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Analysis of the hydrometeorological data using the Fractal dimension estimation

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***Abstract* — This paper deals with an evaluation of a hydrometeorology data from sites of a different land use and mainly from the places with a different leaf area index. A few methods of an estimation of the Fractal Dimension of a hydrometeorology time series, which is based on the original principle of coastline length measuring, were developed. The first results indicate that developed methods are usable for testing of the relation between measured data and autoregulation functions of the ecosystem.**

Keywords* — *Time series analysis; Fractal Dimension; Small Water Cycle; Microclimatology; Nature autoregulation; Urban Heat Island.

I. INTRODUCTION

The fractal object is an ideal entity and its nature is epistemological – not ontological. There is possible to see partial characteristics of an ideal fractal on a real object. Especially there is possible to see here any degree of a fractality at the hydrometeorological data. The difference between fractal and euclidean object lies in the property which is called self-similarity. Self-similarity is for this purpose repeating the same/or similar theme/shape in a different scales. This property indicates relatively high segmentation of the object. Moreover, this segmentation is possible to see repeatedly in higher and higher enlargement of the scale. This principle is shown Benoit Mandelbrot [1] and Lewis Fry Richardson on the example of measuring the coastline length and in this paper is used for time series.

When the task to show hydrometheorological data from the database [2] appropriately and user-friendly was handle, then the fractal character of this data revealed (Figure 2).

The aim of this contribution is to prepare method for verification of the general hypothesis: "**the fractal properties of hydrometeorological data have a relation to ecosystem functions like an autoregulation**". One of nowadays-discussed autoregulation function of the ecosystem is regulation of the temperature mediated by the transpiration of plants and trees (This principle is referred e.gh. [8], [14]). This contribution is also fundament for research of the autoregulation of the temperature in the Prague in the context of Urban Heat Island UHI theory [13].

II. FRACTAL DIMENSION

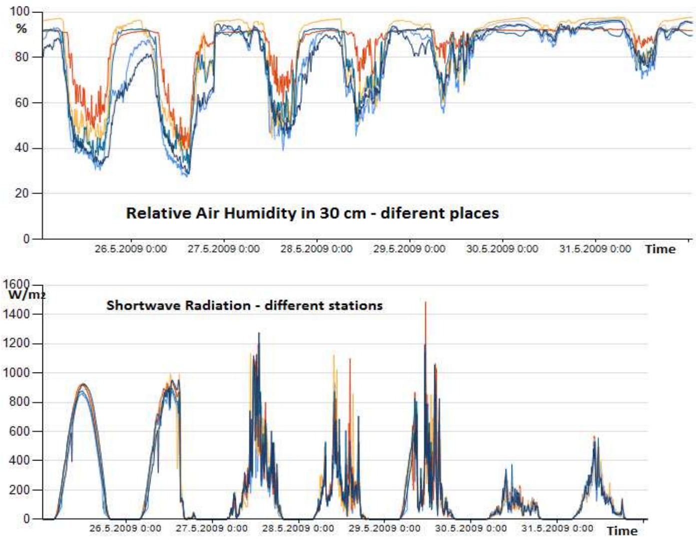
On the example of coastline, length measuring is possible to illustrate, that the relation between use scale (or better-measuring stick size) and the length of a coastline, is fundamental. This relation provides a quantitative estimation of the fractal dimension of the measured object and also provides the qualitative analysis the unusualness and the courses of this relation.

It is clear that for fractal objects the length of coastline increases with decreasing size of measuring stick. The steepness of this relation represents the degree of fractality of a given object.

FIG. 1. The map of meteorological stations placement [2].

There are many methods for Fractal dimension estimation. Herein discuss methods are designated for real fractal objects. Since the fractal theory is primarily intended for geometrical objects, so also Fractal Dimension estimation methods are also primarily intended to the geometrical objects (e.g. Box-counting [3]). Unfortunately, our fractal object is a time series and the previous method is not possible to use directly.

FIG. 2. Graph of measured data (Air humidity and incoming solar radiation) [2].



III. MOTIVATION

This research is a continuation of the project Development of methods for evaluation of flows of energy and matters in the selected Eco-Systems. This project provides us with a database of a hydrometeorological data, which had been collected from 16 meteorological measuring stations (one of them is on the Figure 3) in South Bohemia near Třebon town - Figure 1. Each station had measured a hydrometeorological variables (Figure 2) and few of them was selected for this project – Table 1. Nowadays it continues by the collection of hydrometeorological data in the Prague city.

TABLE 1. Selected hydrometeorological variables from database TOKENELEK [2]

Air Temperature in 2 m and 30 cm above the ground	°C
Relative Air Humidity in 2 m and 30 cm above the ground	%
Absolute Air Humidity in 2m and 30 cm above the ground	g/m3
Incoming Solar Radiation	W/m2
Reflected Solar Radiation	W/m2
Precipitation	mm
Wind speed and wind direction	m/s, DEG

Measuring stations were placed at the different types of sites {field, wet meadow, concrete surface, fishpond, pasture, village} [4]. Measuring in the South Bohemia was more oriented to nature places. Oppositely, the Prague is oriented to the city environment and provide us much representative data for example from the concrete surface.

A. *Autoregulation of the temperature*

The aim of this article is to add a new point of view to the autoregulation in the landscape exploration. Especially the autoregulation of a temperature depends on the water in the landscape and on the state of a Small Water Cycle SWC [5], [6], [7], which is a rival theorem, to the well known Big Water Cycle. SWC works locally at the ground level of the landscape. SWC describe the cycle from the evaporation, specially evapotranspiration, of the water by plants and condensation at the colder places or in more cold time (e.g. night hours) [8].

Note: It is possible to estimate an autoregulation as an ability to regulate given an environmental variable to a given value. For example the temperature is in the nature environment regulated to the temperature which is convenient to a local flora and fauna. For quantification of this effect, two indexes of green-auto regulation IGA was suggested. The first one is based on a correlation between incoming solar radiation and temperature at 30 cm above the terrain.

$$IGA\_C = \mathrm{correl}\,(\mathrm{GR\_income}, \mathrm{Temp\_30}) \tag{1}$$

The second one eliminates the major disadvantage of the previous one, which is the absence of the physical dimension of this indicator.

$$IGA\_F = \frac{GR\_income}{Temp\_30 + 273{,}15} \tag{2}$$

The interpretation of this index is the amount of the incoming solar radiation, which is able to heat the environment about one degree of Kelvin.

The relation between diversity (especially biodiversity) of an ecosystem and its stability and level of the autoregulation is currently discussed. To help to find the relation between fractal characteristics of hydrometeorological variables and stability and the level of the autoregulation is the aim of this work. The partial aim is to develop the reliable method for estimation of Fractal Dimension of mentioned hydrometeorological variables.

B. *Calculate of absolute humidity*

Relative air humidity is dependent on the temperature, therefore, it is not useful pro the analysis. Absolute humidity is a better solution. Absolute humidity can be defined as density in g/m^3 of water vapour from temperature and relative humidity. Water vapour is a gas whose behaviour is similar to an ideal gas at normal atmospheric temperatures. For ideal gas is valid ideal gas law PV = nRT. From this formula, in our case are known variables of temperature (T) and volume (V). For solving n had to be calculated pressure (P). To obtain a variable for P is used Magnus – Teten's approximation [17] which generates saturated vapour pressure as a function of temperature. For calculation with any relative humidity, we have to this formula multiply by $H_R/100$.

$$P = 6{,}112.\frac{17{,}67.T}{T + 243{,}5}.\frac{H_R}{100} \quad (3)$$

The result of this formula is in a number of moles of a substance. The final result is needed to be enumerated in g/m3 so this formula has to be multiplied by 18,02 – molecular weight of water. The final formula is:

$$H_A = \frac{6{,}112.e^{\frac{17{,}67.T}{T+243{,}5}}.H_R.18{,}02}{(T + 243{,}5).8{,}315} \quad (4)$$

FIG. 3. One of meteorological station in South Bohemia [2]

IV. FRACTAL DIMENSION OF TIME SERIES

In general, the idea of calculating of the fractal dimension is derived from a coastline measuring and it's (in general) formulated [8] as:

$$D = \frac{\log N}{\log(1/r)} \quad (5)$$

Where D is the fractal dimension, N is the number of an object's internal homotheties and r is the ratio of homothety.
One of the methods of the estimation of the Fractal dimension of the real object is the Box Counting Method (e.g. [3], [9], [10]). This method is based on the covering of the object by the n-dimensional spheres or cubes of given size ε.

$$D_C = \lim_{\varepsilon \to 0} \frac{\log[C(\varepsilon)]}{\log \frac{1}{\varepsilon}} \quad (6)$$

Where D is the estimation of FD, ε the size of the n-dimensional cubes and $C(\varepsilon)$ is the minimal number of n-dimensional cubes.

Unhappily this method is appointed primarily for the geometric object – not for time series. Its adaptation [11] to a time series induces an additional problem with the choice of a time-size scale. The aim of methods developed herein is to closely respect the principle of a coastline length measuring.

Next method which is potentially possible to use for this data is, for example, Minkowski dimension [15].

Methods which is proposed herein is based on the principle of smoothing of the time series – and changing the coefficient of the smoothing. From the dependency between the length of the curve and smoothing coefficient is the value of Fractal dimension estimated. The primarily smooth course have a low Fractal Dimension because of the smoothing of the smooth object. And the fractal dimension of a linear course is equal to zero. Next is a description of two smoothing methods used for estimation of Fractal dimension – Moving Average and Moving Maximum and Minimum.

V. MOVING AVERAGE (FDMAVG) METHOD

This method, which is primarily developed here [16] and which is continue developed is based on the analogy with the length of the coastline measuring. Similarly, as a measuring stick size is changed in the case of the coastline, or box (or sphere) size ε is changed in the case of the Box-counting method, so the window of the moving average is changed herein.

A. The algorithm.

Firstly, the set of Window sizes EPS = {1, 2, 3, 5, 10, 20, ...} is determined. The Given size of the window is labelled here in according to Box Counting as ε_n and is chosen from the set EPS. For each data row index "k" and data x(k), from X the value AVG(k) of mean with given window size ε_n is calculated for k from 1 to m- ε_{max}, where m is index of the last element.

$$AVG(k) = \frac{1}{\varepsilon}\sum_{i=k}^{k+\varepsilon_n} x_i \quad (7)$$

Next, is calculated the „variance "VAR".
Def: variance VAR is for this purpose the absolute value of a difference between two consequential values of AVG(k). The mean value of the variance VV_AVG(n) for each window size ε_n is after simplification calculated as:

$$VV\_AVG(n) = \frac{1}{\varepsilon(m - \varepsilon_{\max})}\sum_{k=1}^{m-\varepsilon_{\max}} \left| x_k - x_{k+\varepsilon+1} \right| \quad (8)$$

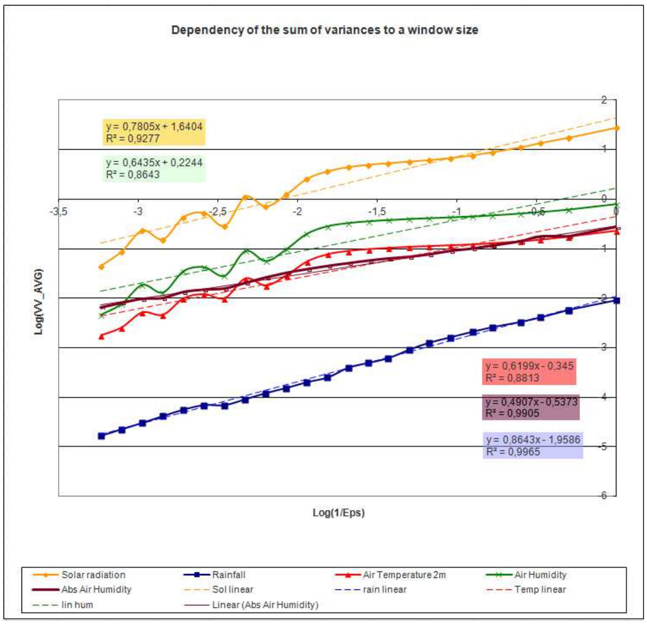
B. Methodology of use FDMAvg algorithm

This method, similarly as others methods for estimation of Fractal Dimension, is sensitive to parameters of the calculation. For reach out the reliable results, there it needs to keep the methodological recommendation and same parameters for analysis which should be compared.

1) Determination of minimal window size (ε_{min})

Since the data are sampled with the period 10 minutes, so a minimal window size has the wide of one sample. The effect of the dynamical characteristic of the measuring instrument is significantly smaller than sample period. Hypothetically, in the case of a smaller sample period than the time constant of measuring instrument, the result of fractal dimension estimation will be related more to the measuring instrument than a measured object.

FIG. 4. Estimation of Fractal Dimension of temperatures at 30 cm and 2 m above the ground -station "9_CIGLEROVSKY_STO_All_Year_2008"



2) Determination of maximal window size (ε_{max})

The determination of the maximum window size is not equally clear as the size of a minimum window. For example, Felix Hausdorff works with 1/5 of the whole range. Although a time series really has a finite length, theoretically can the same series be longer (or newer end). This range significantly affects the calculation of the fractal dimension. The basic data set has a length of one month, so the maximal window size of 1/5 of a month is considered. For correct comparison, the same maximum window size ε_{max} is used also a different (usually longer) data sets.

3) Set the increment of the window size

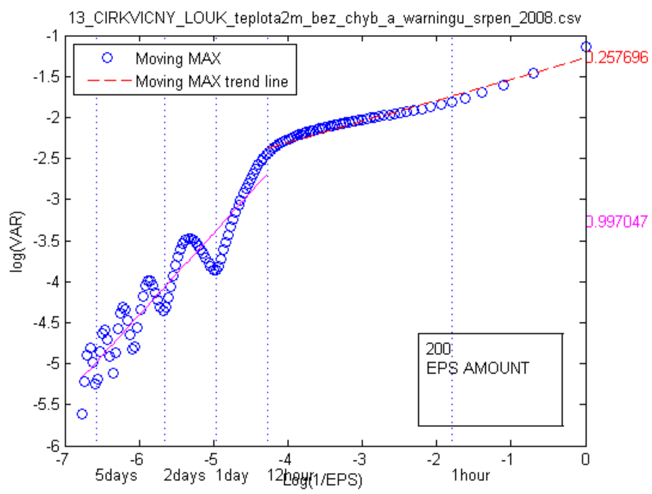
Because the method is based on a calculation of the slope of the line. And the line parameters are estimated by the minimal square method, so we have to ensure a uniform increment of a window size ε in a given (logarithmic) coordinates (Fig. 5). In the opposite case, the impact of the segment of points, which are more close to small values, is significantly bigger than an impact of outlying ones. For elimination of this undesired impact we recalculating the step of ε as an inverse function of "ln (1/ ε)".

The total amount of used window size is primarily not important for calculation because the line parameters influence minimally. A higher density of amount of ε is useful for a visual evaluation of the results. The same set of ε is used for all calculation, which should be compared.

4) Trend line

As it is possible to see the at Fig. 5 the graph contain the turning point (at the ε close to 1 day). This turning point splits up the approximated line into two parts. The first part with the lesser slope represents shorter ε and contains segmentation, which can be a consequence of irregularities at the side of incomes (like an incoming solar radiation or air flux). The second part with a greater slope represents ε longer than 24 hours and contain circadian rhythms (and this is probably causation of a steeper line). For hydrometeorological variables without circadian rhythms (as absolute air humidity or precipitation) is possible to approximate by one line, because there is no turning point (Figure 4). Diagram (like Fig. 5) for ideal sine curve looks like one stair and raising edge is at the ε close to the sinusoid ω. Next waves are probably a reflection of this circadian rhythm. For the analysis of the hydrometeorological data is more interesting the first line (ε smaller than 12 hours), which better represents specifics of the given ecosystem.

FIG. 5. Dependency Log(VAR) on Log(1/EPS).



VI. MOVING MIN&MAX (FDMM) METHOD

This method is very similar as a previous – Moving Average FDMAvg. For analysis is used sums of moving minimums VV_MIN, sums of moving maximums VV_MAX and above all their mean value VV_MM. VV_MIN and VV_MAX have usually similar course. The Bigger difference is possible to observe for sharply asymmetric variables, like a precipitation. Normally the difference is small and we can estimate the Fractal dimension from the VV_MM.

A. *Description of algorithm*

The window size set is same as for method FDMAvg and also other procedures. The minimums MIN(k) and maximums MAX(k) are calculated instead of mean value AVG(k).

$$MIN(k) = \min(x_i); MAX(k) = \max(x_i) \qquad (9)$$

for $i = \{k, \ldots, k + \varepsilon\}$

Next is calculated the „variance" VAR for all rows. And the mean value of these variances VV_MIN(n) and VV_MAX(n) for each window size ε_n.:

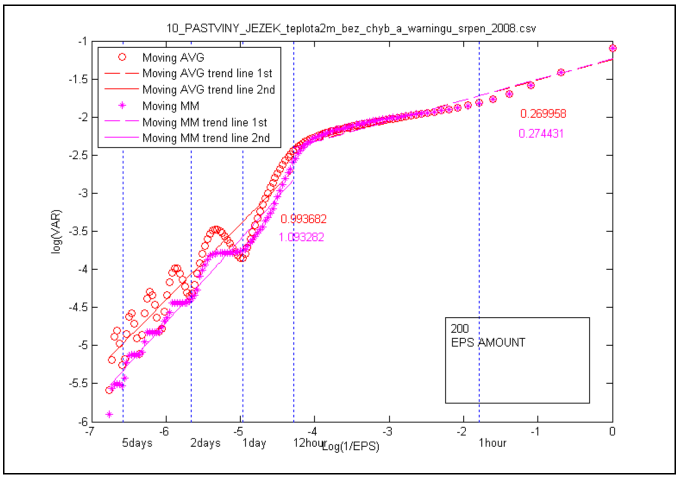
$$VV\_MIN(n) = \frac{1}{m - \varepsilon_{\max}} \sum_{k=1}^{m-\varepsilon_{\max}} \left| MIN(k) - MIN(k+1) \right| \quad (10)$$

$$VV\_MAX(n) = \frac{1}{m - \varepsilon_{\max}} \sum_{k=1}^{m-\varepsilon_{\max}} \left| MAX(k) - MAX(k+1) \right| \quad (11)$$

$$VV\_MM(n) = \frac{1}{2}\left(VV\_MAX(n) + VV\_MIN(n)\right) \quad (12)$$

And finally the estimation of the Fractal Dimension is again deriving from the slope of the line approximating the dependency Log(VV_MM) on Log(1/ε). (Or VV_MAX or VV_MIN).

FIG. 6. The dependency of methods AVG and MM.



VII. PARTIAL RESULTS AND VERIFICATION OF USED METHOD

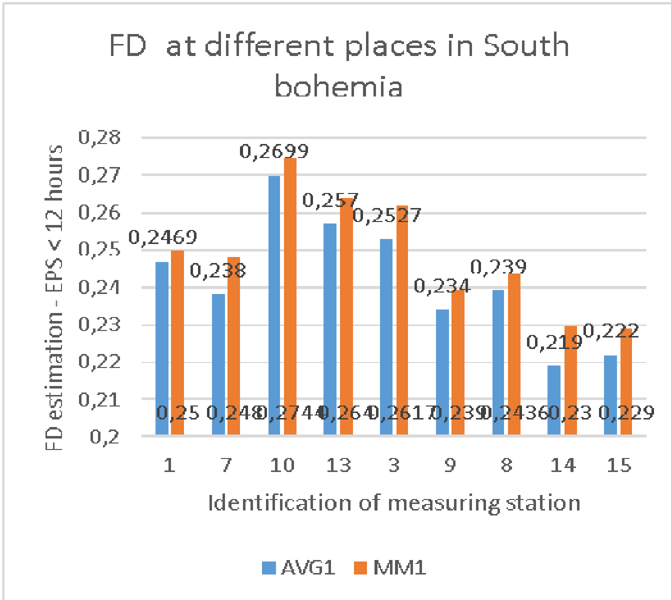
Before calculating has started, the database records had to been checked to some operational errors (in the database had been records which had represented breakdowns states of a measuring device). Besides the basic analysis of data credibility had been made also.

The first series of estimations have been done for a few selected variables on the different sites and is shown on the Figure 7. The Fractal Dimension of a temperature is higher at the locality with higher evapotranspiration (green areas versus concrete area or lake).

This consideration is also supported by the example of results of a Fractal Dimension estimation of Temperatures whose is measured in 2 metres and 30 centimetres above the ground (Figure 4). The Fractal Dimension of variables measured more close to the ground points to influence of a Small Water Cycle. At the other side, the rigorous verification of this conclusion is not aimed of this article and should to be done with consideration of many aspects of microclimatology.

The differences between various hydrometeorological variables are observable at the level of a Fractal Dimension. The level of a dependency is shown in Figure 9.

FIG. 7. Estimation of Fractal Dimension for Temperature in 2m above the ground. Station 1 is in a small town, stations 7 has a concrete surface, stations 10, 13 are from the meadow, stations 3 and 9 are from wet terrain and stations 8, 14 and 15 from the lake.



A. *Evaluation of used algorithms*

As was mentioned above and illustrated in the Fig. 5 the dependency is approximated by two lines – the first one for values of ε smaller than 12 hours and the second line for ε greater than 12 hours.

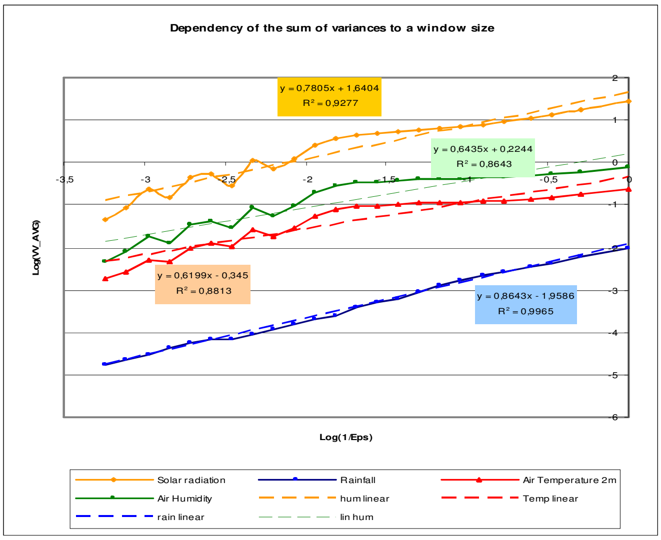
TABLE 2.CORELATION OF RESULTS OF USED METHODS.

correlations	AVG1	AVG2	MM1	MM2
AVG1	1			
AVG2	0,530753	1		
MM1	0,983742	0,400184	1	
MM2	0,609062	0,465935	0,575277	1
IGA_F	0,363197	0,363197	0,388108	-0,23202
AGA_C	0,824767	0,638179	0,824696	0,75013

AVGs and MMs indexes correspond with this designation 1 and 2 (the first and second part of the line). There is a strong correlation (0,98) between both methods AVG1 and MM 1 (index 1 indicates smaller ε). Moreover, the difference between them is only constant. Others correlations are weaker. It leads to an idea, that the qualitative analysis of the fractal properties indicates unique properties of a real system.

The correlation TABLE 2 shows also a significant correlation (0,83) between an indicator of autoregulation AGA_C and all of FD estimation methods – more significantly with the first part (indexed as 1).

FIG. 9. Estimation of Fractal Dimension for different variables from station 9 CIGLEROVSKY_STO_july_2008, FDMAvg method.



VIII. CONCLUSION

Two methods for an calculating of the Fractal Dimension of hydrometeorology variables are described in this article. Measuring sites of a different land use were choice for their possible comparison. Mentioned methods of calculation of Fractal Dimension are based on the relation between smoothed length of the course of the given hydrometeorological variable and independent smoothing parameter. For smoothing of given courses are used the moving average as an analogy to a coastline measuring and moving maximum and minimum as an analogy to the envelope approach to FD calculation. Both methods had been developed at the level of algorithm and at the level of methodology, which containing procedure of algorithm parameters setting. Finally, methods were mutually compared and correlation between AVG1 and MM1 method was proved. Moreover, the indicator of autoregulation of the temperature in the ecosystem was developed and the correlation between this index of autoregulation of temperature and fractal dimension estimation of the temperature by AVG1 and MM1 method is also significant (0,82 and 0,83). Developed methods are usable for the analysis of hydrometeorology variables and for a testing of the relation with autoregulation functions of the ecosystem. Especially autoregulation of the temperature in the vegetation (TABLE 2), which is solved in the context of an effect of the Urban Heat Island and in the context of a new approach to the smart cities [18].

IX. ACKNOWLEDGEMENT

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