent heat flux. Describing model considers only with conduction and convection. Heat conduction takes place at the molecular level when the bordering molecules with different temperatures interact with each other as there is no temperature equalization. With heat transfer caused by heat conduction, the temperature of a material is a function of the spatial coordinates and the time, reflecting the simplified [3] Fourier - Kirchhoff heat equation (2.1), where a is the coefficient of thermal

$$\frac{\partial t}{\partial \tau} = a\left(\frac{\delta^2 t}{\delta x^2} + \frac{\delta^2 t}{\delta y^2} + \frac{\delta^2 t}{\delta z^2}\right) = a\nabla^2 t \tag{2.1}$$

conductivity. For a stationary one dimensional thermal field equation (2.1) can be overridden by the form of expressing heat flux H (2.2),

$$H = \lambda_T S \frac{\partial t}{\partial x} \tag{2.2}$$

i.e. the heat transferred per time unit through a surface S, where λ_T is the thermal conductivity of the material $[\mathrm{Js^{-1}m^{-1}K^{-1}}]$. The convective heat transfer occurs in liquids and gases due to the dependence of fluid density on temperature. At different temperature liquids have different densities, so natural flow and mutual mixing and balancing of temperature appear. This process is called natural convection, but if the fluid is set in motion for example by pump or fan, it is a forced convection. Heat flow of flowing fluid is given by (2.3),

$$H = Q\rho c_p T \tag{2.3}$$

where c_p is specific heat capacity of liquid$[\mathrm{Jkg^{-1}K^{-1}}]$, Q is volume flow $[\mathrm{m^3h^{-1}}]$ and T is liquid temperature [K]. Heat transmission between liquid separated by solid is called passage of heat. It consists of three processes: Transfer of heat from the cooled fluid, heat conduction through solid substance and heat transfer from the surface to heated liquid. Heat flow from one fluid to another is expressed (2.4).

$$H = kS\Delta T; \quad k = \frac{1}{\frac{1}{\lambda_1} + \frac{\delta}{\lambda_T} + \frac{1}{\lambda_2}} \tag{2.4}$$

Coefficients λ_1 and λ_2 are the heat transfer coefficients on both sides of e.g. plane wall$[\mathrm{Wm^{-2}K^{-1}}]$, δ is the thickness of wall [m], λ_T is the thermal conductivity of wall $[\mathrm{Wm^{-1}K^{-1}}]$ and S is contents of wall $[\mathrm{m^2}]$. Coefficient k is called thermal transmittance $[\mathrm{Wm^{-2}K^{-1}}]$ and it represents the amount of heat which passes from one fluid to the other through a unit area (1 m2) per time unit (s) of fluid temperature difference of 1 K. Denominator members of the coefficient k are also called thermal resistance. Description of the overall dynamic characteristic of thermal system is expressed by the equation based on the law of conservation of thermal energy. It defines that the increase rate of temperature of the object is proportional to the amount of heat introduced into the object of conduction, convection, radiation, burning and phase change. Heat flows entering the object with the equation taking a sign "+", heat flows emerging from the object have a sign "-". In the description of the system, it is necessary to write as many equations (2.5) as much heat capacity the system has, i.e. for all the parts that are able to accumulate thermal energy. Equation

$$\mathrm{mc_p} \frac{\mathrm{dT}}{\mathrm{dt}} = \sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{H_i} \tag{2.5}$$

(2.5) is the base of physical descriptions of selected parts of the office building. As an example model was selected a group of six offices in three floors, with five offices in contact with the glass facade and one surrounded by the internal environment as shown in *Fig. 1*. Heat flows entering every office are represented by heat transfer to the adjacent room, forced convection of air supply handling unit and constant thermal performance of convector. Heat flows extending from each of the offices are represented by the heat transfer to the outside and to the adjacent cooler rooms and by exhausted air of the air handling unit. The internal temperature of adjacent offices (not described in the model), as well as the outside temperature, is considered to be a constant. The airflow entering and leaving the office is constant. Performance of convectors or warm water heaters is also considered to be a constant in the description and together with external temperature is regarded as a disturbance in the system. This multivariable system is described by a system of six differential equations. From a regulatory perspective the room air temperature is considered as an output variable and the supply air temperature is control variable at constant volume flow. It is necessary to write six differential equations. For brevity only two of them are listed here: (2.6) – central room and (2.7) – room on the left from central room. Indices in the equations are named with the position of rooms $k \in \{C, L, R, T, B, I\}$ (*C*-central, *L*-left, *R*-right, *T*-top, *B*-bottom, *I*-internal room).

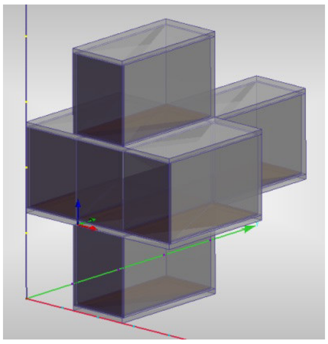

Fig. 1: Described model of six neighbouring rooms

$$\begin{aligned} c_p\rho V_C \frac{dT_C}{dt} &= P_C + c_p\rho Q_C(T_{IC} - T_C) + \\ &k_o S_{C1}(T_o - T_C) + k_B S_{C2}(T_B - T_C) + \\ &k_S S_{C3}(T_I - T_C) + k_B S_{C4}(T_T - T_C) + \\ &k_S S_{C5}(T_L - T_C) + k_S S_{C6}(T_R - T_C) \end{aligned} \tag{2.6}$$

$$\begin{aligned} c_p\rho V_L \frac{dT_L}{dt} &= P_L + c_p\rho Q_L(T_{IL} - T_L) + \\ &k_o S_{L1}(T_o - T_L) + k_B S_{L2}(T_V - T_L) + \\ &k_S S_{L3}(T_V - T_L) + k_B S_{L4}(T_V - T_L) + \\ &k_S S_{L5}(T_V - T_L) + k_S S_{L6}(T_C - T_L) \end{aligned} \tag{2.7}$$

The first member in each equation corresponds to the constant power of convector, the second member to heat convection, the third to heat transfer to the outside environment (except equation for internal room) and fifth other to heat transfer to the adjacent room. T_o is outside temperature, T_V is constant temperature of neighbouring non-described room, c_p is specific heat capacity of air, ρ is density of air, V_k is volume of individual rooms, Q_k is constant airflow of incoming and outgoing air, S_{ki} is surface of walls, P_k is performance of convector or warm water heater, $\mathrm{k_o}$ is heat transfer coefficient of glass, k_B is heat transfer coefficient of concrete plate (concrete, insulation, wooden floor), k_S is heat transfer coefficient of walls. After editing and merging constants, system of equations can be entered in the state space $\dot{x} = \mathrm{A}x + \mathrm{B}u$; $y = \mathrm{C}x + f$ (2.8),

$$\dot{x} = Ax + Bu + f$$
$$y = Cx$$
$$A = \begin{pmatrix} -K_{CC} & K_{CL} & K_{CR} & K_{CT} & K_{CB} & K_{CI} \\ K_{LC} & -K_{LL} & 0 & 0 & 0 & 0 \\ K_{RC} & 0 & -K_{RR} & 0 & 0 & 0 \\ K_{TC} & 0 & 0 & -K_{TT} & 0 & 0 \\ K_{BC} & 0 & 0 & 0 & -K_{BB} & 0 \\ K_{IC} & 0 & 0 & 0 & 0 & -K_{II} \end{pmatrix};$$
$$x = \begin{pmatrix} T_C \\ T_L \\ T_R \\ T_T \\ T_B \\ T_I \end{pmatrix}; \; u = \begin{pmatrix} T_{IC} \\ T_{IL} \\ T_{IR} \\ T_{IT} \\ T_{IB} \\ T_{II} \end{pmatrix}; \; C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}; \qquad (2.8)$$
$$B = \begin{pmatrix} b_{CA} & 0 & 0 & 0 & 0 & 0 \\ 0 & b_{LA} & 0 & 0 & 0 & 0 \\ 0 & 0 & b_{RA} & 0 & 0 & 0 \\ 0 & 0 & 0 & b_{TA} & 0 & 0 \\ 0 & 0 & 0 & 0 & b_{BA} & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{IA} \end{pmatrix}; \; f = \begin{pmatrix} K_{PC} + K_C \\ K_{PL} + K_L \\ K_{PR} + K_R \\ K_{PT} + K_T \\ K_{PB} + K_B \\ K_{PI} + K_I \end{pmatrix}$$

where f is the vector of disturbances, $\boldsymbol{x} = T_k$, $\boldsymbol{u} = T_{Ik}$ and $\boldsymbol{x} = \boldsymbol{y}$. State space model is shown only to clarify the description of thermal system and it will not be used in the creation of predictive controller. By substituting constants $k_o = 0.7400\ \mathrm{Wm^{-2}K^{-1}}$, $k_B = 0.7254\ \mathrm{Wm^{-2}K^{-1}}$, $k_s = 1.0385\ \mathrm{Wm^{-2}K^{-1}}$, $P_k = 1153\ \mathrm{W}$, $Q_k = 100\ \mathrm{m^3h^{-1}}$, $T_o = 10\ °\mathrm{C}$, $T_V = 18\ °\mathrm{C}$ and room dimensions $a = 5.5$ m; $b = 2.5$; $c = 3.5$ m state space (2.8) acquires numerical expression (2.9).

$$A = \begin{pmatrix} -19 & 3.46 & 3.46 & 1.73 & 1.73 & 1.57 \\ 3.46 & -19 & 0 & 0 & 0 & 0 \\ 3.46 & 0 & -19 & 0 & 0 & 0 \\ 1.73 & 0 & 0 & -19 & 0 & 0 \\ 1.73 & 0 & 0 & 0 & -19 & 0 \\ 1.57 & 0 & 0 & 0 & 0 & -19.34 \end{pmatrix} 10^{-4};$$
$$B = \begin{pmatrix} 5.82 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5.82 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5.82 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5.82 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.82 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5.82 \end{pmatrix} 10^{-4}; \qquad (2.9)$$
$$C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}; f = \begin{pmatrix} 0{,}0211 \\ 0{,}0330 \\ 0{,}0330 \\ 0{,}0327 \\ 0{,}0327 \\ 0{,}0347 \end{pmatrix}$$

From the numerical expression of matrix A and vector b can be noticed that a temperature change in individual rooms caused by an adjacent room (coefficients K_{kC} and K_{Ck}) is identical in order to influence of actuating variable (coefficients b_{kA}). When we compare the influence of the adjacent room and influence of action control at constant flow, an influence of actuating variable is more than two times greater than the influence of the adjacent room. If it is necessary to increase the room temperature by one degree, and only supply air (one degree of Celsius higher than room temperature) participates in heating, one almost half of the needed power would be supplied from the adjacent room, provided that temperature in the adjacent room is also higher by one degree of Celsius. Adjacent room, of course, would be cooled. Increasing the heat flow of incoming air (by increasing flow rate or increasing of temperature difference between room temperature and temperature of incoming air), the impact of control action increases and the effect of adjacent rooms decreases. At the specified flow (100 m3/h) and dimensions of office (48.125 m3), the air is exchanged about 2 times per hour, which is within the normally used range. Since the office neighbours with four other offices, the surrounding effect on a given room compared with the actuating variable is not insignificant. However, it is important to note that heat gains are only affective until there is temperature balance between offices whereas the actuating variable is limited only by the limit value of supply air temperature. From (2.9) can be deduced major impact of failure represented by the vector f. It consists of constants K_{Pk}, which includes effect of performance of warm water heater and K_k, which includes effect of outside temperature and constant ambient temperature of rooms, which are not included in the model. The impact of these constants seems to be incomparably higher then impact of acting control, but constant K_{Pk} has to cover energy loss to the outside, so during the heating season a component of the vector f has most visible impact. In case of inability of heating elements to cover heat loss to the environment vector f takes negative value.

The question is how to manage the interconnected system without negative effect on temperature in the neighbouring room and especially in cases, where the heat transfer coefficient between the rooms take higher values, for example when glass walls are used. Respecting limited computing capacities of zone regulators was used an input-output linearization to simplify MIMO system.

III. THERMAL SPREAD AND MODEL OF THERMAL SYSTEM

Multivariable system containing mutual relations is decoupled in terms of regulation if each of the input control variables affects only one output controlled variable. Efforts to regulate each output of a multivariable system independently is the natural whereas the interaction of output quantities can lead to system instability, overshooting, waste of resources required for the control action, unsuccessful maintain the required values. Independence of each output variable of MIMO system is ensured by introducing a suitable compensator, in literature known as Decoupling control system. From (2.6) and (2.7) and the state description (2.8) it is possible to note that the state variables (T_C; T_L; T_R; T_T; T_B; T_I) are also outputs of the system. By derivation of the output variables we get the functional dependence of the input variables (T_{IC}; T_{IL}; T_{IR}; T_{IT}; T_{IB}; T_{II}). The principle of separation of the individual pairs of input-output lies in the elimination of expressions containing other state variables appearing in the first derivation (as the system is the relative order of one). This principle is used in the input-output linearization where the input signal u is designed to eliminate the non-linear members and the system seems to outwards linear. Since the described system is linear, the effort is to design input u to eliminate interconnections in the system. For designed input v when equality $\dot{y} = \dot{x} = v$ is valid, transfer functions of six decoupled systems would have the form $F_i(s) = \frac{Y_i(s)}{V_i(s)} = \frac{1}{s}$. If it is desirable the system to behave as a system of first order with time constant T and gain K, transfer function will have form $F_i(s) = \frac{Y_i(s)}{V_i(s)} = \frac{K_i}{T_i s+1}$. From equality $T_i \dot{y} + y = K_i v$ and $\dot{y} = \dot{x}$ for the inputs u without respecting failures, next equations are valid. For brevity only two of them are listed here: (3.1) – central room and (3.2) – room on the left.

$$u_1 = \frac{K_1}{T_1 b_{SA}} v_1 - \frac{(1 - T_1 K_{SS})}{T_1 b_{SA}} y_1 - \frac{K_{SL}}{b_{SA}} y_2 - \frac{K_{SP}}{b_{SA}} y_3 - \frac{K_{SH}}{b_{SA}} y_4 - \frac{K_{SD}}{b_{SA}} y_5 - \frac{K_{SB}}{b_{SA}} y_6 \tag{3.1}$$

$$u_2 = \frac{K_2}{T_2 b_{LA}} v_2 - \frac{K_{LS}}{b_{LA}} y_1 - \frac{(1 - T_2 K_{LL})}{T_2 b_{LA}} y_2 \tag{3.2}$$

Substituting (2.6) and (2.7) into (3.1) and (3.2) (substitution is between 6 equations, only two are listed), the system of sixth order will act as six independent linear systems of first order with failure f. The important task is to choose the right time constants $T_i$and gain K_i. It is possible to say that the choice is arbitrary. However, if the constants were chosen incorrectly, the controller with a limited control action would not be able to achieve the desired value of output quantify. Therefore, the constants are selected so that the transfer function describes the system as it would not be influenced by any links or disturbance. From (3.1) and (3.2) expressions for time constants of the gains (3.3)

$$c_p \rho V_x \frac{dT_x}{dt} = c_p \rho Q_x (T_{Ix} - T_x) \longrightarrow$$

$$\frac{T_x(s)}{T_{Ix}(s)} = \frac{Y_i(s)}{U_i(s)} = \frac{1}{\frac{V_x}{Q_x} s + 1} \longrightarrow K_i = 1;\ T_i = \frac{V_x}{Q_x} \tag{3.3}$$

can be written for $k \in \{C, L, R, T, B, I\}$ and $i \in \langle 1; 6 \rangle$. For a given model, time constant has value T_i = 1718.8 s. *Fig. 2* and *Fig. 3* shows the time response of the output variables (temperature of all surrounding offices). It is possible to observe the jump of the desired temperature of central office from 21°C to 22°C in time 2.5 10^4 s. Also required temperatures of surrounding rooms are changing sequentially in time 1.3; 1.6; 2; 3.5 and 4 10^4 s. The impact of control variables on room temperature of surrounding rooms is compared. In this case, especially the impact on central office. As it is shown in *Fig. 2*, any change of desired temperature of adjacent room has an effect of fluctuation of temperature of the central room. By using decoupling and optimizing parmeters of PI regulators in *Fig. 3* it is not possible to see any negative effects of the interconnection of system on controlled variables.

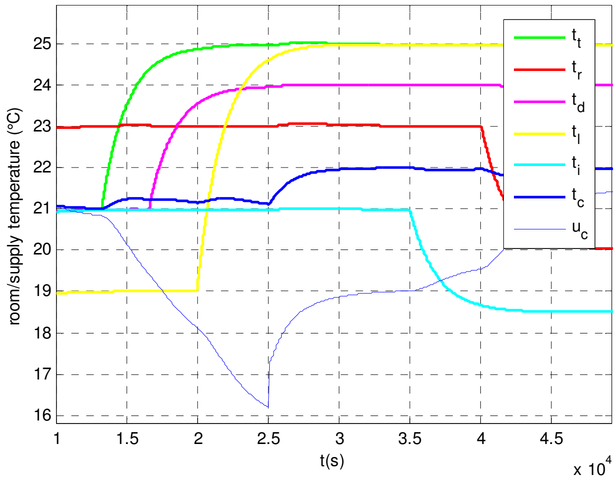

Fig. 2: Comparison of time response of room temperature without using decoupling

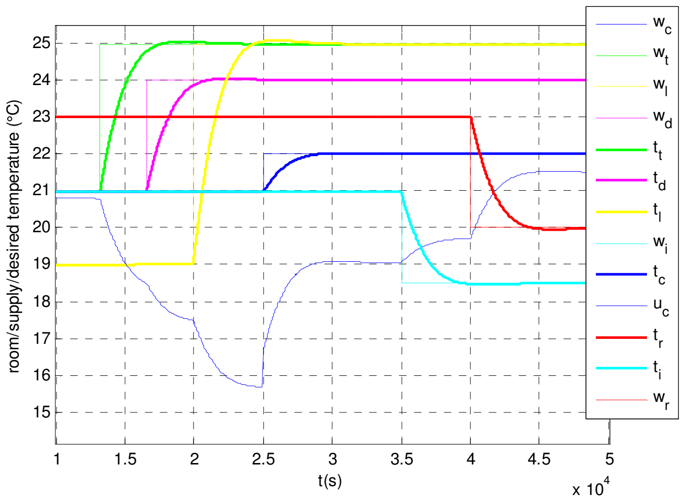

Fig. 3: Comparison of time response of room temperature using decoupling

Apparently it is not a significant overshoot of the temperature, but at reduced flow (for example, at the time of attenuation when the control variable does not have so strong influence of the room) temperature overshoots are getting bigger. Also with poorly selected parameters of zone controller, temperature oscillations of adjacent rooms caused by change of room state can have a negative impact on the actuator control circuit. Decoupled zone regulation can have a positive impact on the quality of regulation, energy costs and comfort, especially in buildings, where bounds are more evident (glass partitions, walls with high coefficient of thermal transmittance).

IV. Predictive Control

From previous figures is evident, that decoupled PI regulators are able to control room temperature of MIMO system with eliminating coupling between neighbour rooms.

There is a question why should decoupled system be regulated by predictive control? The main problem of room temperature control is slow dynamics which usually causes problems connected with temperature comfort and efficiency of used energy sources. Simple zone regulators are not able to ensure the required temperature of office exactly in time when people start or finish working. Often set time program of zone regulator starts to heat or cool office several hours before work shift. Since the optimal time needed for achievement required temperature respecting constraints of the system should be calculated automatically, MPC regulator seems to be a suitable choice. Objective of predictive controller is to find such sequence of future acting control *ch* (horizont control) to achieve the future desired response of system for *ph-ps* steps ahead, where *ph* is predictive horizont and *ps* is initial horizont. Mostly is a cost function defined as minimizing the sum of quadrates of future deviation at respecting constraints of input and output variables and the sum of quadrates of future command increment. From this reason it is necessary to predict the future behaviour of system for *ph* steps ahead in current time. Prediction of output of system is based on the model system and it is function of past inputs and also inputs, which is going to be used. For linear systems predicted response of the output can be obtained by the sum of two components. The first is a system response on past values of inputs provided that the last value of control will remain unchanged in the future (free response) and the second is system response on the estimated future value of inputs (forced response). In each sampling period the all future control sequences are calculated but only the first value of sequence is applied. In the next step of sampling new value of output is measured and the entire calculation is repeated with the values shifted by one step.

One of the most spread predictive algorithm is generalized predictive control GPC, which is based on parametric input/output model. [9] Consider a class of systems whose input/output behaviour can be described in the following CARIMA models (4.1), where u(t) is control variable, y(t) is

$$A(z^{-1})y(t) = B(z^{-1})u(t-1) + v(t) + w(t) \tag{4.1}$$

$$D(z^{-1})v(t) = C(z^{-1})\xi(t) \quad where:$$

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{na} z^{-na}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_{nb} z^{-nb}$$

$$C(z^{-1}) = 1 + c_1 z^{-1} + \cdots + c_{nc} z^{-nc}$$

$$D(z^{-1}) = 1 + d_1 z^{-1} + \cdots + d_{nd} z^{-nd}$$

control variable, y(t) is output, v(t) represents an internal failure, w(t) represents modelling failure, $\xi(t)$ is the perturbation acting as a zero mean white noise. Transfer function represents regulated system $P^u(z^{-1}) = \frac{z^{-1}B(z^{-1})}{A(z^{-1})}$. In the special case $D(z^{-1}) = 1 - z^{-1}$ integration component is explicitly incorporated into the design. Prediction of output is based on solving two Diophantine equations (4.2)

$$\begin{gathered} C(z^{-1}) = A(z^{-1})D(z^{-1})E(z^{-1}) + z^{-j}F_j(z^{-1}) \\ E_j(z^{-1})B(z^{-1}) = C(z^{-1})G_j(z^{-1}) + z^{-j}H_j(z^{-1}) \end{gathered} \tag{4.2}$$

(see [7,8,9] for details). Prediction of the output for *j* steps ahead is then obtained from (4.3).

$$\hat{y}(t+j/t) = G_j(z^{-1})D(z^{-1})u(t+j-1) + y_0(t+j/t)$$

$$\begin{aligned} C(z^{-1})y_0(t+j/t) &= H_j(z^{-1})D(z^{-1})u(t-1) \\ &+ F_j(z^{-1})y(t) \end{aligned} \tag{4.3}$$

It is composed of two parts: The first depends on sequence of future command increment (mentioned forced response) and the second depends on the data available at the time *t* (mentioned free response). Objective of design GPC is minimization of following cost function (4.4),

$$\begin{aligned} &J(t, ph, ch, sh, \lambda_y, \lambda_u) = \\ &\sum_{j=sh}^{ph} \lambda_y(j)\,[\hat{y}(t+j/t) - y^*(t+j)]^2 \\ &\qquad + \sum_{j=1}^{ch} \lambda_u(j)(\Delta u(t+j-1))^2 \end{aligned} \tag{4.4}$$

where $\Delta u(t) = u(t)(1 - z^{-1})$, $y^*(t+j)$ is known required signal and λ_y, λ_u are weighting coefficients. Vector of command increment *U(t+ch-1)* which minimizes the cost function is given by (4.5),

$$\begin{aligned} U(t+ch-1) = -\left[G_1^T \Lambda_y G_1 + \Lambda_u\right]^{-1} G_1^T \Lambda_y (Y_0(t) \\ - Y^*(t+ph)) \end{aligned} \tag{4.5}$$

where matrix $G_1$is reduced matrix G from (4.2) which contains elements of pulse characteristic of the system (see [10] for details). From vector (4.5) will be applied only first element during every iteration. By knowing polynomials $A(z^{-1})$, $B(z^{-1})$ of each decoupled system and future occupancy profile with required office temperature is possible ensure earlier reaction on changing conditions in the office. *Fig. 4* shows the difference between simple PI regulator and MPC regulator,

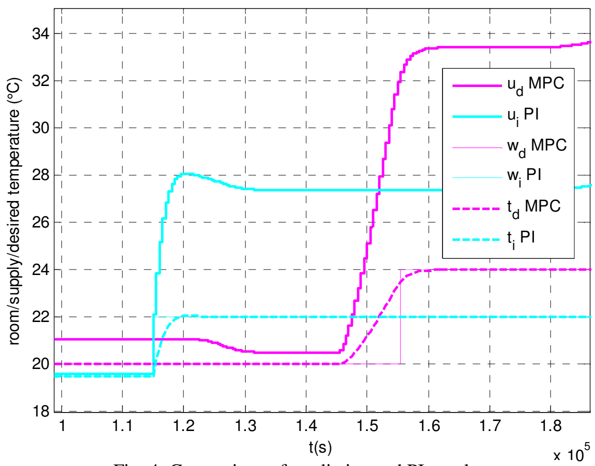

Fig. 4: Comparison of predictive and PI regularor

which has information about future occupancy of modelled office described in chapter 2. Occupancy profile is a signal which activates a component of cost function which penalizes control deviation. It means that during unoccupied period of office is penalized only command increment. Change of occupancy is visible for regulator within predictive horizont. Thanks to the introduction of occupancy signal, predictive regulator started to focus on achievement of required temperature exactly in time, when it is necessary [8]. It is obvious that MPC regulator started to control earlier than PI regulator, but due incorrectly set weighting coefficients λ_y, λ_u, control deviation is still appearing mainly. Literature offers several approaches how to change the cost function to eliminate this fact, for example penalization of acting control, not the increment $\Delta u(t)$[8]. Our approach focuses on fact, that increasing of impact on coefficient, which penalizes control deviation can cause instability of the system. From vector of command increment (4.5) only the first value is applied and control law is expressed by (4.6),

$$\Delta u(t) = -\sum_{j=sh}^{ph} \gamma_j (y_0(t+j/t) - y^*(t+j)) \qquad (4.6)$$

where gain coefficients γ_j for *j=sh…ph* are elements of the first row of matrix $\left[G_1^T \Lambda_y G_1 + \Lambda_u\right]^{-1} G_1^T \Lambda_y$. This control law can be expressed by standard linear form (4.7) of RST control law (see [8] for details).

$$S(z^{-1})\Delta u(t) + R(z^{-1})y(t) = T(z^{-1})y^*(t+j) \qquad (4.7)$$

Polynomials $S(z^{-1}), R(z^{-1})$ and $T(z^{-1})$ contains gain coefficients γ_j and matrixes from the solution of Diophantine equations (4.2). The roots of the characteristic equation of closed control circuit (4.8) determine the stability of system.

$$A(z^{-1})S(z^{-1}) - B(z^{-1})R(z^{-1}) = 0 \qquad (4.8)$$

We can observe that characteristic equation (4.8) also consists of polynomials $S(z^{-1}), R(z^{-1})$ which contain gain coefficients. *Fig. 5* shows what consequence can have an incorrect choice of weighting coefficients λ_y, λ_u for example, in an effort to reduce control deviation at the beginning of the occupancy period.

To achieve minimal control deviation at the beginning and end of the occupancy period without problems with stability, it is necessary to involve modified occupancy profile. At the beginning, it is important to ensure that designed weighting coefficients do not cause instability of the system. When it is impossible to improve the quality of regulation by setting of weighting coefficients or by changing of predictive or control horizont size it is possible to shift optimal solution from time *t* to time max *t-ch or to time t+ch* in case that the regulator has information about the future occupancy profile. The algorithm is clearer from *Fig. 6*.

Predicted regulator with *ch*=30, *ph-ps*=30, and sampling period 500 s predicts change of desired room temperature in time *t*=1.35 10e5 s. *Fig. 6* shows sequence of predicted output variable in time *t*=1.255 10e5 s (19 steps before change

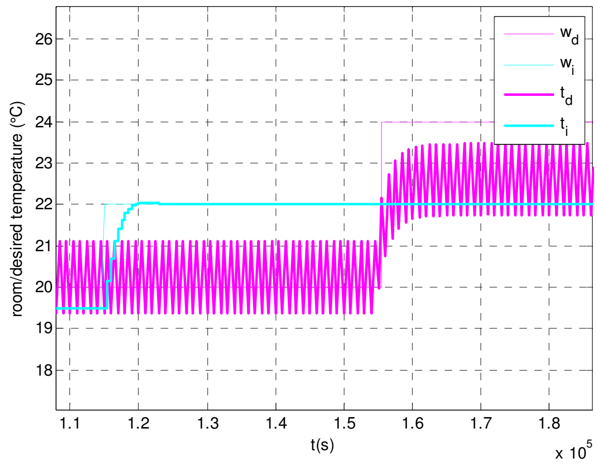

Fig. 5: The instability caused by increasing the weighting coefficients

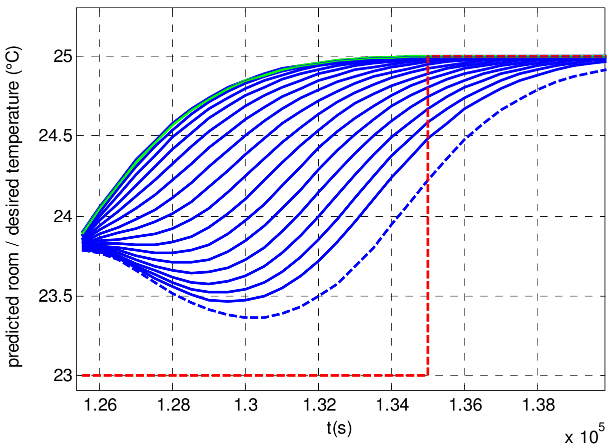

Fig. 6: Shifing of predicted output

of desired temperature). The sequence is computed in a cycle of 30 steps with similar weighting coefficients with shifted future step of desired temperature. It is not necessary to increase predictive horizont and increase size of matrices which inversion has to be computed, it is necessary only to compute them several times. Additional cost function, which penalizes mainly control deviation, but from real predicted step of required temperature.(in our case from time t=1.35 10e5 s – from the period of occupancy) then select the optimal time response and control output starts to effect. *Fig. 6* demonstrates an extreme situation, when weighting coefficient λ_y actuating to control deviation is undervalued. It is possible to observe the first prediction from sequence which is apparently deformed (blue dashed line). It is due the fact that normal prediction would not begin to work. The time response of room temperature is marked with green colour and is obvious that main criterion has been met. The same problem appears when the period of occupancy ends. Simple predictor would start to decrease supply temperature before people leave the office. In this case is necessary sequentially compute predicted time respond of room temperature also after the future end of the occupancy period. *Fig. 7* shows time response of decoupled systems controlled by predictive regulators.

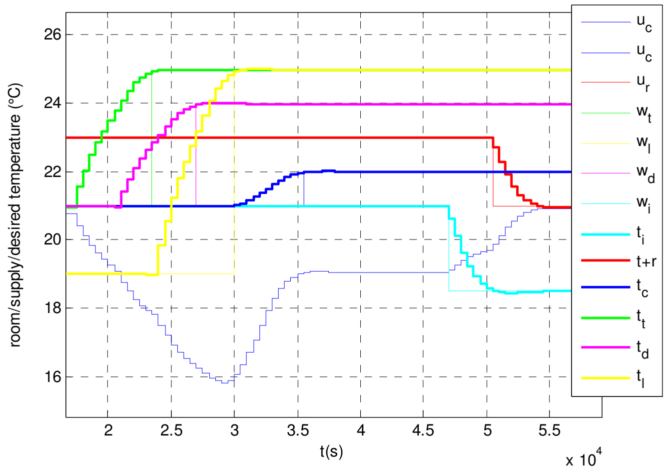

Fig. 7: Comparison of time response of room temperature using MPC decoupled regulator

Each of these regulators does not need prediction of neighbouring room temperature to predict own control. After every control intervention generated by predictive controller, components which ensure independence of each system is added, up to (3.1). These components include a temperature of neighbouring room and information about thermal conductivity of the wall and so on. This is a big advantage over methods like centralized predictive control or distributed predictive control [8] which are based on computing prediction from the state space model of the whole system. Inclusion of constraints to the system means to solve the problem of quadratic programming with standard form (4.9).

$$\begin{aligned} &\min x^T H x - g^T x \\ &Cx \geq c \end{aligned} \qquad (4.9)$$

Where C is a matrix of constant, c is the vector of constraints, which is necessary to actualize every period of sampling, H is Hesse matrix and g is a vector of the gradient which includes free response of the system (see [21] for details). Solving the problem of quadratic programming is well-known issue, but in our approach is addition in actualizing of matrix C, which defines the constraints of the input variable and control output. If the first element of predictive output without constrains is generated, it is sent to every regulator, especially to part which is responsible to ensure independence of the system. Here is calculated constraint for control signal v from (3.1) / (3.2). Constraints for final control u is often between 12 °C and 40 °C. Then actualized value of constraint of signal v is built into the recalculation of prediction via (4.9). When each zone regulator built control signal u according to (3.1) / (3.2) is recalculated predictive control v limited such that also addition from neighbouring rooms do not exceed primary defined restriction (mentioned range 12 °C and 40 °C).

CONCLUSION

Control of the indoor environment requires the analysis of the used technologies not only in terms of the impact on the quality of management, energy efficiency, but also in terms of a direct impact on humans. When controlling temperature, humidity, flow velocity, pressure and air quality, different kinds of limits appear that technologist and designer have to respect to ensure thermal comfort for people or technology. The result of solution of presented issue should be an optimized control structure which is able to provide the desired state of the indoor environment.

Zone control in recent office buildings usually uses terminal control components for preparing centrally distributed air with a constant temperature and relative humidity. These regulatory elements (active cooling beams, fan coils, etc.) together with central heating systems provide thermal and hygienic comfort. Often, however, without respecting of interconnections and interactions of the various parts of the building. The intention of the vast majority of investors, owners and users are reducing energy requirements for operation of buildings. This trend results in an ever-increasing number of installed consumption meters and sensors which monitor an actual energy requirement of the building. By using the network interface of zone controllers it is possible to monitor the actual room temperature, desired temperature and partly also disturbances which enter the controlled system (door and window contacts, attenuation). These collected data are used for elimination of interactions between control action on the adjacent room and thus save energy and increase comfort. Due to predictive regulator with known occupancy profile is possible to eliminate unwanted human intervention to setpoints and save energy.

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