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Robust Nonlinear Model Predictive Control with Reduction of Uncertainty via Dual Control

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***Abstract*—Dual control is a technique that solves the trade-off between using the input signal for the excitation of the system excitation signal (probing actions) and controlling it, which results in a better estimation of the unknown parameters and therefore in a better (tracking or economic) performance. In this paper we present a dual control approach for multi-stage robust NMPC where the uncertainty is represented as a tree of possible realizations. The proposed approach achieves implicit dual control actions by considering the future reduction of the ranges of the uncertainties due to control actions and measurements. The region of the uncertainties is described by the covariance of the parameter estimates. The proposed scheme does not require a priori knowledge on the relative importance of the probing action compared to the optimal operation of the system, as employed in other approaches. Simulation results obtained for a semi-batch reactor case study show the advantages of dual NMPC over robust (multi-stage) NMPC and adaptive robust NMPC, where the scenario tree is updated whenever a new measurement information is available.**

I. Introduction

Linear model predictive control (MPC) is widely used in the process industries because of its ability to handle multivariate systems with constraints. It uses a linear model to predict the future evolution of the system and solves an open-loop optimization problem over a finite horizon at each sampling time. Only the control input that is obtained for the first sampling period is applied to the system. At the next sampling time, the optimization problem is reinitialized based on the information available from the system and solved again, thus providing feedback [1]. If the system exhibits nonlinear behavior, a nonlinear model can be used in the optimization problem and the resulting control scheme is known as nonlinear model predictive control (NMPC).

One of the major challenges for the wide-spread use of NMPC is the handling of uncertain behaviour of the real plant. Several NMPC approaches have been presented in the last years that address this problem, commonly known as robust NMPC. The min-max approach [2] performs the optimization for the worst-case realization of the uncertainity from a given uncertainty set. The tube-based methods [3], [4] solve the nominal control problem and also include an ancillary controller that makes sure that the evolution of the real system stays in a tube that is centered around the nominal solution. The multi-stage NMPC [5] approach, which models the uncertainties by a tree of discrete scenarios and applies multi-stage optimization strategies, represents a promising strategy for NMPC-based control of systems under uncertainty because it takes into account the presence of feedback in the future but formulates the optimization problem as an open-loop optimization. An inherent disadvantage of all these robust schemes is that they result in conservatism when compared to the case where perfect information (without uncertainty) about the system is available because they need to provide cautious control commands to avoid violations of constraints or unstable closed-loop behaviour. The scenario-tree representation of the multi-stage NMPC makes it possible to adapt the future control inputs to the observations. This means that the future control actions act as recourse variables without assuming a fixed feedback structure in contrast to [2] which can improve the performance significantly as shown in [6]. We follow the multi-stage approach in this work.

The measurement information that is available in the future can be incorporated to improve the knowledge about the system, thereby reducing the conservatism of the robust approaches. This is known as adaptive control [7]. To obtain measurements that provide useful information, it is often necessary to excite the system, which can in turn decrease the performance of the closed-loop system. This poses a challenging optimization problem that was discussed for the first time under the name of dual control by Feldbaum [8]. Dual control has two contradictory goals: applying probing inputs (excitation signals) that maximize the information obtained from the measurements of the uncertain system and applying inputs that optimize the closed-loop performance.

The rigorous solution of the dual control problem based on dynamic programming is usually intractable [7], therefore different approximations have been proposed in the literature to solve the dual control problem. These are classified into explicit and implicit dual control methods [9]. The objective function used in the explicit methods consists of two terms. The first term represents the control performance whereas the second term represents the reward for the increased information with respect to the future control performance [10]. The drawback of this approach is that the relative importance of the two terms must be chosen a priori. The implicit dual control methods try to solve the original problem by introducing several approximations. For a review on the dual-control methods the reader is referred to [9].

In this paper, we focus on implicit dual control using robust multi-stage NMPC in the presence of constant, but uncertain, model parameters. A dual-control scheme using a multi-stage MPC and Ensemble Kalman filter for a linear model was proposed in [11]. This method considers the predictions obtained along the scenarios as the future measurements and updates the covariance matrix of the Ensemble Kalman filter and thus achieves a dual action. A drawback of this method is that if the initial covariance is large, then none of the predicted future confidence regions obtained along the scenarios might include the true values of the uncertain parameters. In [12] we proposed an implicit dual control formulation by taking into account the future reduction of the uncertainty provided by the future probing actions. This method assumes that the least-squares estimate of the uncertain parameters stays constant along the prediction horizon.

The new method proposed in this work computes the future reduction of the uncertainty in the model parameters along the prediction horizon by computing the future parameter covariance matrix using an approximate estimate of the uncertain parameters along the prediction horizon. This eliminates the assumption that the least-squares estimate remains constant along the prediction horizon as in [12]. The range of the future uncertainty is computed such that the true value of the uncertain parameters is enclosed in the scenario tree over the prediction horizon.

The remainder of the paper is structured as follows. Section II describes the multi-stage NMPC approach. Section III presents an adaptive robust NMPC scheme and Section IV presents the proposed dual robust NMPC method based on the multi-stage approach with implicit consideration of the future reduction of the uncertainty. Section V discusses the results obtained using different robust NMPC approaches for a chosen case study. The paper is concluded in Section VI.

II. Multi-stage Robust NMPC

Multi-stage NMPC [5] is a robust NMPC strategy that describes the evolution of the uncertainty by discrete scenarios as shown in Fig. 1. Each branch of the tree represents a realization of the uncertainties that are considered. The main advantage of such a formulation is that it can be explicitly considered that the new information will be available in the future and that the future control inputs can be adjusted accordingly. This significantly improves the performance of robust NMPC in comparison to traditional open-loop min-max approaches [6].

The multi-stage optimization problem is rigorously valid only for uncertainties that can assume a finite number of values. For the general nonlinear and continuous case, this formulation does not guarantee robust constraint satisfaction

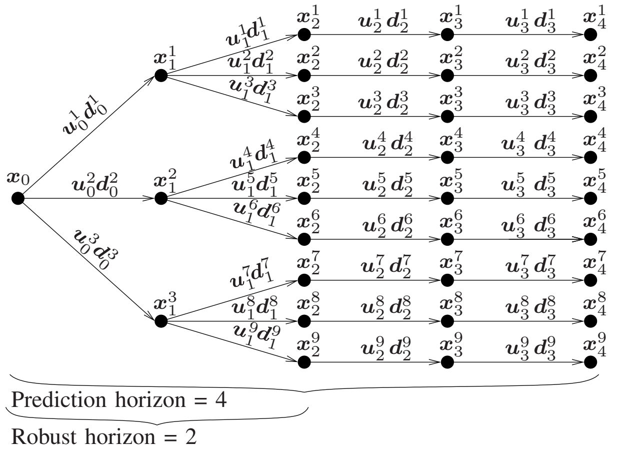


Fig. 1: Scenario tree representation of the uncertainty evolution for multi-stage NMPC.

for the values of the uncertainty that are not explicitly included in the scenario tree. Nonetheless, very often a scenario tree generated using the combinations of the maximum, minimum and nominal values of the uncertainty provides very good results [5]. A rigorous guarantee of robust constraint satisfaction of the multi-stage approach can be obtained by combining it with reachability analysis [13].

The decisions taken at a given node in the scenario tree must be the same because the future realizations of the uncertainties are not known (e.g. in Fig. 1 $\boldsymbol{u}_0^1 = \boldsymbol{u}_0^2 = \boldsymbol{u}_0^3; \boldsymbol{u}_1^1 = \boldsymbol{u}_1^2 = \boldsymbol{u}_1^3; \ldots$). This can be enforced via the so-called non-anticipativity constraints.

A. Formulation of multi-stage robust NMPC

We assume a discrete-time uncertain nonlinear model of the system under control that can be written as:

$$\boldsymbol{x}_{k+1}^j = \boldsymbol{f}(\boldsymbol{x}_k^{p(j)}, \boldsymbol{u}_k^j, \boldsymbol{d}_k^j). \tag{1}$$

The n_x-dimensional state vector ($\boldsymbol{x}_{k+1}^j$) at stage $k+1$ and position (realization) j in the tree is obtained as a function of the parent state ($\boldsymbol{x}_k^{p(j)}$) in the tree, the n_u-dimensional control input $\boldsymbol{u}_k^j$ and the realization of the uncertainty which is denoted by the n_d-dimensional vector $\boldsymbol{d}_k^j$. The tree has s branches that leave each node, given by $\boldsymbol{d}_k^{j-s\lfloor \frac{j}{s} \rfloor} \in \{\boldsymbol{d}_k^1, \boldsymbol{d}_k^2, \ldots, \boldsymbol{d}_k^s\}$ which represents one of the possible combinations of the a priori given minimum, nominal and maximum values of the uncertain parameters. In order to avoid the exponential growth of the tree, we assume that the uncertainty remains constant after a certain point in time called the robust horizon (N_r) until the end of the prediction horizon N_p. To simplify the notation, the set of occurring indices (j,k) in a given node of the tree is denoted by I, and all states and control inputs that belong to the i-th scenario are denoted by $\boldsymbol{X}_i$ and $\boldsymbol{U}_i$.

The optimization problem that is solved at each sampling time in the multi-stage NMPC approach reads as:

$$\min_{\boldsymbol{x}_k^j, \boldsymbol{u}_k^j, \forall (j,k) \in I} \quad \sum_{i=1}^{N} \omega_i J_i(\boldsymbol{X}_i, \boldsymbol{U}_i), \tag{2a}$$

subject to:

$$\boldsymbol{x}_{k+1}^{j} = \boldsymbol{f}(\boldsymbol{x}_{k}^{p(j)}, \boldsymbol{u}_{k}^{j}, \boldsymbol{d}_{k}^{j}), \qquad \forall\,(j,k+1) \in I, \tag{2b}$$

$$\boldsymbol{g}(\boldsymbol{x}_{k+1}^{j}, \boldsymbol{u}_{k}^{j}) \le 0\ , \qquad \forall\,(j,k+1) \in I, \tag{2c}$$

$$\boldsymbol{u}_{k}^{j} = \boldsymbol{u}_{k}^{l} \text{ if } \boldsymbol{x}_{k}^{p(j)} = \boldsymbol{x}_{k}^{p(l)}, \qquad \forall\,(j,k),(l,k) \in I, \tag{2d}$$

where N is the number of scenarios, ω_i and J_i are the weights and the costs of each scenario resp., which are defined as:

$$J_i(.) := \sum_{k=0}^{N_p-1} L(\boldsymbol{x}_{k+1}^{j}, \boldsymbol{u}_{k}^{j}),\ \forall \boldsymbol{x}_{k+1}^{j} \in \boldsymbol{X}_i, \boldsymbol{u}_{k}^{j} \in \boldsymbol{U}_i, \tag{3}$$

with $L(\boldsymbol{x}_{k+1}^{j}, \boldsymbol{u}_{k}^{j})$ being the stage cost. $\boldsymbol{g}(\boldsymbol{x}_{k+1}^{j}, \boldsymbol{u}_{k}^{j})$ denotes the constraint functions, which have to be satisfied at each node in the tree and (2d) denotes the non-anticipativity constraints. The approach presented in [14], can be used to enforce the stability of the multi-stage NMPC but this issue is not considered in this paper.

III. Adaptive Robust NMPC

Adaptive control is based on the idea that if some information about the system (e.g. in the form of measurements) is available, then it can be used to improve the closed-loop performance of the controller. The measurement information can be used to obtain an estimate of the uncertain parameters ($\hat{\boldsymbol{d}}_{N_m}$) e.g. using least-squares estimation (LSE), where N_m stands for the number of past measurements. A confidence region for the parameter estimate can be obtained using the Fisher information matrix (FIM) if we assume that information on the measurement noise is white Gaussian noise [15]

$$\mathbf{F}_{0,N_m}^{\boldsymbol{d}} := (\mathbf{P}_{0,N_m}^{\boldsymbol{d}})^{-1} \approx \sum_{k=0}^{N_m} \boldsymbol{s}_k^T \mathbf{Q} \boldsymbol{s}_k, \tag{4}$$

where $\mathbf{P}_{0,N_m}^{\boldsymbol{d}}$ gives an upper bound on the parameter covariance matrix of the parameter vector $\boldsymbol{d}$ using measurements from time 0 to N_m, $\mathbf{F}_{0,N_m}^{\boldsymbol{d}}$ represents the FIM, $\mathbf{Q}$ is the inverse of the covariance matrix of the measurement noise, and $\boldsymbol{s}_k$ represents the matrix of the sensitivities of the outputs w.r.t. the parameters.

The FIM can be used to obtain the confidence region which is centered at the least-squares estimate of the parameter vector ($\hat{\boldsymbol{d}}_{N_m}$) and is given by

$$(\boldsymbol{d} - \hat{\boldsymbol{d}}_{N_m})^T \mathbf{F}_{0,N_m}^{\boldsymbol{d}} (\boldsymbol{d} - \hat{\boldsymbol{d}}_{N_m}) \le \underbrace{n_d\, F_{dist}^{\alpha}(n_d, N_m - n_d)}_{F_s}, \tag{5}$$

where F_{dist} is a quantile of the Fisher distribution, and α stands for the desired confidence level (normally 95% or 99%). Lower ($\underline{\boldsymbol{d}}_{N_m}$) and upper ($\overline{\boldsymbol{d}}_{N_m}$) bounds on the uncertain parameters can be obtained from the previously known bounds on the uncertain parameters ($\underline{\boldsymbol{d}}_{N_m-1}, \overline{\boldsymbol{d}}_{N_m-1}$) and the projection of the confidence region on the parameter axes:

$$\underline{\boldsymbol{d}}_{N_m} := \max\left(\underline{\boldsymbol{d}}_{N_m-1}, \hat{\boldsymbol{d}}_{N_m} - \operatorname{diag}^{\frac{1}{2}}(F_s \mathbf{P}_{0,N_m}^{\hat{\boldsymbol{d}}_{N_m}})\right), \tag{6a}$$

$$\overline{\boldsymbol{d}}_{N_m} := \min\left(\overline{\boldsymbol{d}}_{N_m-1}, \hat{\boldsymbol{d}}_{N_m} + \operatorname{diag}^{\frac{1}{2}}(F_s \mathbf{P}_{0,N_m}^{\hat{\boldsymbol{d}}_{N_m}})\right), \tag{6b}$$

where the operator $\operatorname{diag}(\cdot)$ returns a vector of the diagonal elements of a matrix. The max and min operators prevent the overestimation of the bounds because the uncertain parameters are assumed to be constant. The scenario tree can be updated based on the lower bound ($\underline{\boldsymbol{d}}_{N_m}$), the nominal value ($\hat{\boldsymbol{d}}_{N_m}$) and the upper bound ($\overline{\boldsymbol{d}}_{N_m}$) and then the updated problem (2) can be solved at the next sampling time.

Adaptive robust NMPC can considerably improve the performance of robust multi-stage NMPC as the width of the scenario tree (with respect to the values of the uncertainties that are considered) narrows down based on the available information [16]. However, it is not a dual approach as it does not take into account explicitly that probing actions can improve the estimation accuracy thus resulting in a further improvement in the performance.

IV. Dual Robust NMPC

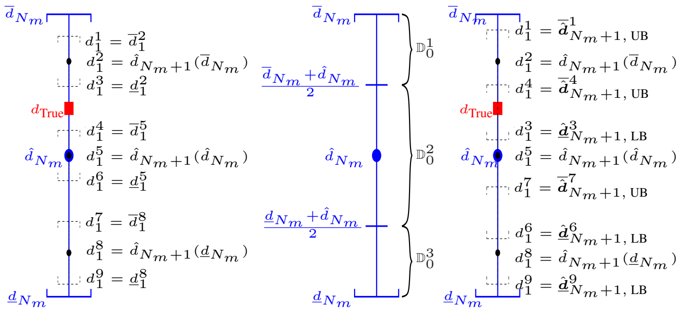
Dual robust NMPC aims at striking a balance between the optimizing control inputs and the probing actions by predicting the impact of the probing actions on the optimal cost. The scenario-tree formulation perfectly fits this purpose because it allows us to treat the uncertainty as being reduced in the future in a straightforward manner. The proposed dual robust NMPC updates the scenario tree along the prediction horizon based on estimates of the confidence region of the future parameter estimations.

A major obstacle for the computation of the future bounds on the uncertain parameters is that the future measurements and the corresponding future parameter estimates are not known. This issue was resolved in [12] by assuming that the least-squares estimate, obtained using all the available measurements, remains constant along the prediction horizon. A more accurate, yet still approximate, approach is to embed the nonlinear least-squares estimation problem into the prediction by enforcing the optimality conditions of the parameter estimation problem via constraints. This results in a computationally very expensive problem, therefore we propose an alternative, computationally less expensive, estimation of the uncertain parameters in the future which can be embedded in the predictions.

A. Approximation of the future parameter uncertainties

The basic idea behind our approach is to subdivide the range of the uncertainties in the future into intervals according to which the future control inputs can be adapted. In the approximation, we use an approximate computation of the optimal parameter estimate in the situation where the estimation based on the available measurements, $\hat{\boldsymbol{d}}_{N_m}$ as well as an estimation at time N_{m+k}, $\hat{\boldsymbol{d}}_F$, which is based on the information from N_m to N_{m+k} are available. Then the nonlinear estimation problem to compute the optimal parameter estimate at time N_{m+k} can be approximated [17] as:

$$\min_{\hat{\boldsymbol{d}}_{N_m+k}} [\hat{\boldsymbol{d}}_{N_m+k} - \hat{\boldsymbol{d}}_{N_m}]^T \mathbf{F}_{0,N_m}^{\hat{\boldsymbol{d}}_{N_m}} [\hat{\boldsymbol{d}}_{N_m+k} - \hat{\boldsymbol{d}}_{N_m}] + [\hat{\boldsymbol{d}}_{N_m+k} - \hat{\boldsymbol{d}}_F]^T \mathbf{F}_{N_m,N_m+k}^{\hat{\boldsymbol{d}}_F} [\hat{\boldsymbol{d}}_{N_m+k} - \hat{\boldsymbol{d}}_F], \tag{7}$$



(a) (b) (c)

Fig. 2: Illustration of the scenario-tree update at stage $k = 1$ for one uncertain parameter.

where $\mathbf{F}^{\hat{\boldsymbol{d}}_{N_m}}_{0,N_m}$, gives the contribution of the past measurements gathered until the current time point, analogically to the arrival cost of moving horizon estimator [1], and $\mathbf{F}^{\hat{\boldsymbol{d}}_F}_{N_m,N_m+k}$ gives an estimate of the contribution of the future measurements from the current time point until the kth step in the prediction horizon. The optimal solution of the unconstrained quadratic optimization problem (7) is:

$$\hat{\boldsymbol{d}}_{N_m+k}(\hat{\boldsymbol{d}}_F) := \left(\mathbf{F}^{\hat{\boldsymbol{d}}_{N_m}}_{0,N_m} + \mathbf{F}^{\hat{\boldsymbol{d}}_F}_{N_m,N_m+k}\right)^{-1} \mathbf{F}^{\hat{\boldsymbol{d}}_{N_m}}_{0,N_m}\hat{\boldsymbol{d}}_{N_m} + \left(\mathbf{F}^{\hat{\boldsymbol{d}}_{N_m}}_{0,N_m} + \mathbf{F}^{\hat{\boldsymbol{d}}_F}_{N_m,N_m+k}\right)^{-1} \mathbf{F}^{\hat{\boldsymbol{d}}_F}_{N_m,N_m+k}\hat{\boldsymbol{d}}_F. \quad (8)$$

Note that $\hat{\boldsymbol{d}}_F$ is an unknown quantity here, and that $\hat{\boldsymbol{d}}_{N_{m+k}}$ is a function of $\hat{\boldsymbol{d}}_F$.

B. Scenario-tree update along the prediction horizon

The scenario tree of the dual robust NMPC is built assuming that the future parameter estimates $\hat{\boldsymbol{d}}_F$ take the nominal, lower and upper bound values of the uncertain parameters. For an estimate based on the future information at node (j,k) in the scenario tree, $\hat{\boldsymbol{d}}_{N_m+k}(\boldsymbol{d}^j_k)$ gives the approximate parameter estimate that will be obtained at the node (j,k). The lower and upper bounds on the uncertain parameters at node (j,k) can be obtained from Eq. (6) using the Fisher information matrix $\mathbf{F}^{\hat{\boldsymbol{d}}_{N_m+k}(\boldsymbol{d}^j_k)}_{0,N_m+k}$, where $\hat{\boldsymbol{d}}_{N_m+k}(\boldsymbol{d}^j_k)$ is obtained from Eq. (8) but these bounds cannot be used to update the uncertainty bounds in the scenario tree because there might be a case where none of the bounds obtained at time k include the true value of the uncertain parameter as illustrated in Fig 2a. The blue line indicates the projected confidence regions that were obtained from the past measurements for one parameter, the dashed black lines indicate the predicted future confidence regions obtained and the red square indicates the true parameters. It can be seen that none of the predicted confidence regions encloses the true value of the uncertain parameter. This has to be avoided by over-approximating the predicted lower and upper bounds of the uncertain parameters. The box approximation of the past confidence region is split into s regions and each region $(\mathbb{D}^j_{k-1})$ is allocated to a scenario as shown in Fig 2b. The center of the confidence region $(\hat{\boldsymbol{d}}^j_{N_m+k,\,\text{LB}}$ and $\hat{\boldsymbol{d}}^j_{N_m+k,\,\text{UB}})$ that is used to obtain the lower and upper bounds on the uncertain parameters at the node (j,k) are given by

$$\hat{\boldsymbol{d}}^j_{N_m+k,\,\text{LB}} = \hat{\boldsymbol{d}}_{N_m+k}(\tilde{\boldsymbol{d}}_{\text{LB}}),\ \hat{\boldsymbol{d}}^j_{N_m+k,\,\text{UB}} = \hat{\boldsymbol{d}}_{N_m+k}(\tilde{\boldsymbol{d}}_{\text{UB}}), \quad (9)$$

where $\tilde{\boldsymbol{d}}_{\text{LB}}$ and $\tilde{\boldsymbol{d}}_{\text{UB}}$ represent the parameter inside the region $(\mathbb{D}^j_{k-1})$ which gives the maximum perimeter of the box approximation of confidence region obtained at the node (j,k), and are given by

$$\arg \min_{\tilde{\boldsymbol{d}}_{\text{LB}},\, \tilde{\boldsymbol{d}}_{\text{UB}} \in \mathbb{D}^j_{k-1}} \mathbf{1}^T \hat{\boldsymbol{d}}_{N_m+k}(\tilde{\boldsymbol{d}}_{\text{LB}}) - \mathbf{1}^T \hat{\boldsymbol{d}}_{N_m+k}(\tilde{\boldsymbol{d}}_{\text{UB}}) \quad (10)$$

where $\mathbf{1}$ is a vector of 1 of the length n_d. The scenario tree of the dual NMPC is updated using the new nominal value, lower and upper bounds of the uncertain parameters along the prediction horizon until $k < N_r - 1$. An example of this procedure is shown in Fig. 2c.

V. Case Study

The exothermic semi-batch reactor benchmark problem from [18] is adapted to illustrate the advantages of the proposed approach via simulation studies. The chemical reaction that takes place in the reactor is given by $A + B \rightarrow C$. The nonlinear dynamics of the volume of the reactor (V), the concentration of the reactant A (C_A), reactant B (C_B) and product C (C_C) can be obtained from [18], in addition to this we also consider the dynamics of the temperature of the reactor (T) and the jacket (T_{J}) and is given by

$$\dot{T} = \frac{u}{V}(T_{\text{in}} - T) - \frac{\alpha U(T - T_{\text{J}})}{\rho V c_p} - \frac{K c_A c_B H}{\rho c_p}, \quad (11)$$

$$\dot{T}_{\text{J}} = \frac{\dot{Q}_{\text{K}} + \alpha U(T - T_{\text{J}})}{\rho V_{\text{J}} c_p}, \quad (12)$$

where $U(= 0.027 + 0.022\,V)$ denotes the surface area of the reactor that is covered by the reaction mixture, $V_{\text{J}}(= 2.22\,\text{L})$ denotes the volume of the jacket and $\alpha(= 534384\,\text{J}\,\text{K}^{-1}\text{h}^{-1}\text{m}^{-2})$ denotes the heat-transfer coefficient between the reactor and the jacket. The true value of the reaction rate constant $(K = 1.11 \times 10^{-7}\,\text{L}\,\text{mol}^{-1}\text{h}^{-1})$ and the reaction enthalpy $(H = -327\,\text{J}\,\text{mol}^{-1})$ are considered to be uncertain ($\pm 40\%$ w.r.t. their nominal values chosen as $1.21 \times 10^{-7}\,\text{L}\,\text{mol}^{-1}\,\text{h}^{-1}$ and $-355\,\text{J}\,\text{mol}^{-1}$ resp.). The feed rate $(u \in [0, 35]\text{Lh}^{-1})$ and the cooling capacity $(\dot{Q}_{\text{K}} \in [-9, 0] \times 10^6\,\text{J}\,\text{h}^{-1})$ act as control inputs. $T_{\text{in}}(= 300\,\text{K})$ and $c_{B\,\text{in}}(= 2.75\,\text{mol}\,\text{L}^{-1})$ represent the temperature and the concentration of reactant B entering into reactor. The initial conditions of the states are given by $\boldsymbol{x}_0 = (V_0,\, C_{A,0},\, C_{B,0},\, T_0,\, T_{\text{J},0})^T = (3,\, 2,\, 0,\, 325,\, 325)^T$, the measured quantities are given by $\boldsymbol{y} = (c_A,\, c_B,\, T,\, T_{\text{J}})^T$ and the corresponding standard deviations of the measurement noise are given by $(0.01,\, 0.01,\, 0.1,\, 0.1)^T$.

The control task is to maximize the amount of product C that is produced along the prediction horizon while respecting the constraint on the reactor temperature $(322\,\text{K} \leq T \leq 326\,\text{K})$ and volume of the reactor $(V \leq 7\,\text{L})$. The nonlinear optimization problem was implemented using CasADi [19] and Ipopt [20] as explained in [12].

A. Simulation results

Simulations were carried out using the three different robust NMPC strategies for the case study described above. In all these cases a scenario tree is generated that considers all possible combinations of the maximum, minimum and nominal values of the uncertain parameters with a robust horizon $N_r = 3$ and a prediction horizon $N_p = 5$. All the scenarios in the scenario tree are equally weighted ($\omega_i = 1/N$). The sampling time of the NMPC is chosen as $0.05\,\mathrm{h}$. The simulations are carried until $0.5\,\mathrm{h}$. The confidence level used for bounding the confidence region is chosen as 95%. Simulation analysis showed that the objective function to maximize the mass of product C (n_{C}) produced along the prediction horizon provides the best results for the control task in hand.

Fig. 3 shows the results obtained using multi-stage NMPC, adaptive robust NMPC and dual robust NMPC for the case when the reaction rate is 10% smaller and the reaction enthalpy is 10% larger than the nominal value. The optimal operation is to feed as much as possible while respecting the constraints. It can be seen from the figure that a very small amount of reactant B (u) is fed into the reactor when using multi-stage NMPC because of the tight specification of the reactor temperature (T). The amount of reactant fed into the reactor can be increased with the help of adaptive robust NMPC because the scenario tree of the multi-stage NMPC is continuously updated, when better estimates of the uncertain parameters has been obtained.

The amount of feed fed into the reactor can be further improved with the help of dual robust NMPC. The dual robust NMPC is aware of the fact that the future inputs will improve the estimations of the uncertain parameters, which reduces the conservativeness (back-off from the temperature constraint) and this effect of the future inputs is taken into account in their optimization. This can be seen from Fig. 4 which shows the confidence regions of the estimated parameters obtained at time $0.25\,\mathrm{h}$ using the adaptive robust and the dual robust NMPC. The parameters are scaled such that they vary between 0 to 1 within their bounds. It can be seen from the figure that the confidence region obtained using the dual robust multi-stage NMPC is substantially smaller than the confidence region obtained using the adaptive robust multi-stage NMPC. The reduction of the uncertainty along the prediction horizon is illustrated in Fig. 5, which shows the predicted box approximation of the confidence region (dashed rectangles) obtained at time $k = 1$ for the scenarios built using the lower bound on the reaction rate and the nominal value, the lower, and upper bounds on the reaction enthalpy (filled dots) as explained in Sec. IV-B. Similarly, the confidence regions for the other scenarios (not shown in the figure) were obtained such that the union of the predicted confidence regions covers the entire past confidence ellipse. It can be seen from the figure that the new nominal parameters (squares) that are used in the updated scenarios move towards the past confidence region because the dual NMPC takes the influence of both the past and the future measurements into account as mentioned in Sec. IV-A. The scenario tree is adapted using the predicted confidence regions along the prediction horizon. This results in less conservative control inputs and thus in an improved performance. The performance gain of the dual robust NMPC relative to the adaptive robust NMPC is ≈15%.

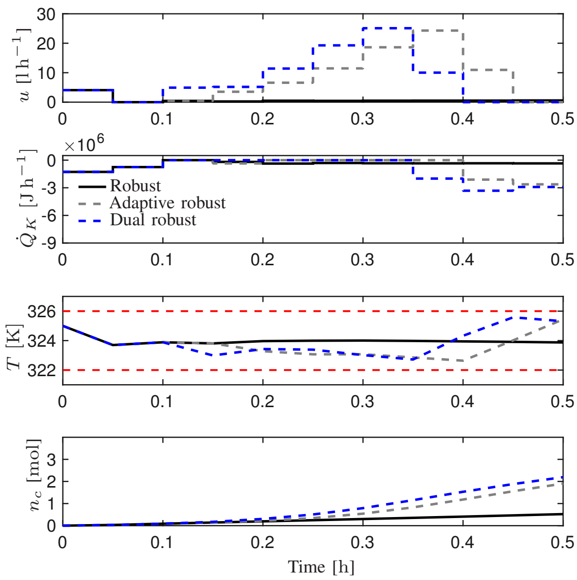


Fig. 3: Input feed, cooling capacity of the jacket, reactor temperature and moles of product C for one batch and different controllers.

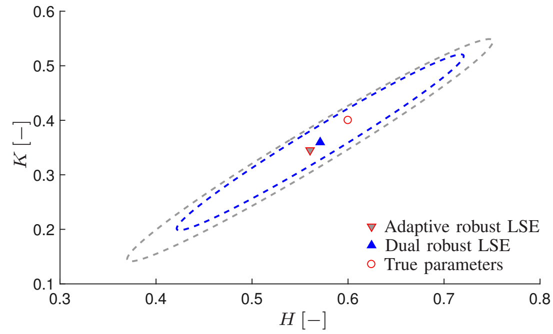


Fig. 4: Confidence ellipsoids after $0.25\,\mathrm{h}$ of batch time and least square estimates obtained using adaptive robust NMPC (gray dashed) and dual robust NMPC (blue dashed).

In order to evaluate the performance of the dual NMPC, the simulations were repeated for 100 random realizations of the uncertain parameters. Fig. 6 shows a histogram plot of the number of moles of product C produced by the different robust NMPC strategies. It shows that adaptive robust multi-stage NMPC outperforms robust multi-stage NMPC whereas dual multi-stage NMPC outperforms the adaptive robust multi-stage NMPC. The price to pay for the increased performance is a higher computational effort. The multi-stage NMPC and the adaptive robust multi-stage NMPC can be solved in 5 s and 7 s per step on the average while dual robust NMPC takes around

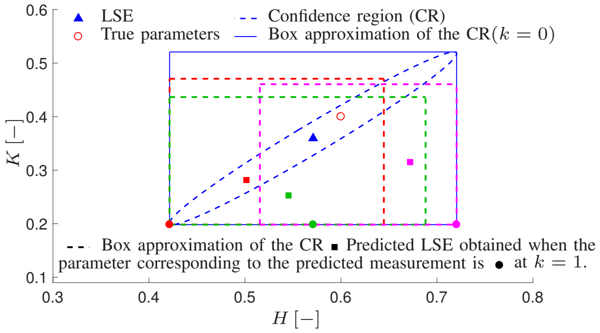


Fig. 5: Confidence regions considered at $k = 0$, 1 along the prediction horizon.

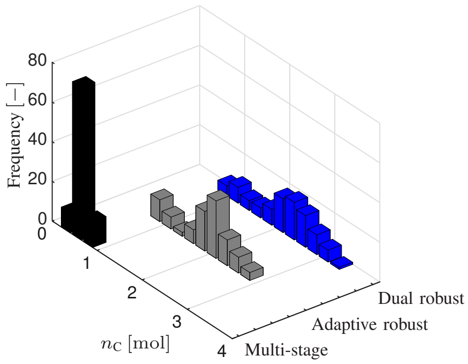


Fig. 6: Number of moles of product C produced by multi-stage, adaptive robust and dual robust NMPC .

120 s per step because of the simultaneous computation of sensitivities and future confidence regions. The proposed dual NMPC does not consider that the LSE remains constant along the prediction horizon as in [12] and the union of the predicted future confidence region obtained along the prediction horizon covers the entire current confidence region in contrast to [11], hence it results in a controller that is robust w.r.t. estimation errors.

VI. Conclusion

This paper proposes a dual robust NMPC scheme that is based on the multi-stage formulation which reduces the conservatism of the robust control actions since it considers that the future control inputs can be adapted according to the future observations and also considers the future reduction of the uncertainty. A priori knowledge on the relative importance of the probing actions on the control goal is not required. The proposed dual NMPC makes sure that the entire current confidence region is enclosed along the prediction horizon resulting in a robust controller when compared to other approaches. It was applied to a simulated semi-batch reactor and the simulation results show the advantages of the proposed method over other robust NMPC strategies.

In our future work, we will investigate guaranteed parameter estimation methods to identify the bounds on the uncertain parameters [21] instead of relying on the approach based on approximation of the covariance matrix using the FIM.

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