Input shaping solutions for drones with suspended load: first results

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Abstract—In this paper our first achievements are reported on application of input command shapers for control of quadcopters with suspended load. Simulation results are presented for a free 2DoF quadcopter. The flight control system, consisting of two PID controllers and of a static nonlinearity mapping the propellers thrusts to particular degrees of freedom, is augmented by input shapers in the feedforward path. Properties of the resulting control law are presented and further research proposals are elaborated.

I. INTRODUCTION

Signal shaping is a well known technique for compensating the undesirable oscillatory modes of various mechanical systems. Starting from the Smith's posicast [1], the topic has received a considerable attention in the control theory and directly led to many engineering applications. Various types of shapers have been proposed and investigated by Singer, Seering, Singhose, et al., [2], [3]. Next to the zero-vibration (ZV) shaper of an analogous structure as the posicast, they developed more robust zero-vibration-derivative (ZVD) shaper and extra insensitive (EI) shaper [4]. These were followed by multi-modes shapers tuned to two or more selected flexible modes, [5], [6], [7], [8], [9]. In discrete time-domain, the signal shaping was first addressed in [10], the papers [8], [11] followed soon. Robustness analysis of signal shaping techniques was performed first in [12], related substantial more recent reports are [3], [13], [14] and [15]. On the application side, the shapers are particularly involved in controlling flexible devices, e.g. flexible manipulators and cranes [16], industrial robots [17], [18] etc.

Structurally different input shapers have been introduced by the authors' team [19], [20], where instead of a lumped delay, a distributed delay is considered in the shaper structure. Next to the smoothening effect at the signal accommodation part brought by the delay distribution, the retarded characteristics of the shaper spectrum can be considered as an implementation benefit, particularly, if the shaper is implemented within a closed loop system [21].

In this paper we focus on application of the classical ZV shaper and its recently proposed distributed alternative (the DZV shaper) within a control system of a drone with suspended swinging load. Simulation results are presented for the simple case of a feedforward shaper interconnection, which leads to suppression of "operator-induced-oscillations" of the swinging payload. Application of inverse feedback shapers, which can in addition address oscillations due to atmospheric disturbances like gusts or turbulences, is a subject of further research and it is not described in this report. This work is related to the results achieved recently by research group of prof. Singhose (GATECH), focused on an application of input shapers for oscillatory suppression modes of payload on autonomous and radio controlled helicopters [22], [23]. Even though the similarity between helicopter and quadcopter dynamical properties is obvious, the transformation of the results still provides a number of challenging tasks.

The paper is organized as follows. The preliminaries on input shaping are presented in Section II. Section III is devoted to the simulation model of our setup. The flight control system is described in section IV. Shapers used for our demonstrations are developed in Section V. Simulation results are discussed in Section VI. Further research directions and concluding remarks are given in Section VII.

II. PRELIMINARIES ON INPUT SHAPING

A general form of a zero vibration shaper is as follows

$$u(t) = Aw(t) + (1 - A) \int_0^\vartheta w(t - \eta) dh(\eta),$$
 (1)

where w and u are the shaper input and output, respectively. The parameters are the gain $A \in [0, 1]$ and the time delay with a shape determined by $h(\eta)$, which is a non-decreasing function over the interval $\eta \in [0, \vartheta]$ with the boundary values h(0) = 0 and $h(\vartheta) = 1$.

The transfer function of the shaper is given by

$$S(s) = A + (1 - A)G(s)$$
(2)

where $G(s) = \mathcal{L} \{g(\eta)\}$, with $g(\eta) = \frac{dh(\eta)}{d\eta}$ being the impulse response of the delay. The zeros of (1) are determined as the roots of the equation S(s) = 0.

In majority of applications, the input shaper is linked with a system in a serial connection in order to shape its input so that the system's oscillatory modes are not induced. In spectral domain synthesis of the shaper, its dominant couple of zeros is placed at the position of the system pole $r_{1,2} = -\beta \pm j\Omega, \beta =$ $\omega \zeta, \Omega = \omega \sqrt{1 - \zeta^2}$, where ζ, ω are the damping and natural frequency of the mode to be compensated.

A. Classical ZV shaper with a lumped delay

The most common zero vibration shaper [2], [3], denoted as ZV, involves a lumped delay with

$$h(\eta) = \begin{cases} 0, & \eta < \vartheta \\ 1, & \eta \ge \vartheta \end{cases} ,$$
 (3)

with the impulse response $g(\eta) = \delta(\eta - \vartheta)$, where $\delta(\eta - \vartheta)$ denotes the time shifted Dirac impulse. The delay transfer function is given by

$$G(s) = e^{-s\vartheta}.$$
 (4)

The shaper synthesis is done by placing the dominant zero $s_{1,2}$ of the shaper at the position of $r_{1,2} = -\beta \pm j\Omega$ providing the parameters [12]

$$A = \frac{e^{\frac{\beta}{\Omega}\pi}}{1 + e^{\frac{\beta}{\Omega}\pi}}, = \frac{e^{\frac{\zeta}{\sqrt{1-\zeta^2}}\pi}}{1 + e^{\frac{\zeta}{\sqrt{1-\zeta^2}}\pi}}$$
(5)

$$\vartheta = \frac{\pi}{\Omega} = \frac{\pi}{\omega\sqrt{1-\zeta^2}}.$$
(6)

B. DZV Shaper with a distributed delay

In [24], [19], as an alternative to the ZV shaper, the DZV shaper with equally distributed delay was introduced with

$$h(\eta) = \begin{cases} 0, & \eta \in \tau \\ \frac{1}{\vartheta}(\eta - \tau), & \eta \in [\tau, \vartheta] \\ 1, & \eta > \vartheta \end{cases}$$
(7)

and the transfer function

$$G(s) = \frac{1}{(\vartheta - \tau)s} (e^{-s\tau} - e^{-s\vartheta}).$$
(8)

Again, the synthesis is done by placing the zero $s_{1,2}$ of the shaper at the position of $r_{1,2} = -\beta \pm j\Omega$, see [20] for the fully analytic parametrization procedure.

The main benefit of the DZV shaper is the retarded character of its spectrum, compared to the neutral character of the spectrum of ZV shaper. This can be utilized particularly in the inverse implementation of the shaper within the closed loop [25], [21] where the high frequency zeros of the shaper are projected to the closed loop poles. Thus, the neutrality of ZV shaper with infinitely many zeros located very close to the stability boundary is a considerable stability risk, which is not the case if DZV shaper is applied. Other benefits of the DZV shaper are the smother transient at the accommodation stage of the signal and the better robustness at the higher frequency range. On the other hand, the response time of DZV shaper is considerably longer than the response time of ZV shaper.

III. MODEL OF THE QUADCOPTER

In this work, a two dimensional model of a quadcopter with suspended load at the center of gravity is considered as shown in figure 1. The mass of the copter frame is denoted as M and the mass of the payload suspended on a rope of a length r is given as m.



Fig. 1. Basic schema of copter with payload

Based on the physical analysis, the quadcopter model has been derived in the following form

$$(M+m)\ddot{x} - mR\dot{\phi}^2\sin\phi + mR\dot{\phi}\cos\phi = F_x \qquad (9)$$

$$(M+m)\ddot{y} + Mg + mg\cos\phi - mR\phi^2\cos\phi = F_y \quad (10)$$

$$mR^{2}\phi + mRg\sin\phi + mR\ddot{x}\cos\phi - mR\ddot{y}\sin\phi = 0, \quad (11)$$

where x and y denote the position of the quadcopter, ϕ is the angle of the suspended load. The quadcopter pitch angle α is given by

$$\ddot{\alpha} = \frac{l}{2I} \left(F_1 - F_2 \right) \tag{12}$$

with the action forces F_1 and F_2 . The forces are transformed to

$$F_x = (F_1 + F_2)\sin\alpha \tag{13}$$

$$F_y = (F_1 + F_2)\cos\alpha \tag{14}$$

used in (9)-(10), where F_1 and F_2 are forces given by propellers, I is the moment of inertia of the whole frame and l is the length between propellers axes.

Equations (9)-(11) have been linearized and transformed to state space model

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{15}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{16}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{mg}{M+m} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -\frac{g}{r} - \frac{mg}{r(M+m)} & 0 \end{bmatrix}$$
(17)

$$\mathbf{B} = \begin{bmatrix} 0 & 0 \\ mr + \frac{1}{M+m} & 0 \\ 0 & 0 \\ 0 & \frac{1}{M+m} \\ 0 & 0 \\ -\frac{m}{M+m} - \frac{1}{r(M+m)} & 0 \end{bmatrix}$$
(18)
$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$
(19)

where $\mathbf{x} = [x, \dot{x}, y, \dot{y}, \phi, \dot{\phi}]^T$, $\mathbf{u} = [F_x, F_y]^T$ and $\mathbf{y} = [\dot{x}, \dot{y}, \dot{\phi}]^T$.

Due to structural properties and functioning of the control system, the second part of the model (12)-(14) cannot be linearized.

Simulation results of this model implemented in Matlab-Simulink are shown in figures 2 and 3. The following parameters are considered: $M = 0.4kg, m = 0.1kg, I = 3^{-3}kgm, r = 1.5m, l = 0.4m$

As can be seen from the responses to the step-wise changes of the input forces, the dynamical properties are oscillatory. It is mainly done by the coupling between the quadcopter and the suspended load.



Fig. 2. Linearized model validation F_x



Fig. 3. Linearized model validation F_y

IV. FLIGHT CONTROL SYSTEM

The primary objective is to control the velocities in x-y axis. This is done by two PI controllers determining the desired forces in x and y directions

$$F_x(s) = \frac{r_{x0}s + r_{xi}}{s}e_x(s),$$
 (20)

$$F_y(s) = \frac{r_{y0}s + r_{yi}}{s}e_y(s) + \frac{g}{M+m},$$
 (21)

where the control errors are given by $e_x = \dot{x}_{set} - \dot{x}$ and $e_y = \dot{y}_{set} - \dot{y}$ with the velocity set-points \dot{x}_{set} and \dot{y}_{set} . Parameters of the controllers are $r_{x0}, r_{xi}, r_{y0}, r_{yi}$.

In order to transform the above desired forces to the actual forces of the propellers, the following nonlinear transformation is used

$$F_1(t) = \frac{F}{2} + \Delta F, \qquad (22)$$

$$F_2(t) = \frac{F}{2} - \Delta F, \qquad (23)$$

$$F(t) = \sqrt{F_x^2 + F_y^2},$$
 (24)

where ΔF is the output of the PD controller of the pitch angle

$$\Delta F(t) = r_{\alpha 0} \left(\alpha_{set} - \alpha \right) - r_{\alpha d} \dot{\alpha}, \qquad (25)$$

with the parameters $r_{\alpha 0}$ and $r_{\alpha d}$ and the setpoint $\alpha_{set} = \operatorname{atan} \frac{Fx}{Fy}$.

The whole control system scheme including the model is shown in figure 4. The detailed control scheme is then given in subsequent figure 5.

Let us note that this simple structure control system has been selected due to the planned implementation at the physical model of the quad-copter. Due to this fact, the number of tunable parameters is relatively small, which does not allow full assignment of the rather complex dynamical properties of the copter-payload system. Therefore, the input shaper will be applied to pre-compensate the oscillatory mode.

V. COMPENSATION OF OSCILLATORY MODE BY INPUT SHAPERS

The subsequent step in the control system design is application and parametrization of the input shapers to precompensate the oscillatory mode of the system that can be nicely seen in the responses shown in Fig. 2, 3 for the system alone and in figure 6 for the controled system. Two input shaper cases are considered here i) classical ZV shaper $S_{ZV}(s) = A_1 + (1 - A_1)e^{-s\tau_1}$ with a lumped delay, and ii) DZV shaper $S_{DZV}(s) = A_2 + (1 - A_2)\frac{1 - e^{-s\tau}}{sT}e^{-s\tau_2}$ with a distributed delay.

The shapers has been applied in a feedforward manner to filter the reference signal \dot{x}_{set} and tuned as described in Section II. First, the shapers were tuned to compensate the undamped oscillatory mode of the payload itlesf, which is determined as $\Omega = \sqrt{g/r} = 2.56s^{-1}$. The shaper parameters were obtained as $A_1 = 0.5, \tau_1 = 1.2285$ for ZV and $A_2 = 0.4738, \tau_2 = 0.9213, T = 0.6142$ for the DZV.

Note however that if the mass of the payload is substantial compared to the mass of the drone, mechanical coupling of those two dynamics will occur and will alter the frequency and damping of the target mode. As a result, performance of the shaper can be expected to deteriorate. This will be considered as the second case in the simulation study presented below. For this case, the linearized closed loop spectrum have been determined and the mode responsible for the oscillations have



Fig. 4. System overview



Fig. 5. Flight control lan structure



Fig. 6. Set-point response of the feedback system without a shaper with payload weight $m=0.1 kg\,$



Fig. 7. Set-point response of the feedback system with a ZV shaper tuned for the nominal frequency of oscillations, with payload weight m = 0.001 kg



Fig. 8. Set-point response of the feedback system with a DZV shaper tuned for the nominal frequency of oscillations, with payload weight m = 0.001 kg



Fig. 9. Set-point response of the feedback system with a ZV shaper tuned for the nominal frequency of oscillations, with payload weight m=0.1kg

been identified as $s_{1,2} = -0.036 \pm 2.54j$ ($\Omega = 2.54s^{-1}$, $\zeta = 0.0143$). The shaper parameters for this case were obtained as $A_1 = 0.5112$, $\tau_1 = 1.237$ for ZV and $A_2 = 0.485$, $\tau_2 = 0.9265$, T = 0.6185 for the DZV.

VI. SIMULATION RESULTS

First we consider the shapers tuned to the nominal frequency of the payload, i.e. considering the uncoupled case, and a lightweight payload of m = 0.001 kg. Simulation results are presented in Figure 7 for ZV shaper and in Figure 8 for DZV shaper. Notice that the attenuation of the payload oscillations is close to the ideal case for both the shaper cases. It is due to the fact that the light weight load does not influence the dynamics of the copter, and thus the oscillatory mode is close to the one of the uncoupled case.

However, when the payload mass is increased to 0.1kg, substantial coupling occurs due to mechanics as explained in the section above, and also due to actions of the flight control system which now reacts to the payload motion - since this motion affects substantially also the motion of the drone itself to which the flight controller is attached, unlike in the



Fig. 10. Set-point response of the feedback system with a DZV shaper tuned for the nominal frequency of oscillations, with payload weight m = 0.1 kg



Fig. 11. Set-point response of the feedback system with a ZV shaper tuned to the oscillatory mode of the coupled dynamics, with payload weight m = 0.1 kg

lightweight-payload case. Resulting responses are in Figure 9 for ZV and Figure 10 DZV. Compared to the previous case, significant performance loss can be observed.

Finally, we present simulation results for the case when the shapers were tuned to the complete loop consisting of the complex multibody system (drone+payload) and the flight controller attached. The resulting responses with ideal compensation for this coupled case are presented in Figure 11 for ZV and Figure 12 for DZV shapers.

VII. FURTHER RESEARCH AND CONCLUSIONS

Our first achievements on application of input command shapers for control of quadcopters with suspended load are reported in this paper. Simulation results are presented for a free 2DoF quadrocopter. The flight control system, consisting of two PI controllers and of a static nonlinearity mapping the propellers thrusts to particular degrees of freedom, is augmented by the ZV and DZV shaper respectively in the feedforward path. Properties of the resulting control law are presented.



Fig. 12. Set-point response of the feedback system with a DZV shaper tuned to the oscillatory mode of the coupled dynamics, with payload weight m = 0.1 kg

For future research, the following open problems are to be resolved:

- 3D version: a full three-dimensinal simulation model shall be developed.
- Inverse feedback shapers: solutions discussed in [21] shall be applied for our problem. Such a way, the effect of atmospheric disturbances can be handled effectively. Needless to say, turbulences and gusts will be a substantial source of vibrations in any practical applications (unlike the classical case studies with portal cranes or space manipulators) [26], .
- Experimental validation: at present, a simplified laboratory setup exists with a two-degree-of-freedom drone attached to a rail. See [21], [27]. We plan to build a full 6DoF platform for further experiments.

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