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Model Predictive Control of a Combined Electrolyzer-Fuel Cell Educational Pilot Plant

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***Abstract*—In today's era of renewable energy, hydrogen fueled proton exchange membrane (PEM) fuel cells are considered as an important source of clean energy. As the technology is emerging fast, many universities and colleges have adopted fuel cells in their educational program. In this paper, we will present the modeling and control of the fuel cell pilot plant present in *Clean Energy Trainer*, which is used by students and researchers in many universities. The plant under consideration is a laboratory-scale pilot plant designed mainly for verifying the applicability of theoretically studied control strategies on the real-world application. The plant is a series connection of electrolyzer and a PEM fuel cell stack with one input and one output. The control of such a plant is the challenging research problem due to the nonlinearities, slow dynamics, dynamics and physical constraints. The control oriented data-driven model of the plant is developed and validated through a series of experiments. To tackle the electrolyzer-fuel cell control problem, we present a model predictive control (MPC) scheme that can take into account the physical constraints of the plant. In addition to the controller, a disturbance observer is designed to cope with the external disturbances and to avoid adverse effects on the system performance. Subsequently, the developed control scheme is successfully implemented in real-time. Highly satisfactory results are obtained, regarding reference tracking, constraint handling, and disturbance rejection.**

I. INTRODUCTION

Over the last decade, fuel cells and hydrogen energy technologies have received attention in power generation and automobile industries. Use of the fuel cells in these sectors has remarkable advantages like high efficiency, zero emission, no noise and low heat transmission. There are several types of fuel cells and in general, they all work in the same manner. Every fuel cell unit consists of three adjacent segments: the anode, cathode, and electrolyte. Two chemical reactions occur at the interfaces of the three different segments. As a result of these reactions, an electric current is generated by the consumption of fuel and water or carbon dioxide is produced as a by-product [1].

Among the family of fuel cells, the Proton Exchange Membrane (PEM) fuel cell is considered as the most promising one due to the small size, lightweight and quick startup [2]. A good overview of the PEM fuel cell applications can be found in [3]. Apart from the problems of integration and capital cost, the use of PEM fuel cell as a power source also needs a full control of dynamics and thus having a closed-loop control strategy. Many classical and advanced control methods have been proposed for the closed-loop control of PEM fuel cells. A dynamic feedforward controller was proposed in [4] to control air flow rate, proportional-integral (PI) controller in [5] to achieve power density and proportional-integral-derivative (PID) controller in [6] to keep power output at desired reference. The main advantage of these gain-based and rule-based control methods is their simple implementation. However, these controllers fail in handling multi-input multi-output (MIMO) and slow dynamic systems, physical constraints and do not guarantee optimal use of resources.

To improve the efficiency and safety of the PEM fuel cells several authors have proposed the use of advanced control techniques. generalized predictive control (GPC) in [7] for tracking output voltage. A model predictive control (MPC) scheme has also been investigated for the closed-loop control fuel cells. An on-line nonlinear MPC was proposed and the applicability of proposed controller was experimentally validated in [8]. A linear MPC was proposed in [8] and experimentally validated the applicability of proposed controller. A linear MPC was demonstrated to control the oxygen excess ratio [9] and to satisfy the hydrogen's expected quality [10]. All these proposed MPC schemes showed that the MPC outperforms the classical controllers in reference tracking, disturbance rejection, constraint handling and multi-variable control.

Model predictive control is a control strategy that offers attractive solutions for the control of constrained linear or non-linear systems and, more recently, also for the control of hybrid systems. MPC is an optimal control method, where the control action is obtained by solving a constrained finite time optimal control (CFTOC) problem for the current state of the plant at each sampling time. The sequence of optimal control inputs is computed for a predicted evolution of the system states over a finite horizon. However, only the first element of the control sequence is applied and the state of the system is then measured again at the next sampling time. This so-called receding horizon control (RHC) introduces feedback to the system, thereby allowing a compensation of potential modeling errors or disturbances acting on the system. These features in combination with the need of PEM fuel cell operation signify that the application of MPC to PEM fuel cell systems is an attractive solution.

Incorporation of two separate units; a ready to use hydrogen supply unit and a fuel cell unit is a well-known process, used in industry and has been presented in above all the papers, but there are no studies where a water electrolysis unit is incorporated into hydrogen fuel cell system. The plant under consideration is the combined system comprised of electrolyzer and fuel cell, which can be used in portable applications. Off-course this set-up emerge interesting question, is it possible to control such a system? This paper shows that the control of such a pilot plant is possible and answer the question by experimental results.

In this paper, we present a model predictive control strategy to control a laboratory plant called the *Clean Energy Trainer*[1]. The plant is comprised of an electrolyzer connected to the fuel cell stack. The task is to manipulate the voltage to the electrolyzer such that the output voltage from the fuel cell tracks user-supplied references. The hydrogen produced by the electrolyzer is accumulated in a storage tank. However, due to the setup of the particular plant under consideration, it is not possible to manipulate the hydrogen flow rate directly. Instead, the hydrogen flow rate is controlled indirectly by the input voltage to the electrolyzer. Admittedly, this is not a standard way of operating a fuel cell. However, it represents the only setup available by the Clear Energy Trainer. Since such a device is frequently used by many universities in their educational and research processes, we believe this setup is of interest to the community. In addition, it represents a challenging control problem on its own that is further complicated by the presence of physical constraints on the actuators. Moreover, operating conditions like room temperature, humidity, air flow rate, and measurement noise make the control design procedure challenging. To cope with these factors, an optimization-based control strategy is adopted in this paper, together by using a disturbance observer to deduce external unmeasured disturbances.

II. Process Description

A. Fuel Cell Process

Power generation using PEM fuel cell is a continuous process wherein hydrogen is fed to the anode side and oxygen to the cathode side of the cell. Process of generating electric energy from the chemical reactions can be described as follows:

$$H_2 + \frac{1}{2}O_2 \longrightarrow H_2O. \tag{1}$$

In addition to the fuel cell stack, the plant comprised of other subunits such as a voltage regulator to regulate voltage supply of electrolyzer, an electrolyzer to produce hydrogen, canisters to collect generated hydrogen and oxygen, and the load. In practice, the power required for electrolysis process can be obtained from solar or wind energy but in our case, we are using readily available power in the lab as our focus is mainly on the control of pilot plant. In our plant, the voltage to electrolyzer is supplied through USB-data monitor.

B. Experimental Set-up

Considered electrolyzer-PEM fuel cell plant is one of the experiments in laboratory-scale *Clean Energy Trainer Kit* provided by *Heliocentris Energiesysteme GmbH, Germany*. Fig. 1 shows the experimental set-up available in the process control laboratory. This plant consists of several sub-units, the technical specifications of each unit are listed in Table V.

1) Hydrogen Generation: Hydrogen (H_2) is the basic fuel for the stack to generate electricity. Hydrogen can be produced from both renewable and non-renewable sources using a variety of techniques. In our experimental set-up we are producing hydrogen by splitting water. We used electrolyzers to decompose water into hydrogen and oxygen by the electrolysis process. A DC electrical power source is connected to two electrodes of electrolyzers which are dipped into distilled water supplied by two canisters. The electrolyzer is comprised of an anode and the cathode. The water decomposition reactions can be written as

$$H_2O \longrightarrow \frac{1}{2}O_2 + H_2. \tag{2}$$

The generated hydrogen is then collected in another canister connected to the input side of the cell stack.

[1]https://www.heliocentris.com/en/academia/education-products/clean-energy-trainer/

TABLE I. Components and specifications of the electrolyzer-PEM fuel cell set-up [11].

Component Name	Specification	Items
Fuel cell stack	Max. power: 1 W Max. output voltage without load: 4.8 V	1
Cell	Max. power without load: 0.2 W Max. output voltage: 0.96 V	5
Electrolyzer	Max. input voltage: 2 V Power: 1.16 W Hydrogen generation: $5\,\mathrm{cm^3 min^{-1}}$ Oxygen generation: $2.5\,\mathrm{cm^3 min^{-1}}$	2
USB data monitor	Max. power: 10 W Max. current: 5 A Max. voltage: 10 V DC	2
Load (bulbs)	Power of each bulb: 2.4 W	2
Water canister	Volume: $30\,\mathrm{cm^3}$	2
Oxygen canister	Volume: $30\,\mathrm{cm^3}$	1
Hydrogen canister	Volume: $30\,\mathrm{cm^3}$	1
Power source	6 VDC/3.3 A	1

2) Fuel Cell Stack: In general, PEM fuel cell stacks consist of several cells assembled together to meet the power demands of a variety of commercial applications [12]. In our set-up, it consists of five fuel cells to achieve the desired voltage. Continuously generated hydrogen flows to the cell stack through the canister. Inside the stack, hydrogen reacts with atmospheric air to produce electricity and pure water. The performance of fuel cell stack mainly depends on hydrogen flow rate, temperature, air flow rate and humidity [12].

3) Data Monitor: We used two USB data monitors, one of them is used as a voltage source to regulate the input voltage of electrolyzers. It is worth to mention that it shows $5-6$ % (i.e 0.10 V-0.12 V) of error in the output. So, it is mandatory to feed 2.12 V to get 2 V. The another monitor is used to measure the output voltage generated by the cell.

4) MATLAB *Interface:* The device drivers were written to establish the communication between USB data monitor and the host PC. Further, a MATLAB/Simulink block was created to control the whole process through the MATLAB.

III. Electrolyzer-Fuel Cell Plant Model Identification

The model is based on experimental data which takes into consideration the dynamic characteristics of the plant. It describes how the output voltage of fuel cell is affected due to changes in the input voltage of electrolyzers at a constant load. Steps involved in model identification are described below,

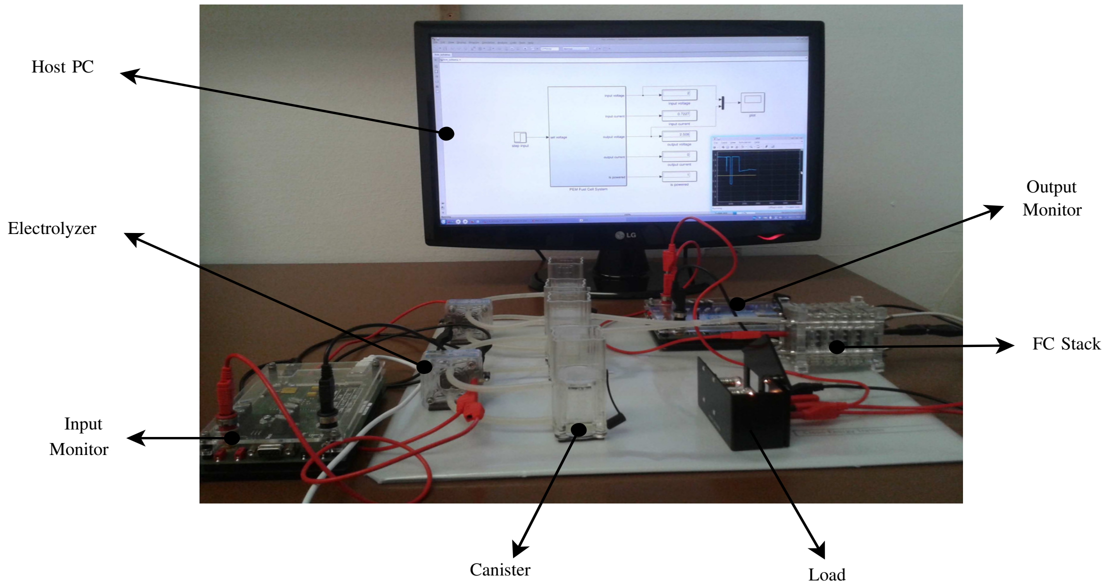

Fig. 1. A combined electrolyzer-PEM fuel cell experimental set-up at the process control laboratory.

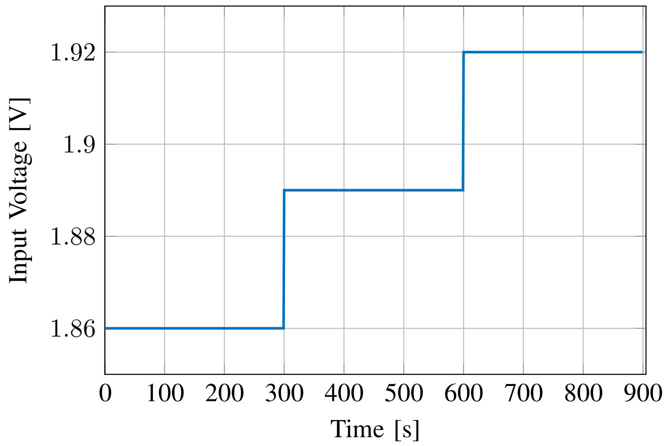

Fig. 2. Input excitation signal for model identification.

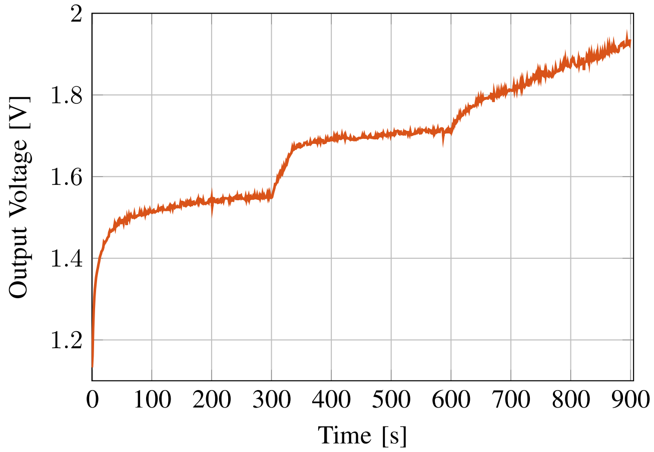

Fig. 3. Measured output response corresponding to input excitation signal.

1) Process variables and operating ranges: For the system identification task we considered input voltage (V_{IN}) as a manipulated variable and output voltage (V_{OUT}) as a measured variable. Despite the fact that electrolyzer operates in the voltage range of $0\,\mathrm{V}$-$2.12\,\mathrm{V}$, but practically it starts to generate hydrogen at $1.80\,\mathrm{V}$. Therefore, we set the operating range of the input voltage as $1.80\,\mathrm{V}$-$2.12\,\mathrm{V}$ which means that electrolyzers operating range is only $0.32\,\mathrm{V}$.

2) Input-output data acquisition: To obtain the system time constant and the gain, input signal was set as shown in Fig. 2. Input is applied in staircase form with a step change of $0.03\,\mathrm{V}$. Two data monitors were used to acquire the electrolyzers input voltage and fuel cells output voltage at the sampling time of $0.5\,\mathrm{s}$. A load of $4.8\,\mathrm{W}$ is connected throughout the experiment and data points were stored for identification and validation purpose. The measured output response of the plant is shown in Fig. 3.

3) Data pre-processing: Measured data were accompanied by undesired noise and other anomalies. To improve the identification, measured data were normalized before used in MATLAB's System Identification Toolbox.

4) Model selection: The function "`n4sid`" of the MATLAB's System Identification Toolbox and the measured data were utilized in estimating a set of discrete time state space models ranging from first order to fifth order. The function"`n4sid`" uses numerical subspace algorithm to identify state space model of the system. All the models were targeted for "prediction" focus and disturbance model estimation. Table II shows the values of data fitting, final prediction error (FPE) and mean square error (MSE) for different models. The detailed description about FPE and MSE is given in [13]. It can be observed that there is a very small difference in data fitting and error values among the five models. Considering the model complexity and data fitting values, the first order is selected which shows approximately the same dynamics as that of the fifth-order model as there is only 1% difference in fit.

TABLE II. VALUES OF DATA FITTING EFFECTIVENESS FOR THE DIFFERENT MODELS.

Model Order	Fit To Estimated Data	FPE	MSE
1	95.61 %	3.924×10^{-5}	3.90×10^{-5}
2	96.57 %	2.415×10^{-5}	2.394×10^{-5}
3	96.64 %	2.324×10^{-5}	2.293×10^{-5}
4	96.73 %	2.256×10^{-5}	2.187×10^{-5}
5	96.84 %	2.107×10^{-5}	2.080×10^{-5}

5) Parameter estimation: In this step, parameter estimation of the best fit prediction model was carried out and as a result of that, we obtained an linear-time invariant state space model of plant in discrete time given as

$$x_{t+T_s} = 0.9960x_t + 0.0083(u_t - u_{ss}), \tag{3a}$$
$$y_t = x_t + y_{ss}, \tag{3b}$$

where $x \in \mathbb{R}^n$ is the state variable, $u \in \mathbb{R}^l$ is the control input variable and $y \in \mathbb{R}^m$ is the output variable. Moreover, the constants $u_{ss} = 1.84\,\text{V}$ and $y_{ss} = 1.71\,\text{V}$ are the steady state values of input and output respectively.

6) Model validation: In this task, the estimated model (3) is validated with measured data to check if model in (3) fits appropriately to the real system. To show the ability of the model to predict future dynamics, we compared measured output with 10-step-ahead prediction output of the estimated model. Fig. 4 shows the response of normalized measured output and 10-step-ahead prediction output. The percentage of fit is a statistical measure of how well the predicted output matches with the measured output. The percentage of fit was utilized as a criteria to select the best prediction model to represent plant behavior. The cost function utilized to determine the goodness of fit was Normalized root mean square (NRMS) and we obtained 90.66 % of data fitting for 10-step-ahead prediction.

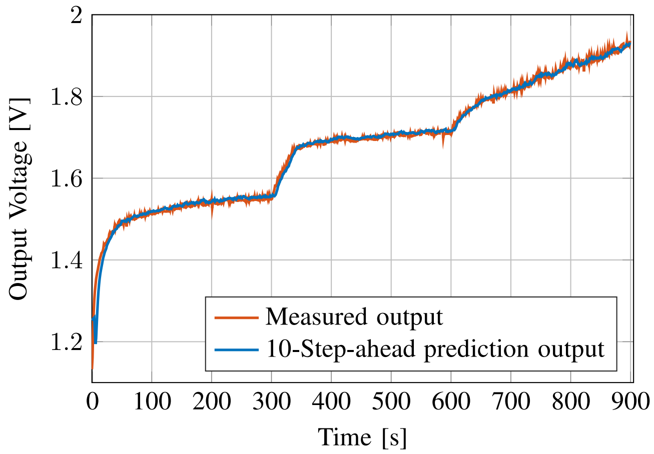

Fig. 4. Measured and 10-step-ahead prediction output.

IV. CONTROL PROBLEM

To devise appropriate control strategy for experimental set-up, it is essential to understand the nature of control problem first. The closed-loop control scheme is shown in the Fig. 5. This control loop scheme is simplified, thus, we combine the observer and MPC into one block. The control objective is to control desired output voltage (V_{OUT}) of fuel cell stack by manipulating the input voltage of electrolyzers (V_{IN}). The idea is to obtain optimal input voltage by solving an optimization problem at each sampling time considering the prediction model in (3), the objective function (reference tracking) and constraints on input. In this work, we are more focused towards the plant control rather than improving the efficiency. In our experimental set-up, the hydrogen demand is fulfilled by generating hydrogen by water electrolysis using electrolyzers which have 76% degree of energetic efficiency whereas the total degree of energetic efficiency of a fuel cell including electrolyzers is around 48% [11]. The plat is influenced by operating conditions like temperature, humidity and air flow rates. In simulations, this works perfect, but in practice, there is always plant-model mismatch and to tackle with such a problem in next section we will present disturbance modeling approach.

A. Model Predictive Control

This section features the synthesis of MPC strategy. Design of the MPC strategy is two-fold. First, we present the design of a disturbance model and second, we formulate the MPC problem.

The main principle of disturbance modeling approach lies in extending the state space model by one disturbance signal $d_i(t)$ for each plant output signal $y_i(t)$. A simple Luenberger observer is designed to estimate the disturbance signal. Formally, the extended state space model can be expressed as

$$x_{t+T_s} = Ax_t + Bu_t, \tag{4a}$$
$$y_t = Cx_t + Du_t + Fd_t, \tag{4b}$$
$$d_{t+T_s} = d_t. \tag{4c}$$

The matrix F is a user defined matrix, usually equal to identity matrix of appropriate dimensions. Define a new vector of states as

$$x_{\text{e}} = \begin{bmatrix} x \\ d \end{bmatrix}, \tag{5}$$

and

$$A_{\text{e}} = \begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix}, \quad B_{\text{e}} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \tag{6a}$$
$$C_{\text{e}} = \begin{bmatrix} C & F \end{bmatrix}. \tag{6b}$$

After state space model in (4) with definitions in (5) and (6) we can formulate an extended state space model, given by

$$x_{\text{e},t+T_s} = A_{\text{e}}x_{\text{e},t} + B_{\text{e}}u_t, \tag{7a}$$
$$y_t = C_{\text{e}}x_{\text{e},t} + Du_t. \tag{7b}$$

Based on the extended matrices (6), an observer is designed via pole placement method.

Next, we show how the MPC strategy is formulated as a constrained finite-time optimal control problem with a predic-

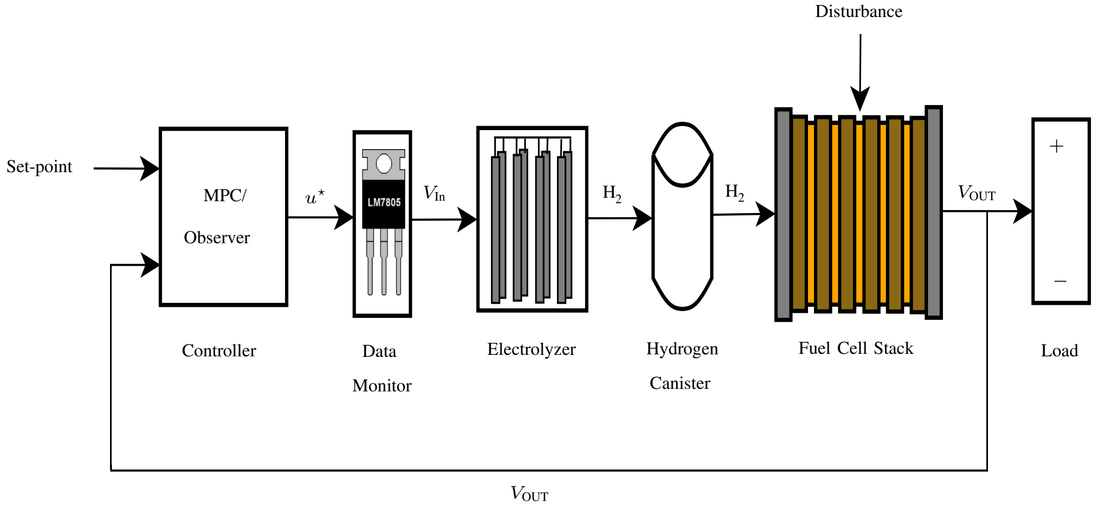

Fig. 5. The closed-loop control configuration with MPC for electrolyzer-PEM fuel cell plant.

tion horizon N and it is given by

$$\min_{u_0,\dots,u_{N-1}} \sum_{k=0}^{N-1} \left(||y_k - r||_Q^2 + ||\Delta u_k||_R^2\right) \tag{8a}$$

$$\text{s.t. } x_{k+1} = Ax_k + Bu_k, \tag{8b}$$

$$y_k = Cx_k + Du_k + Fd_k, \tag{8c}$$

$$d_{k+1} = d_k, \tag{8d}$$

$$\Delta u_k = u_k - u_{k-1}, \tag{8e}$$

$$u_{\min} \leq u_k \leq u_{\max}, \tag{8f}$$

$$x_0 = x(t), \tag{8g}$$

$$d_0 = d(t), \tag{8h}$$

where (8a) is the cost function where are penalized weighted squared 2-norms of respective quantities, i.e., $||z||_M^2 = z^\intercal M z$. Weighting matrices are positive definite matrices and $Q \in \mathbb{R}^{n_x \times n_x}$ and $R \in \mathbb{R}^{n_u \times n_u}$. Next, constraints (8b)- (8f) are enforced for $k = 0, \dots, N-1$. Furthermore, the control action increment is defined as $\Delta u_k = u_k - u_{k-1}$. The reference r is assumed to be constant during whole prediction horizon.

The optimization problem in (8) is a quadratic programming problem (QP) with quadratic objective function (8a) and linear constraints (8b)-(8f). Such a problem can be solved via state of the art solvers like GUROBI or CPLEX. The QP (8) is initialized at

- $x_0 = x(t)$ as the current estimation of the states in (4),
- $d_0 = d(t)$ as the current estimate of unmeasured disturbances,
- $r = r(t)$ as the user defined value of the reference,
- $u_{-1} = u(t - T_s)$ as the value of previous control action.

By solving the QP (8) initialized at mentioned values, we obtain the open-loop sequence of control inputs $u_0, \dots, u_{N-1}$, from which value of u_0 is applied as a control action to the plant. By repeating this procedure every sampling instant, we achieve closed-loop control and enforce stability. To such a strategy is often referred to as a Receding Horizon Policy [14].

V. Experimental Results

In this section, the experimental results of MPC scheme implemented for electrolyzer-fuel cell plant control problem and performance index values are presented. The developed MPC scheme is tested for reference tracking, disturbance and constraints handling.

The linear MPC employing an on-line optimization method was implemented in MATLAB using multi-parametric toolbox (MPT) [15] with prediction horizon N equal to 10, input manipulation weights R equal to 1 and target error cost Q equal to 100. Constraint on input voltage $u_{\min}$ and $u_{\max}$ was set to 1.81 V and 2.12 V respectively.

Initially, input and output were kept at steady state values and then a step change in a reference is given. Fig. 6 shows the output response of the plant for variable reference. In first step change from 1.71 V to 1.81 V, output took 40 s to reach the desired reference which is due to the large increase in the reference. In the subsequent steps, output took less than 20 s to achieve reference. Fig. 7 shows the manipulated variable

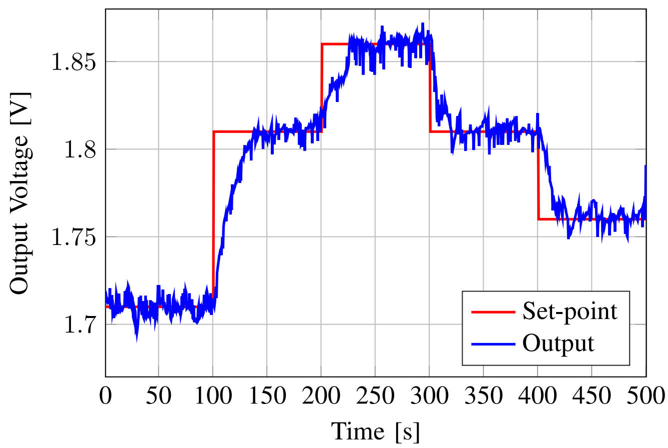

Fig. 6. Output response of the electrolyzer-PEM fuel cell system controlled by the linear MPC.

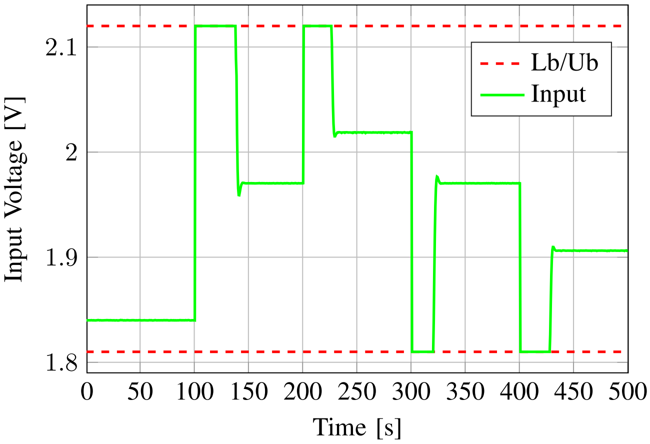

Fig. 7. Control input profile of electrolyzer-PEM fuel cell system controlled by the linear MPC.

profile. It can be seen that as the reference increases there is an increase in the input voltage to generate more hydrogen and subsequently produce more voltage from the fuel cell. We can observe that the input voltage lies within the constraints.

In order to see the performance of MPC strategy, we used quantitative values of settling time (τ_s), mean squared error (MSE), Integral Square Error (ISE), integral absolute error (IAE) and integral squared control efforts (ISCE). Table V shows the values of performance indices obtained for experimental results.

The settling time of the output voltage is different at each step change in reference and depends on step size. The average settling time of four step changes is given in the table and other indices are calculated for the whole trajectory. In terms of error between desired and measured output, MPC shows very less error which in turn shows that MPC drives output very close to the reference. In terms of ISE and IAE, MPC shows satisfactory performance. To show the input voltage required by the electrolyzer to achieve desired output voltage reference trajectory we calculated the controller efforts ($\sum \Delta u^2$).

TABLE III. PERFORMANCE OF THE ELECTROLYZER-PEM FUEL CELL PILOT PLANT.

τ_s	MSE	ISE	IAE	ISCE
28 s	0.0002	0.2811	8.8625	0.1859

VI. CONCLUSIONS

The control oriented data driven model and the model predictive controller for the combined electrolyzer-fuel plant called *Clean Energy Trainer* [11] used in many universities and colleges has been developed. The designed MPC uses parameter adaptation to electrolyzer input voltage. Moreover, the disturbance observer has been designed to mitigate the effect of pilot plant's operating conditions. The control strategy has been tested and validated through the series of experimental results. The results of electrolyzer-fuel cell plant controlled by the constrained linear MPC showed highly satisfactory results for variable reference tracking, disturbance rejection, and handling physical constraints on electrolyzer input voltage.

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