Gaussian Trajectories in Motion Control for Camless Engines

Benedikt Haus, Paolo Mercorelli Institute of Product and Process Innovation Leuphana University of Lueneburg Volgershall 1, D-21339 Lueneburg, Germany Tel.: +49-(0)4131-677-5571, Fax: +49-(0)4131-677-5300 Email: {haus,mercorelli}@leuphana.de

Abstract—In the last few years, variable engine valve control has attracted a lot of attention because of its ability to reduce pumping losses and increase torque performance over a wider speed and load range. Variable valve timing also allows control of internal exhaust gas recirculation, thus improving fuel economy and reducing NOx emissions. One of the most important issues in this context is to track suitable variable (optimized in terms of engine speed and load) motion profiles for the intake and exhaust valves. This can be achieved using dedicated actuators for the valves instead of a traditional camshaft. This contribution considers a new kind of actuator for this purpose and its control for motion tracking in the context of camless systems. However, this paper's main intention is to introduce a method of generating variable engine valve trajectories that are based on Gaussian curves and exemplarily provide the reader with information on how to exploit their favorable mathematical properties for control design purposes. As a demonstration of this kind of curve's variability, a delay-compensating phase-adaptive feedforward action is derived from a linear model description of the actuator. Simulations show the effectiveness of a simple heuristic delay-estimation algorithm in combination with the mentioned feedforward action.

Index Terms—Engine applications, servo hydraulic systems, trajectory generation

I. INTRODUCTION

Conventional valve actuators with camshafts often have no variability of the valve trajectory. It is the same in all speed and load ranges. Implementation of the Miller or Atkinson Cycle could achieve improvements in fuel consumption and exhaust emission. This approach, however, would lead to a power reduction as well. Therefore, by charging the filling phase of the cylinder via a compressor or a turbocharger, the power reduction in comparison with conventional internal combustion engines could be compensated. This compromise can be circumvented with a variable valve timing and stroke as proposed in this contribution. Fig. 1 shows the full potential variability of the engine valve trajectory. While the general properties of the curve are always the same (closed - open closed), there are different parameters relevant to the application. These are mainly the maximum valve lift, the opening duration and the opening timing. Of course, the general shape of the curve must be considered as well, since it is implicitly defined by the (traditionally fixed) camshaft profile. In dependence of engine speed and load, these parameters

Nils Werner Faculty of Mechanical Process Engineering and Maritime Technology University of Applied Sciences Flensburg D-24943 Flensburg, Germany Email: nils.werner@hs-flensburg.de

must be changed in an optimal way to achieve the goals of reduction in fuel consumption, a cleaner combustion and more power. Full variability can be achieved by employing



Fig. 1. Classification of the variable parameters of the valve trajectory actuators instead of the traditional camshaft. If these can be controlled to track user-defined trajectories, all parameters can be changed. These are usually electromagnetic actuators or combinations with pneumatic or hydraulic servo-systems. The presented actuator consists of a piezo-electric actuator with a hydraulic displacement amplifier, driving a servo valve that is in turn driving one or more double-acting hydraulic cylinders. This architecture combines the advantages of piezo actuators (speed and precision) with those of hydraulic systems (high power and compactness of the actuator itself). The actuator can be seen as two decoupled subsystems:

- Piezo actuator with hydraulic displacement amplifier
 - input signals: input voltage of the piezo actuator
 - output signal: position of the valve spool
- Servo valve and hydraulic cylinder with the engine valve
 - input signal: position of the valve spool
 - output signal: position of the engine valve.

Obviously, retroactive effects caused by the hydraulic subsystem, which are acting on the valve spool position, are not considered in the overall model as the authors deem them negligible in satisfying accuracy, based on experimental experience. In [1], an optimization-based polynomial trajectory generation algorithm for variable valve control in camless combustion engines is presented. However, in practical applications, schemes with as few parameters as possible are helpful. [2] gives an overview of different analytic expressions of elementary trajectories, e.g. in an exponential form. Using Gaussian bell curve-like trajectories like those in [3], [4], the number of parameters to achieve full variability can be reduced to three. The paper is structured in the following way. Section II gives some details on the modeling of the proposed actuator, which is divided in two decoupled subsystems. Section III explains how the Gaussian trajectories are generated using analytically evaluated equations. The simple heuristic time delay estimation algorithm that was developed in [5] is described in section IV. Section V shows how to deploy the phase correction in the trajectory generation to yield an adaptive feedforward action. Simulations and a conclusion close the paper.

II. DESCRIPTION OF THE WHOLE SYSTEM AND SOME SPECIFICATIONS

The actuator described in the previous section is modeled as three submodels: The double-acting hydraulic cylinder (see Fig. 3), the hydraulic displacement amplifier (which is shown in Fig. 2) and a five milliseconds time delay of the signal $x_2(t)$ (the valve spool position) between the two former parts, representing the (so far, unmodeled) hysteresis caused by friction. The cylinder is modeled in a linearized form and described by simple differential equations while the displacement amplifier model is modeled in a SISO state space form. As detailed in [6], the corresponding state space parameters are:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}V_z(t), \qquad \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t), \qquad (1)$$

$$\mathbf{x}(t) = \begin{bmatrix} x_2(t) \\ \dot{x}_2(t) \end{bmatrix}, \qquad \mathbf{A} = \begin{bmatrix} 0 & 1 \\ A_{21} & -\frac{D_{VS}}{m_{VS}} \end{bmatrix}, \text{ with }$$

$$A_{21} = \left(\frac{K_{FL2}^2}{K_{FL1} + K_{FL2}} - K_{VS} - K_{FL2} + \frac{\left(\frac{K_{FL1} \cdot K_{FL2}}{K_{FL1} + K_{FL2}}\right)^2}{K + K_x + K_{FL1} - \frac{K_{FL1}^2}{K_{FL1} + K_{FL2}}}\right) \cdot \frac{1}{m_{VS}},$$

$$\mathbf{B} = \frac{A_1}{A_2} \begin{bmatrix} 0 \\ \frac{D_x \cdot K_x \cdot \frac{K_{FL1} \cdot K_{FL2}}{K_{FL1} + K_{FL2}}} \\ \frac{(K + K_x + F_{FL1} - \frac{K_{FL1}^2}{K_{FL1} + K_{FL2}})}{\left(\frac{K + K_x + F_{FL1} - \frac{K_{FL1}^2}{K_{FL1} + K_{FL2}}\right)m_{VS}} \end{bmatrix}, \mathbf{C}_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

where A_1 is the surface of the piezo-side piston and A_2 that on the servo valve side. The hydraulic cylinder (driving the input and exhaust valves) is depicted in Fig. 3 and can be modelled in the following way, as described in [7]. In general, it is a nonlinear system [8]. However, for constant pressures, the volumetric flow into and out of the cylinder $Q_{th}(t)$ can be approximated to be proportional to the length of the opening slit that equals $x_2(t) - \overline{x}_2$:

$$Q_{th}(t) = (x_2(t) - \overline{x}_2(t)) \cdot K_{SP}, \qquad (2)$$

where the model parameter K_{SP} described in [6] is used. It includes hydraulic fluid density, tank pressure, system pressure of the hydraulic aggregate as well as some parameters of the 4/3 way servo valve. As described in [6], the differential equations describing the valve motion are linearized and simplified using several approximations:

$$\dot{y}(t) = K_H Q_{th} \tag{3}$$



Fig. 2. Hydraulic displacement amplifier (equivalent linear mechanical model)



Fig. 3. Double-acting hydraulic cylinder and the servo valve

Here, K_H is a constant model parameter, depending on the steady-state behavior of the hydraulic cylinder.

III. REALISATION OF THE VARIABLE TRAJECTORIES

The desired engine valve trajectory is chosen as a Gaussian

$$y_d(t) = H \cdot e^{-\left(\frac{mt+a}{b}\right)^2} , \qquad (4)$$

which has favourable properties and belongs to the class of C^{∞} functions. The expression mt + a denotes a periodic and resettable ramp function covering the range -360° (crank angle) to 360° . The ramp mt, with $m = n \cdot \frac{360^{\circ}}{60s}$ (where n is the engine speed in RPM and must be kept constant within a period) is implemented in dSpace as a resettable integrator:

$$\int n \cdot \frac{360^{\circ}}{60\mathrm{s}} dt,\tag{5}$$

that is set to 0 as soon as it reaches 720° , implementing a crank angle as typical for four-stroke engines (see Fig. 4). The constant *a* has a value of -360° , so the numerator maintains in $[-360^{\circ}, 360^{\circ}]$, yielding the characteristic bell curve y_d .

To increase the valve opening duration, one can simply freeze the crank angle in implementation, achieving a behaviour like that shown in Fig. 5, resulting in a change of the valve profile like depicted in Fig. 6. The constant $b = 65^{\circ}$ is a parameter that controls the aperture of the Gaussian, different manifestations can be seen in Fig. 7. The parameter is proportional to the full time width of the valve curve at a height of $\frac{H}{2}$ with a factor of $\frac{m}{2\sqrt{\ln 2}}$. Here, H = 10 mm denotes the maximum valve lift. The main advantage of this symbolic approach function is its infinite differentiability which is beneficial for calculating the time derivatives of the desired trajectory for use in the control structure, e.g., the first time derivative

$$\dot{y}_d(t) = -2m \frac{mt+a}{b^2} \cdot \underbrace{H \cdot e^{-\left(\frac{mt+a}{b}\right)^2}}_{y_d(t)} .$$
(6)



Fig. 4. Graphical presentation of the automatically resettable ramp function used in the Gaussian, in the case of 8000 $\rm min^{-1}$



Fig. 5. Holding the ramp function at 360 $^{\circ}$ results in a prolonged opening duration of the valve at the peak position



Fig. 6. Demonstration of the consequence of holding the ramp function at 360 $^\circ$



Fig. 7. Valve trajectory with different parameters for the opening time

Fig. 8 shows the shape of the generated trajectory in the worst case of n = 8000 RPM as well as the relevant region of its frequency spectrum. Here, frequency components larger than 500 Hz can be neglected in good accuracy. A Fourier analysis like this can be helpful when using approximations in order to simplify a linear model, which is often necessary to invert the system to design a feedforward action. A simple Bode plot of the original and the simplified model, in combination with the trajectory's spectrum, will tell if the simplification is acceptable. In this paper, however, the subsystem model in question (the hydraulic displacement amplifier) is differentially flat (see section V), so no further model order reductions are necessary and the feedforward action design exploits the flatness property.



Fig. 8. Desired valve trajectory and its frequency content (FFT) at $n_e=8000 \ RPM$

IV. REALISATION OF A VARIABLE PHASE SHIFTING CONTROL BASED STRATEGY

The time delay, which is caused by an unmodeled hysteresis related to friction, must be determined to allow for heuristic phase compensation. For this purpose, the algorithm first presented in [5] that can be seen in Fig. 9, is used. It is based on a comparison of the current valve lift with the reference signal. The time that passes between the reference trajectory reaching a specified threshold, and the obtained one reaching it, too, is determined by using an integrator, very similarly to the ramp generation in section III. The resulting time is multiplied by a factor of $-6^{\circ} \cdot n$, with n being the speed of the engine, to yield the phase correction c in degrees (in analogy to the crank angle). The implementation which can be seen in in Fig. 9 results from (5). The phase calculation algorithm is activated when an Enable Bit (see Fig. 9) is set to true. A reasonable calculation-starting condition to be chosen is $y_d \leq 1 \text{ mm } \land y \leq 1 \text{ mm}$ to ensure that the whole phase difference is captured and not only a fraction of it. The result is the necessary phase correction (hence the negative sign) in degrees. The phase correction is added to the periodical ramp, resulting in mt + c, which maintains inside the crank angle interval $[0^\circ, 720^\circ]$ (see next section).



Fig. 9. Block diagram of the phase correction algorithm

V. PHASE-ADAPTIVE FEEDFORWARD CONTROL

Exploiting the infinite differentiability of (4), a flatnessbased feedforward action of the hydraulic cylinder can be derived. In the last decade, a large number of papers on control applications using flatness-based control techniques was published. Trajectory tracking and feedforward control are simple if the output to be controlled is *flat*. Flatness-based control has already been used to address soft landing problems, see [9] and [10]. A dynamical system is differentially flat if it is possible to find a set of outputs (the number of output variables is equal to the number of inputs) that allow for an algebraic parametrisation of all state variables and all input variables using those flat outputs and their derivatives. Considering a system with n state variables $\mathbf{x} \in \mathbb{R}^n$, and m inputs $\mathbf{u} \in \mathbb{R}^m$, the system is differentially flat, see [11], if the outputs $\mathbf{y} = \mathbf{y}(\mathbf{x}, \mathbf{u}, \dot{\mathbf{u}}, \dots, \mathbf{u}^{(p)}) \in \mathbb{R}^m$ and their time derivatives algebraically define both the states and the inputs according to

$$\mathbf{x} = \mathbf{x}(\mathbf{y}, \dot{\mathbf{y}}, \dots, \mathbf{y}^{(q)}), \quad \mathbf{u} = \mathbf{u}(\mathbf{y}, \dot{\mathbf{y}}, \dots, \mathbf{y}^{(q+1)}).$$
(7)

Introducing a phase correction c, the desired trajectory is generated as:

$$y_d(t,c) = H \cdot e^{-\left(\frac{mt+a+c}{b}\right)^2},\tag{8}$$

and its derivative, expressed with a time and phase dependent function c_1 is:

$$\dot{y}_d(t,c) = c_1(t,c)y_d(t) = -2m\frac{mt+a+c}{b^2}y_d(t)$$
 (9)

while the corresponding acceleration is

$$\ddot{y}_d(t,c) = c_2(t,c)y_d(t) = \frac{2m^2(2(mt+a+c)^2 - b^2)}{b^4}y_d(t).$$
(10)

In order to obtain a feedforward action, eqs. (2,3) are inverted. With the practical design choice of $\bar{x}_2 = 0$, the flat feedforward action on the cylinder part of the system results as

$$x_{2,d}(t,c) = \frac{\dot{y}_d(t,c)}{K_H K_{SP}} \Rightarrow \dot{x}_{2,d}(t,c) = \frac{\ddot{y}_d(t,c)}{K_H K_{SP}}.$$
 (11)

Exploiting the multiplicative properties of eqs. (9) and (10):

$$\dot{x}_{2,d}(t,c) = \frac{c_2(t,c)}{c_1(t,c)} x_{2,d}(t,c) = -\frac{m(2(mt+a+c)^2-b^2)}{b^2(mt+a+c)} x_{2,d}(t,c).$$
 (12)

Now, the model of the subsystem corresponding to the displacement amplifier, (1), can be inverted after a Forward Euler discretization, using sampling time T_s , and inserting the calculated phase-corrected desired values for x_2 and its derivative:

$$V_{z}(n-1) = pinv(\mathbf{B})T_{S}^{-1}(\mathbf{x}_{2}(n) - (\mathbf{I} + \mathbf{A}T_{S})\mathbf{x}_{2}(n-1)),$$
(13)

where $pinv(\mathbf{B}) = (\mathbf{B}^T \mathbf{B})^{-1} (\mathbf{B})^T$ and the necessary approximation $V_z(n) = V_z(n-1)$. It is to be noticed that $(\mathbf{B}^T \mathbf{B})$ is just a scalar in this SISO case.

VI. SIMULATION RESULTS

Fig. 10 shows the simulation results in the case of minimum speed (of the considered motor cycle engine) of 2000 RPM, which can be considered as the worst case since friction hysteresis delay-like effects are most prevalent at the lowest speed. The seemingly arbitrary choice of a 5 ms delay corresponds to the real friction effects seen in experiments (concurrent measurements will be published soon). The final value of the calculated phase correction angle in this case is -57.94° . Actually the introduced 5 ms delay corresponds to $-\frac{360^{\circ}}{60} \cdot n \cdot t = -6^{\circ} \cdot 2000 \frac{1}{s} \cdot 5$ ms = -60° . This 3.4 % deviation from the true value is, in the authors' opinion, negligible. It corresponds to a remaining 0.17 ms time lag which is very small compared to the trajectory period of 60 ms at 2000 RPM. As demonstrated in Fig. 10, satisfying tracking of both the position and the velocity of the valve is achieved, together with a soft landing. In fact, guaranteeing a soft landing prolongs the life of the actuator and reduces acoustic noise, thus it is usually an important requirement in actuator control design.



Fig. 10. Valve position, velocity and crank angle correction at 2000 RPM

Please note that the first, uncompensated, cycle is is used to calculate the necessary phase correction as described above. The PI controller that is necessary to feedback-control the valve position is activated at t = 50 ms, thus the relatively small offset, that is due to an inherently imperfect model inversion done in eq. (13), is corrected.

VII. CONCLUSION

To summarize, the paper presents a simple yet effective way to generate engine valve trajectories. Gaussian trajectories have the following favorable properties:

• Infinite differentiability, which greatly helps when working with flatness-based approaches,

- Low number of parameters, in this case only three,
- Parameters that are defined in analogy to traditional engines with a camshaft (crank angle, etc.) and can be changed freely, and a
- Rapid decline towards high frequencies in the signal spectrum, which entails fewer bandwidth-related challenges for model simplifications.

Of course, they are suitable for other areas as well, e.g. $\int y_d(t)dt$ for robotic motion planning like in [2]. The proposed actuator, that can be used to achieve fully variable combustion engine valve control, is modelled, including a time delay to be compensated, and controlled to track the valve trajectory. The adopted models are linearizations of the nonlinear models described in [8], [7]. These simplified models have shown to be sufficient for position control purposes in the speed ranges relevant for combustion engines, since the very fast oil pressure dynamics (which is the main challenge when dealing with nonlinear models) can be neglected.

REFERENCES

- A. Fabbrini, A. Garulli, and P. Mercorelli. A trajectory generation algorithm for optimal consumption in electromagnetic actuators. *IEEE Transactions on Control Systems Technology*, 20(4):1025–1032, July 2012.
- [2] L. Biagiotti and C. Melchiorri. Trajectory Planning for Automatic Machines and Robots. Springer Berlin Heidelberg, 2008.
- [3] E. Bayo and B. Paden. On trajectory generation for flexible robots. *Journal of Robotic Systems*, 4(2):229–235, 1987.
- [4] Z. Rymansaib, P. Iravani, and M. N. Sahinkaya. Exponential trajectory generation for point to point motions. In 2013 IEEE/ASME International Conference on Advanced Intelligent Mechatronics, pages 906–911, July 2013.
- [5] B. Haus, P. Mercorelli, and N. Werner. Hysteresis compensation in a piezo-hydraulic actuator using heuristic phase correction of periodic trajectories. In *IECON 2016 - 42nd Annual Conference of the IEEE Industrial Electronics Society*, pages 565–570, Oct 2016.
- [6] B. Haus, P. Mercorelli, and N. Werner. Modelling of a Servo Piezo Mechanical Hydraulic Actuator and its Feed-Forward Control. In *Proc.* 6th International Conference on Modelling, Identification and Control, pages 117–122, Melbourne, Australia, 2014.
- [7] B. Haus, P. Mercorelli, and N. Werner. A robust adaptive self-tuning sliding mode control for a hybrid actuator in camless internal combustion engines. In Quanmin Zhu and Ahmad Taher Azar, editors, Advances and applications in Sliding Mode Control Systems, chapter IV. Series: Studies in Computational Intelligence, Vol. 576, Springer-Verlag publishing, 2014.
- [8] B. Haus, H. Aschemann, P. Mercorelli, and N. Werner. Nonlinear Modelling and Sliding Mode Control of a Piezo-Hydraulic Valve System. In Proc. 21st International Conference on Methods and Models in Automation and Robotics, pages 442–447, Miedzyzdroje, Poland, 2016.
- [9] R.R. Chladny and C. R. Koch. Flatness-Based Tracking of an Electromechanical VVT Actuator with Disturbance Observer Feed-Forward Compensation. *IEEE Transactions on Control Systems Technology*, 16(4):652–663, 2008.
- [10] S.K. Chung, C.R. Koch, and A.F. Lynch. Flatness-based feedback control of an automotive solenoid valve. *IEEE Transactions on Control Systems Technology*, 15(2):394–401, 2007.
- [11] M. Fliess, J. Levine, and P. Rouchon. Flatness and defect of nonlinear systems: Introductory theory and examples. *International Journal of Control*, 61(6):1327–1361, 1995.