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Dynamic Compensator for Reference Tracking Realized on FPGA

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***Abstract*— The paper deals with a practical implementation of a digital observer-based reference tracking state controller for a laboratory plant. The state controller is designed to track reference commands that can be described by linear differential equations with constant coefficients; this type of controller is known as Command Generator Tracker (CGT) [1]. The resulting dynamic compensator is implemented to control speed of a laboratory plant using FPGA.**

***Keywords*— optimal tracking, command generator tracker, motion control, FPGA**

I. Introduction

Optimal control of linear systems with respect to quadratic performance index (LQ problem) is one of the key problems of the modern control theory, and is widely used mainly in motion systems control. In its basic version, a state-feedback LQ regulator (LQR) guarantees that states of a controllable dynamic system are driven to zero from arbitrary initial conditions. In case of other versions (LQ output regulator, LQ tracking), each of the specific problems can be reformulated as LQR design and solved using the standard design procedure. Asymptotic stability and required performance of transients are achieved by a proper choice of weighting matrices of the quadratic performance index. An important prerequisite for using LQR is availability of all plant states for feedback; otherwise, an observer has to be applied to provide state estimates that are used or implementation of the LQ controller instead of real states. The observer can be designed as a deterministic (Luenberger observer) or a stochastic one (Kalman filter). In practice, the stochastic observer is the most frequently used version estimating the state variables from available measurements of the plant output corrupted by process and measurement noises specified by their statistical properties. A combination of an observer and a state controller is called a dynamic compensator.

Tracking a reference input signal by a plant output is one of the most important control tasks called tracking or servo design problem. In case of setpoint tracking the regulator can be converted into a tracker by adding additional feedforward terms; in case of tracking a non-constant reference, the feedforward terms generally contain also its derivatives. A powerful tracker design technique that automatically yields the precompensator required to guarantee proper tracking for a large class of command inputs is the command generator tracker (CGT) based on incorporating the model of the reference dynamics into the control system [1]; according to [2], this tracking design methodology is denoted "error space approach".

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The paper deals with the design of a digital observer-based command generator tracker (DCGT) for a motion control application. Based on measured output (motor speed), states of the plant are estimated using a deterministic observer; the optimal tracker is designed independently. The resulting dynamic observer is implemented on FPGA.

The Field Programmable Gate Arrays (FPGA) are configurable circuits of very large-scale integration (VLSI) able to integrate various logical and control functions. Compared with microprocessor based (software) solutions, the FPGA based hardware realizations of control algorithms are by several orders faster and able to control plants with fast dynamics; also they are more compact, cost-effective and thus cheaper.

II. Command Generator Tracker (CGT) Design

CGT design [1,2,3] is a powerful methodology resulting from the internal model principle; according to it, for proper asymptotic tracking the plant model has to include non-asymptotically stable modes of the reference signal. If it is not the case, the plant model has to be augmented so as to include modes of the reference signal that are not at the same time modes of the plant. A properly designed CGT guarantees a zero steady-state tracking error for a broad class of command inputs describable by linear differential equations with constant coefficients (3) which encompasses polynomial signals (step, ramp, …), and harmonic signals.

Consider the linear state-space model.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

An optimal tracker is a state controller which guarantees that the performance output (generally not equal to the measured output y(t))

$$z = Hx \tag{2}$$

tracks the reference input describable by a d-th order linear differential equation with constant coefficients

$$r^{(d)} + a_1 r^{(d-1)} + \ldots + a_d r = 0,$$
$$\dot{r}(0) = r_{10}, \ddot{r}(0) = r_{20}, \ldots, r^{d-1}(0) = r_{(d-1)0} \tag{3}$$

Characteristic polynomial corresponding to (3) can be defined

$$\Delta(s) = s^d + a_1 s^{d-1} + \ldots + a_d \tag{4}$$

which can be expressed in the controllability canonical form called command generator system; for example if d=3, the state-space model of the command generator system is

$$\dot{\rho} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} \rho = F\rho$$
$$r = [1 \quad 0 \quad 0]\rho \tag{5}$$

Define the tracking error as follows

$$e = r - z = r - Hx \tag{6}$$

and let us express (3) using (4) as

$$\Delta(s) r = 0 \tag{7}$$

Using (7), (6) can be expressed as

$$\Delta(s)e = \Delta(s)r - \Delta(s)Hx = -H\xi \tag{8}$$

A modified state vector can be defined using (7)

$$\xi = \Delta(s)x = x^{(d)} + a_1 x^{(d-1)} + \ldots + a_d x. \tag{9}$$

Now, (8) can be expressed in the observability canonical form

$$\dot{\varepsilon} = F\varepsilon + \begin{bmatrix} 0 \\ -H \end{bmatrix} \xi \tag{10}$$

where $\varepsilon = [e\ \dot{e} ... e^{(d-1)}]$. Applying the error dynamics (9) to (1) we obtain the dynamics of ξ

$$\dot{\xi} = A\xi + B\mu \tag{11}$$

where μ is the modified control input

$$\mu = \Delta(s)u = u^{(d)} + a_1 u^{(d-1)} + \ldots + a_d u \tag{12}$$

Joining (10) and (11) we obtain the augmented state model

$$\frac{d}{dt}\begin{bmatrix} \varepsilon \\ \xi \end{bmatrix} = \begin{bmatrix} F & \begin{matrix} 0 \\ -H \end{matrix} \\ 0 & A \end{bmatrix} \begin{bmatrix} \varepsilon \\ \xi \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} \mu \tag{13}$$

In this way the original tracking problem has been converted to the regulator with the tracking error (6) is regulated to zero.

Standard poleplacement [2] or LQR design can be used to obtain the resulting feedback

$$\mu = -[K_e \ K] \begin{bmatrix} \varepsilon \\ \xi \end{bmatrix} \quad \text{or} \quad \Delta(s)u = -K_e \varepsilon - K\Delta(s)x \tag{14}$$

From (14) the control input for the original system is obtained by a simple manipulation as follows

$$\Delta(s)(u + Kx) = -K_e \varepsilon \tag{15}$$

The corresponding CGT transfer function contains the internal model of the reference input generator [1].

$$\frac{(u + Kx)}{\varepsilon} = \frac{-K_e}{\Delta(s)} = -\frac{K_1 s^{d-1} + ... + K_{d-1}s + K_d}{s^d + a_1 s^{d-1} + \ldots + a_d} \tag{16}$$

III. DIGITAL COMMAND GENERATOR TRACKER (DCGT) DESIGN

The digital CGT (DCGT) is a design procedure is similar to the continuous-time one. The augmented model is build from the digitized command generator and digitized plant model.

Consider a discrete-time state-space model of the plant to be controlled

$$x_{k+1} = A_d x_k + B_d u_k$$
$$y_k = Cx_k \tag{17}$$

where (A_d, B_d, C) is the discrete-time counterpart of the continuous-time state space model (A,B,C) obtained according to the following formulas (using the sampling period T):

$$A_d = e^{AT}; B_d = A^{-1}(e^{AT} - I)B \tag{18}$$

The output matrices C and H do not change with digitization.

The digital command generator matrix F_d (a digital counterpart of (6)) results from digitization of (5) using the relation $z_i = e^{s_i T}$ where s_i, i=1,…,d are roots of the characteristic polynomial (5):

$$\Delta(z) = \prod_{i=1}^{n} (z - z_i) = z^d + a_1 z^{d-1} + ... + a_d =$$
$$= 1 + a_1 z^{-1} + a_2 z^{-2} ... + a_d z^{-d} \tag{19}$$

Similarly as in the continuous-time version, following relations hold [3]

$$\Delta(z) r_k = 0 \tag{20}$$

$$\Delta(z)e_k = \Delta(z)r_k - \Delta(z)Hx_k = -H\xi_k \tag{21}$$

where $\xi_k = \Delta(z)x_k$

To derive dynamics of the modified discrete-time model, $\Delta(z)$ is applied to the state equation

$$\xi_{k+1} = A_d \xi_k + B_d \mu_k \tag{22}$$

where $\mu_k = \Delta(z) u_k$

The resulting augmented discrete-time system is

$$\begin{bmatrix} \varepsilon_{k+1} \\ \xi_{k+1} \end{bmatrix} = \begin{bmatrix} F_d & \begin{matrix} 0 \\ -H \end{matrix} \\ 0 & A_d \end{bmatrix} \begin{bmatrix} \varepsilon_k \\ \xi_k \end{bmatrix} + \begin{bmatrix} 0 \\ B_d \end{bmatrix} \mu_k$$

$$\varepsilon_k = [e_k \;\; e_{k+1} \ldots e_{k+d-1}]^T \tag{24}$$

When the state of the augmented system converges to zero, a zero steady-state tracking error is guaranteed (asymptotic tracking). If LQR approach is used to design DCGT, the discrete-time output equation has the form

$$\upsilon_k = \begin{bmatrix} I & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} \varepsilon_k \\ \xi_k \end{bmatrix} \tag{25}$$

and the discrete-time performance index for the augmented plant becomes

$$J_d = \sum_{k=0}^{\infty} \upsilon_k^T Q_d \upsilon_k + \mu_k^T R_d \mu_k \tag{26}$$

where the digitized weighting matrices are obtained $Q_d = QT$, $R_d = RT$ [1].

Applying the standard LQ procedure to the augmented plant considering (24) and (25), the state feedback output regulator is designed. The control law is obtained in the form

$$\mu_k = \Delta(z) u_k = -[K_e \quad K] \begin{bmatrix} \varepsilon_k \\ \xi_k \end{bmatrix} = -K_e \varepsilon_k - K \Delta x_k \,. \tag{27}$$

The compensator transfer function for implementation according to Fig. 1 is as follows:

$$\frac{u_k + K x_k}{e_k} = -\frac{K_1 z^{d-1} + \ldots + K_{d-1} z + K_d}{z^d + a_1 z^{d-1} + \ldots + a_d} \tag{28}$$

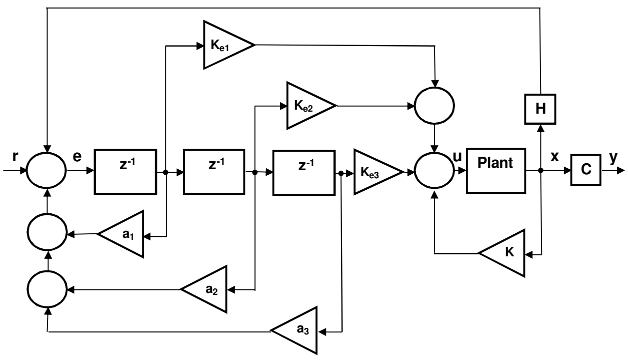

Fig. 1. Implementation of the DCGT (for d=3)

DIGITAL OUTPUT INJECTION OBSERVER DESIGN

The observer is a dynamic system described by

$$\begin{aligned} \hat{x}_{k+1} &= A_d \hat{x}_k + B_d u_k + L_d (y_k - C\hat{x}_k) = \\ &= (A - LC)\hat{x}_k + B_d u_k + L_d y_k \end{aligned} \tag{29}$$

where $\hat{x}_k$ is the estimate of x_k. Dynamics of the estimation error $\tilde{x}_k = x_k - \hat{x}_k$ is given by

$$\tilde{x}_{k+1} = (A_d - L_d C)\tilde{x}_k = A_{0d} \tilde{x}_k \tag{30}$$

It is required that the estimation error (30) vanishes with time, hence the matrix A_{0d} has to be asymptotically stable; in the discrete-time case this means that its eigenvalues have to be located within the unit circle. If the plant is observable, L_d can always be selected so that all observer poles are at z=0. This is called deadbeat observer and guarantees that the estimation error decays exactly after n sample periods.

Next, the digital observer-based command generator tracker will be designed for a laboratory plant and implemented using FPGA.

IV. DESIGN OF DYNAMIC COMPENSATOR FOR THE LABORATORY PLANT

5.1 Description and modeling of the plant

The SISO laboratory plant consists of a cascade connection of two DC motors modules (Fig. 5, green box). Each DC motor module has input range 0 – 10 V, and includes electronic components for the motor drive and for transfer data from the optical incremental encoder in the range 0 – 10 V. The cascade is realized by subtracting the output of the second module (multiplied by an unknown gain lower than 1) from the input of the first module.

Measured I/O identification data are depicted in Fig. 2.

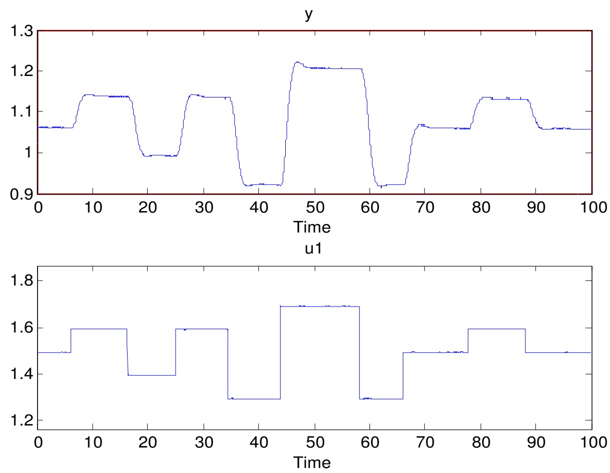

Fig. 2. Measured I/O identification data of the plant

Mathematical model which fits the best the measured data has been chosen in form of a 2nd order transfer function with complex poles and a time delay

$$G(s) = \frac{0.7003}{0.5215s^2 + 1.032s + 1} e^{-0.375s}$$

The discrete-time transfer function was obtained using sampling time T=D=0.375s.

$$G(z) = \frac{0.3725\, z^{-1} + 0.05713\, z^{-2}}{1 - 1.29 z^{-1} + 0.4761\, z^{-2}} z^{-1}$$

The respective state-space model to be used for the DCGT design is

$$A_d = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -0.4761 & 1.29 \end{bmatrix} \qquad B_d = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.0571 & 0.0732 & 0 \end{bmatrix} \qquad D = 0$$

5.2 Observer-based DCGT design

For the SISO laboratory plant, a digital command generator tracker will be designed to track the step reference input. Characteristic polynomials of a step signal for continuous- and discrete-time cases are respectively

$$\Delta(s) = s \quad \Rightarrow \quad \Delta(z) = z - 1 = 1 - z^{-1} \tag{31}$$

Next, the augmented system is built according to (24):

$$\begin{bmatrix} e_{k+1} \\ \xi_{1k+1} \\ \xi_{2k+1} \\ \xi_{3k+1} \end{bmatrix} = \begin{bmatrix} 1 & -0.0571 & -0.0732 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -0.4761 & 1.29 \end{bmatrix} \begin{bmatrix} e_k \\ \xi_{1k} \\ \xi_{2k} \\ \xi_{3k} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \mu_k$$

$$\upsilon_k = [e_k \quad C\xi_k]^T \tag{32}$$

The weighting matrices in the quadratic performance index (26) so as to guarantee a proper tracking and not to exceed feasible ranges of variables in the future implementation.

$$Q_d = \begin{bmatrix} 100T & 0 & 0 & 0 \\ 0 & T & 0 & 0 \\ 0 & 0 & T & 0 \\ 0 & 0 & 0 & T \end{bmatrix} \qquad R_d = 0.5T \tag{33}$$

Using (33), the resulting Kalman gain matrix was calculated for the augmented LQ problem:

$$K = [K_e \; K_1 \; K_2 \; K_3] = \begin{bmatrix} 5.1097 & -0.2919 & -0.2522 & -1.6831 \end{bmatrix}$$

The digital output injection observer was designed using the poleplacement method; as the deadbeat observer was not feasible for implementation with the below described FPGA setup, the following desired poles of the observer matrix were chosen p=[0.1 0.2 0.3] yielding the output injection matrix

$$L_d = \begin{bmatrix} 6.9381 & 6.056 & 14.4562 \end{bmatrix}^T \tag{34}$$

Simulation model of the observer-based step reference tracker is in Fig. 3.

The designed PI controller is denoted as LQPI in the sequel. Closed-loop time responses of the step reference tracking using DCGT are in Fig. 4.

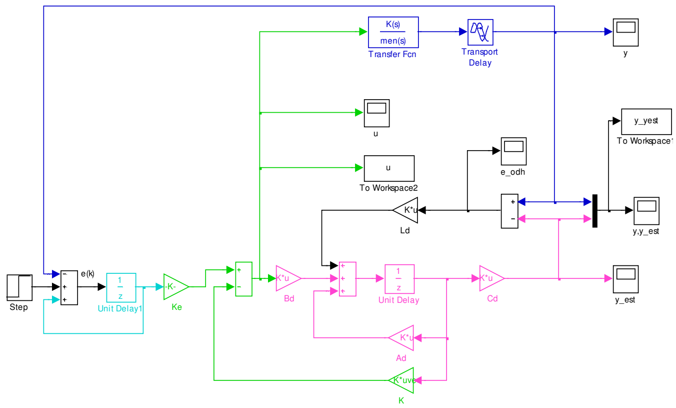

Fig. 3. Observer-based step reference tracking using the DCGT

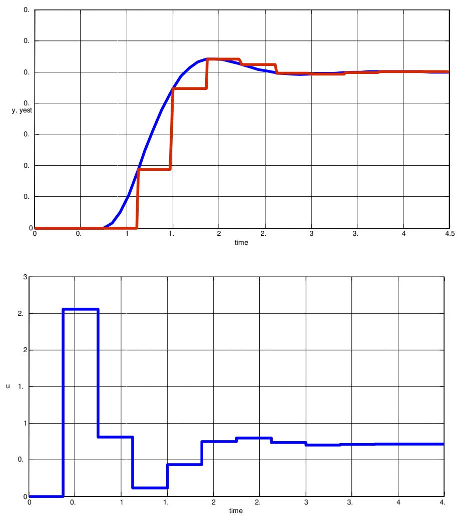

Fig. 4. Step reference tracking using DCGT – simulation results (upper plot: plant and observer outputs, lower plot: control input)

5.3 Hardware implementation on FPGA

Implementation of the dynamic compensator is based on the Artix 7 FPGA included in Digilent Nexis 4 development board [4].

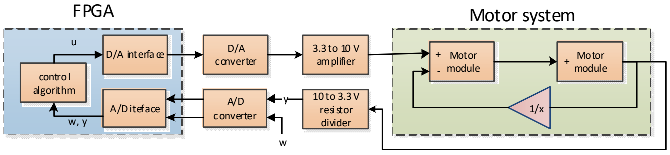

Fig. 5. Block scheme of the control loop

Communication between digital and analog environment is provided by A/D and D/A converters. Both converters have 12-bit width, range 0 – 3.3 V and operate up to a maximum frequency 1MSPS [5]. For spreading and shrinking signals between 3.3V and 10 V range we used a circuit with operational amplifier and voltage resistor divider.

The control algorithm is designed in the VHDL (VHSIC Hardware Description Language). It is composed of five main components (Fig. 6).

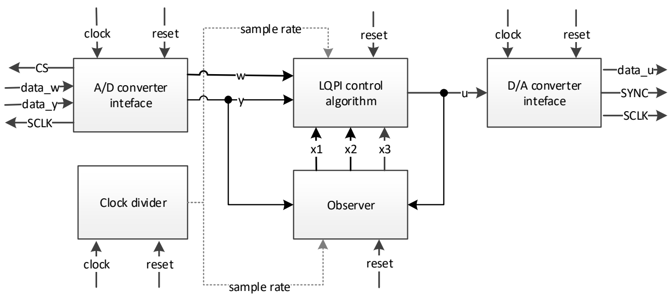

Fig. 6. Block scheme of the control algorithm

Clock divider provides the sample rate for the observer and the LQPI controller. Other clocks are directly connected to the 100MHz system clock.

The DAC and ADC interfaces transmit or receive data from converters. The ADC interface also recalculates 12bit logic vectors to a fixed-point data type. On the other hand, before conversion the DAC interface recalculates fixed-point data type back to the 12bit logic vector.

Widths of the input and output signals were determined; the 12-bit unsigned was chosen and for the input signals w, y and output signals u, because in real hardware application the 12bit A/D and D/A convertors are used.

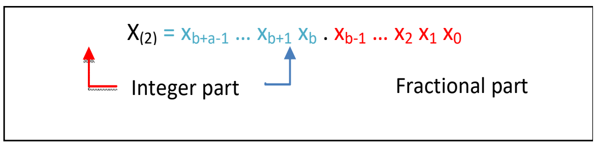

Fig. 7. Fixed point number representation

For implementation of real numbers, the fixed point arithmetic has been used [6]. If a12-bit unsigned fixed point is used, the first two (MSB) bits are reserved for the integer part, and the last ten bits for the fractional part. Such representation is written as X(2,10). Representation X(a,b) has a bits in integer part (in case of signed fixed point with sign bit) and b bits in fractional part (Fig. 7).

In the control loop design, the fixed-point arithmetic range rules have to be respected. The data widths in the fixed-point arithmetic were designed so that there is no possibility of overflow. For example, result of the summation of two N-bit vectors has a N+1-bit range. Result of the summation of vectors with different ranges has range

$$X(a_1,b_1)+X(a_2,b_2)=X(\max(a_1,a_2)+1,\max(b_1+b_2)).$$

Range of product of two unsigned fixed points is

$$U(a_1,b_1)\times U(a_2,b_2)=U(a_1+a_2,b_1+b_2) \quad (36)$$

and the range of a product of two signed fixed points is

$$A(a_1,b_1)\times S(a_2,b_2)=S(a_1+a_2+1,b_1+b_2) \quad (37)$$

Design of LQPI controller block is shown in Fig. 6, implementation of the LQPI block is depicted in Fig. 8.

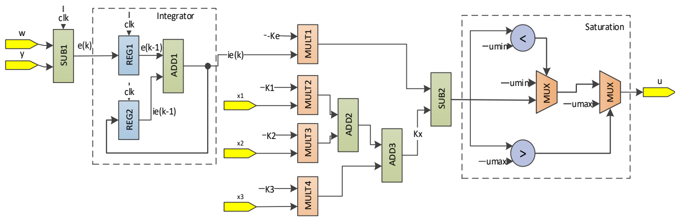

Fig. 8. Structure of LQPI controller

In each sampling period, the control output y_k from the motor system and the set point w_k are loaded. The control error e_k is computed in the block SUB1 where the signal y_k is subtracted from w_k. REG1, REG2 and ADD block work as an integrator. The signal e_k is held in the registry REG1 for one sampling period. Register REG1 output signal is thus e_{k-1}. At the same time, signal ie_{k-1} is recorded at REG2 by latching output of the integrator ie_k. Then, the integrator output is multiplied by a constant $-K_e$. The second branch of the control algorithm can be expressed as

$$Kx_k = K_1x_{1k} + K_2x_{2k} + K_3x_{3k}$$

After the last subtraction in SUB2, the output vector attains the S(11,20) width due to the fixed-point range rules. A saturation block is used to ensure that the control output ranges from 0 to 3.3 V in to 12-bits U(2,10) representation. Saturation is the value and bit range limitation logic that keeps the output in the defined limits. Maximum limit for the output signal is 11.0100110011(2) = 3.2998046875(10) and minimum value is 00.0000000000(2) = 0(10). Implementation of the observer is designed similarly.

View on the experimental setup is shown in Fig. 9, time responses from experiments are depicted in Fig. 10.

V. CONCLUSION

The DCGT design procedure is simple and direct to apply. The resulting compensator includes both feedback and feedforward terms so that both the closed-loop poles and zeros may be affected by varying the state feedback gain matrix K. The method is applicable if the original plant is reachable and the loop transfer function $H(z) = H(zI - A_d)^{-1} B_d$ from u to z has no zeros identical with the roots of $\Delta(z) = 0$. A simple structure of the control law is advantageous for implementation on FPGA. The presented FPGA implementation of the DCGT was realized with a sampling period properly chosen with respect to the plant dynamics; application for plants with fast dynamics is straightforward.

REFERENCES

[1] F. L. Lewis, Applied optimal control & estimation: digital design and implementation. Prentice-Hall Inc., Englewood Cliffs, New Jersey 1992.

[2] G. F. Franklin, J. D. Powell and A. Emami-Naeini, Feedback Control of Dynamic Systems, fifth ed., Pearson Education, Inc., Upper Saddle River, New Jersey, 2006.

[3] A. Kozáková, "Discrete-time asymptotic reference tracking for an industrial robot", in: 23rd International Conference on Robotics in Alpe-Adria-Danube Region, IEEE RAAD 2014; Smolenice, Slovakia; Category number34043; Code 109914

[4] Xilinx Inc., "Artix-7 FPGA Family," http://www.xilinx.com.

[5] Digilent Inc., "Digilent PmodAD1 and PmodDA1" http://digilentinc.com.

[6] D. Bishop, Fixed point package user's guide, http://www.vhdl.org

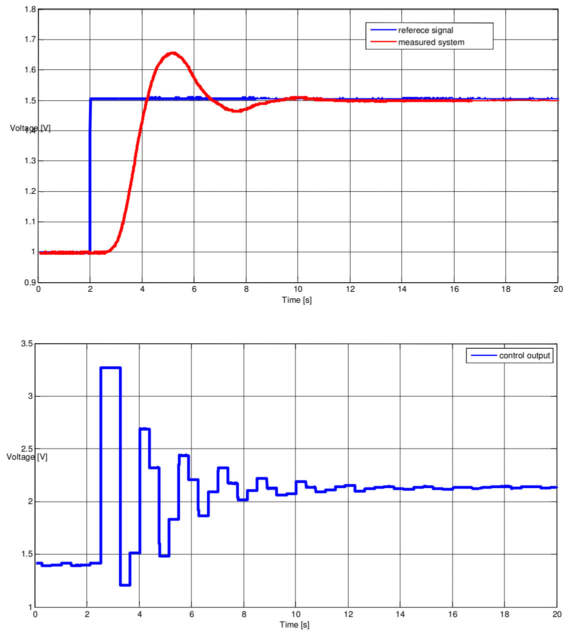

Fig. 9. Step reference tracking using observer-based DCGT implemented on FPGA – experimental results (upper plot: plant and observer outputs, lower plot: control input)

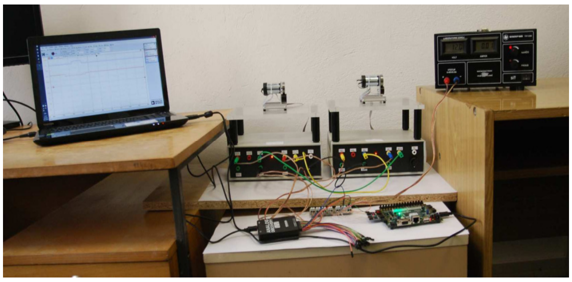

Fig. 10. Hardware realization of the LQPI controller