2017 21st International Conference on Process Control (PC)
June 6–9, 2017, Štrbské Pleso, Slovakia

Intuitionistic Fuzzy Radial Basis Functions Network for modeling of nonlinear dynamics

Yancho Todorov, *IEEE member*
Department of Chemical and Metallurgical Engineering,
Laboratory of Automation and Process Control,
Aalto University,
Helsinki, Finland
yancho.todorov@ieee.org

Margarita Terziyska, *IEEE member*
Department of Informatics and Statistics,
University of Food Technologies,
Plovdiv, Bulgaria
m.terziyska@uft-plovdiv.bg

Petia Koprinkova-Hristova
Institute of Information and Communication Technologies,
Bulgarian Academy of Sciences,
Sofia, Bulgaria
pkoprinkova@bas.bg

***Abstract*—This paper deals with a design methodology for a neural network with improved robust qualities in notion to handling uncertain input data space variations. The proposed network topology combines the simplicity of the radial basis functions networks to interpret or classify data pairs and the abilities of the intuitionistic fuzzy logic to deal with the vagueness of the data space. A simplified gradient optimization procedure as a learning approach for the designed hybrid neural network is proposed. To investigate the effects of the generated structure throughout varying network parameters, the modeling of a two benchmark chaotic time series – Mackey-Glass and Rossler under uncertain conditions is investigated. The obtained results prove the flexibility of the approach and its potentials to cope with data variations.**

Keywords*—Radial Basis Functions Network; neural networks; intuitionistic fuzzy logic; uncertanties; chaotic time series

I. Introduction

In recent years, the neural networks became popular tools in many engineering tasks and economical aspects of our everyday life. They imposed the “brain-style” computation where data streams are being processed within a network of simple connected modules. Each module consist a population of interconnected computational units, called neurons. The individual units perform in parallel, which make the whole concept computationally simple and efficient. On the other hand, the fundamental property of neural networks to learn from “experience” and to discover data relationships and patterns by simply adjusting the strengths of the interconnections called “weights” enables their ability to “program themselves” to perform very complex tasks [1].

Learning from data with uncertain or missing information is an essential requirement for every learning system. When dealing with real systems, features are missing due to unrecorded information or due to occlusion in vision, and measurements affected by noise.

When building regression models, the uncertainty is expected as additive noise, attributed to the dependent variable. This is the most typical case for real systems and assumptions that input features might be uncertain or even missing completely, may lead to model deterioration. In some situations, the problem can be ignored, as we are satisfied with the obtained results, but for real time on-line applications connected to modeling and control of nonlinear systems it may lead to a serious compromise of the whole strategy.

Usually, there are two main reasons why we might want to deal with uncertain input data variations. By one hand, we might be interested to study the underlying relationships as they might have some physical meaning, and from another when the system is non-stationary, the occurring uncertainties might vary over the time [2].

The naive strategy of training networks for all possible input combinations lead to complexity explosion and would require sufficient data for all relevant cases. Therefore, flexible mechanisms for handling of uncertain variations of non-stationary data streams should be employed.

In 60’s Zadeh has introduced the fuzzy set theory as a framework dealing with data vagueness and partial truth by assigning a degree of membership for an element of a universe of discourse [3]. Later on, due to continuous criticism that type-1 fuzzy sets cannot deal with uncertainties, his theory has been extended to type-2 fuzzy sets by the developments of Karnik and Mendel [4-5]. In the beginning of 80’s Atanassov proposed the theory of the intuitionistic fuzzy sets (IFS) by adding a degree of non-membership to Zadeh’s concept. The IFS framework shown a great potential to address more accurately to uncertainty quantification and provides an opportunity to precisely model problems based on existing knowledge and observations [6]. Both of these frameworks, lead to soft computing and approximate reasoning [7].

Besides, the great potential of IFS, few applications exist, as well as, proposed hybrid structures. The combination of ideas from neural networks and IFS is studied in [8]. In [9] the mathematical apparatus necessary to design an IFS Feed Forward Neural Network is given, while an IFS Neural Network with triangular membership functions is described in [10]. An Adaptive Intuitionistic Fuzzy Inference System of Takagi-Sugeno Type is discussed in [11]. Applications to nonlinear modeling and predictive control on the basis of IFS are presented in [12-13]. In [14], a max-min Intuitionistic fuzzy Hopfield neural network (IFHNN) is proposed.

Radial Basis Function (RBF) neural networks have drawn significant attention in the machine learning community due to their strong performance and nice theoretical properties. They represent a classical family of algorithms for supervised learning mostly used for approximation of a target nonlinear function or clustering of data points through explicit use of kernel functions, such as Gaussian ones. The goal of a RBF network is to adjust the centers and the weights of these functions trough appropriate learning strategy. It is easy to see that any input data points can be fitted exactly or classified by allowing every data point to be a center and choosing appropriate coefficients [15].

Unlike, the most neural network topologies the RFB networks lack of a comprehensive mechanism to deal with uncertain variation of the input date space. Usually, this may lead to absorbson of misleading data patterns and deterioration of the network performance that requires a suitable solution to avoid this phenomenon. A possible simple approach is to adopt ideas from the intuitionistic fuzzy logic as discussed in [8], where an application to data clustering is presented.

This paper describes the development of a novel intuitionistic fuzzy RBF network for modeling of complex nonlinear dynamical processes under uncertain conditions. The hidden network layer implements an IFS processing mechanism in order to overcome the vagueness of the input data space. A simple gradient descent approach to train the synaptic links and the parameters of the basis functions is implemented. The potentials of the proposed structure to model nonlinear dynamical systems, as well as the impact on some design parameters is studied by numerical experiments for modeling of benchmark chaotic times series experiencing noisy conditions.

II. INTUITIONISTIC FUZZY RADIAL BASIS FUNCTION NETWORK

A. Intuitionistic Fuzzy Logic

Intuitionistic Fuzzy Set (IFS) *A* over a finite universal set *E* is defined as an object with the following properties [6]:

$$\mathrm{A} = \{(\mathrm{x}, \mu_{\mathrm{A}}(x), \nu_A(x)) \mid x \in X\} \tag{1}$$

where μ_A: $X \rightarrow [0,1]$ and ν_A: $X \rightarrow [0,1]$ are such that $0 \leq \mu_A + \nu_A \leq 1$ and $\mu_A(x)$ represents the degree of membership of $x \in A$, while $\nu_A(x)$ represents the degree of non-membership of $x \in A$. Therefore, for each intuitionistic fuzzy set in *X*, we call $\pi_A(x) = 1 - \mu_A - \nu_A$ the degree on non-determinancy (uncertainty) or hesitation of $x \in A$. This parameter expresses a hesitation degree of weather *x* belongs to *A* or not and it is obviously $0 \leq \pi_A \leq 1$ for each $x \in X$.

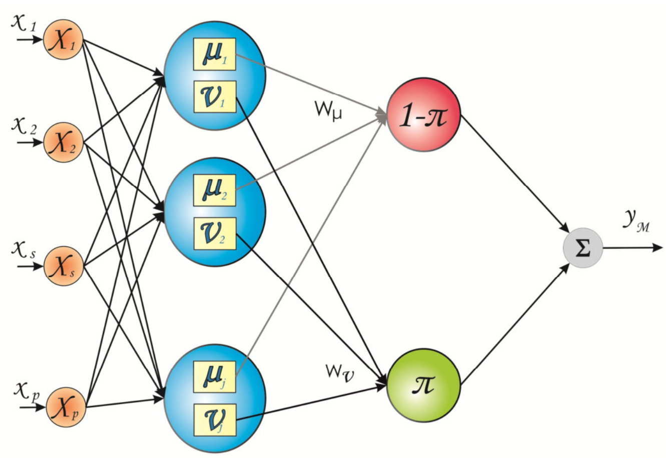
B. Intuitionistic Fuzzy RBF Network

The topology of the proposed Intuitionistic Fuzzy RBF network (IFRBFN) is shown on Fig.1. The IFRBFN is a simple network with one hidden layer, which realize multiple input single output (MISO) structure with RBF neurons. The *first layer* is the input layer. The nodes in this layer only accept the input variables, where x_i is an input value, $i=1:p$ is the number of the inputs of the network and then transmit them to the next layer directly. The *second layer* is a hidden layer, which consist a number of neurons with associated activation function - μ_A and retraction function- ν_A. Both functions, in terms of Gaussian representation can be expressed as:

$$\mu_{ij}(x_i) = \exp\left(\frac{-(x_i - c_{ij})^2}{2\sigma_{ij}^{\ 2}}\right) \tag{2}$$

$$\nu_{ij}(x_i) = \left(1 - \exp\left(\frac{-(x_i - c_{ij})^2}{2\sigma_{ij}^{\ 2}}\right)\right)^k, k \geq 1 \tag{3}$$

Fig. 1. Topology of the proposed Intuitionistic Fuzzy RBF Network.



where, c_j and σ_j are the center and the standard deviation of the Gaussian basis function, $j=1:m$ where *m* is the number of used basis functions in the layer, *k* is a parameter that must be designed. If $k=0$, obviously $\mu_A + \nu_A = 1$ and the hesitation degree π_A also is zero. The schematic representation of an IFS basis function is given on Fig. 2.

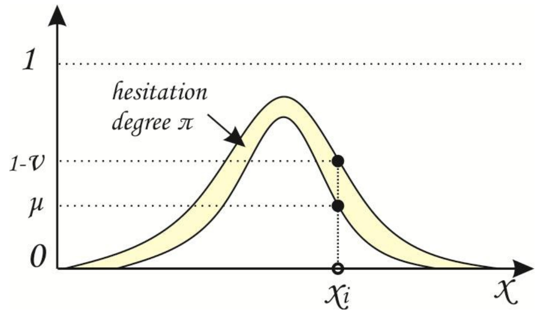
The neurons of the *third layer* perform summation of the outputs from the second layer, along the activation and retraction functions, taking into account the values of the corresponding the synaptic weights and the defined hesitation degrees.

In the *fourth layer*, the output of the network is defined as a sum of its all topological parameters:

$$y(k)=F(\mu_A,\nu_A)=\sum\nolimits_1^j w_{\mu j}\left(f_{\mu_A}\left(w_{01}\sum\nolimits_1^p x_i\right)\right)+ +\sum\nolimits_1^j w_{\nu j}\left(f_{\nu_A}\left(w_{02}\sum\nolimits_1^p x_i\right)\right) \tag{4}$$

$$y(k)=\mathbf{w}(f_{\mu_A}+f_{\nu_A})=\sum\nolimits_1^j(1-\pi_j(x(k)))\mu_j(x(k))w_{\mu j}+ +\sum\nolimits_1^j(\pi_j(x(k)))\nu_j(x(k))w_{\nu j} \tag{5}$$

Fig. 2. Representation of an IFS basis function.



where w_o is a constant weighting of the neuron's net input in order to increase the initial dissimilarity between the neurons and $w_{\mu j}$ and $w_{\nu j}$ are the synaptic links between the second and the third layers.

C. Learning Algorithm of the proposed Intuitionistic NEO-Fuzzy Network

For the proposed IFRBFN structure a simple gradient algorithm, minimizing an error cost term between the real and the modeled system is adopted as a learning approach.

$$E(k)={(y(k)-y_M(k))^2}/{2}={e^2(k)}/{2} \tag{6}$$

During the learning procedure, two groups of parameters are being trained; the vectors of the synaptic weights after the second layer and the centers and the deviations of the activation functions. For calculation of the first group of parameters, the following chain rule notation is employed:

$$\frac{\partial E}{\partial\beta}=\frac{\partial E}{\partial y}\frac{\partial y}{\partial\beta} \tag{7}$$

where β is an adjustable parameter $w_{\mu j}$ or $w_{\nu j}$. Thus, the updating rules for a corresponding synaptic link can be stated as:

$$w_{\mu j}(k+1)=w_{\mu j}(k)+\Delta w_{\mu j}(k)=w_{\mu j}(k)+\eta\left(\frac{\partial E(k)}{\partial w_{\mu j}(k)}\right)= =w_{\mu j}(k)+\eta e(k)(1-\pi_j(k))\mu_j(x(k)) \tag{8}$$

$$w_{\nu j}(k+1)=w_{\nu j}(k)+\Delta w_{\nu j}(k)=w_{\nu j}(k)+\eta\left(\frac{\partial E(k)}{\partial w_{\nu j}(k)}\right)= =w_{\nu j}(k)+\eta e(k)\pi_j(k)\nu_j(x(k)) \tag{9}$$

where η is the learning rate.

The calculation of the second group of parameters by using the following chain rule notation is performed as:

$$\frac{\partial E}{\partial\alpha}=\frac{\partial E}{\partial y}\frac{\partial y}{\partial\mu_A}\frac{\partial\mu_A}{\partial\alpha} \tag{10}$$

where α is a center – c or deviation – σ of a corresponding activation function. Thus, the final recurrent equation are defined as:

$$c_j(k+1)=c_j(k)+\eta e(k)(1-\pi_j(k))w_{\mu j}(k)\frac{\left(w_{0_1}\sum_1^p x_i(k)-c_j(k)\right)}{c_j^2(k)} \tag{11}$$

$$\sigma_j(k+1)=\sigma_j(k)+\eta e(k)(1-\pi_j(k))w_{\mu j}(k)\frac{\left(w_{0_1}\sum_1^p x_i(k)-\sigma_j(k)\right)^3}{\sigma_j^2(k)} \tag{12}$$

III. Numerical Experiments

A. Chaotic time series

Chaos is a dynamical phenomenon, which may be defined using different time series representations. Usually, the chaotic time series are inherently nonlinear and sensitive to initial conditions. On the other hand, they are very often used as practical technique for studying characteristics of complicated dynamics and evaluation of the accuracy of different types of nonlinear models as neural networks.

To investigate the modeling potentials of the proposed IFRBFN structure, two benchmark chaotic time series - Mackey-Glass and Rossler are employed. The Mackey-Glass time series can be described by the following time-delay differential equation:

$$x(i+1)=\frac{x(i)+ax(i-s)}{(1+x^c(i-s))\text{-}bx(i)} \tag{13}$$

where $a=0.2$; $b=0.1$; $C=10$; initial conditions x_0=0.1 and s= 17s.

Another test of the proposed IFRBFN model with Rossler chaotic time series is made. These series are described by the following three coupled first-order differential equations:

$$\frac{\partial x}{\partial t}=-y-z;\quad \frac{\partial y}{\partial t}=x+ay;\quad \frac{\partial z}{\partial t}=b+z(x-c) \tag{14}$$

where $a=0.2$; $b=0.4$; $c=5.7$; initial conditions $x_0=0.1$; $y_0=0.1$; $z_0=0.1$.

B. *Modeling of Chaotic Times Series*

The proposed network is tested in modeling the above-mentioned chaotic times series where a 15% additive noise on the nominal value of the signal is assumed, in order to assess the potentials of the IFS fuzzy logic. The proposed experiments are made for two values of $k=1$ and $k=2$. Due to the lack of extensive research in the IFS domain, there is no specific recipe how to select k. Obviously, the larger values of k will lead to a wider zone of the hesitation degree, thus the selection of k will

Fig. 3. Modelling of a MG time series under 15% input nominal noise, k=1 and without training of the basis functions.

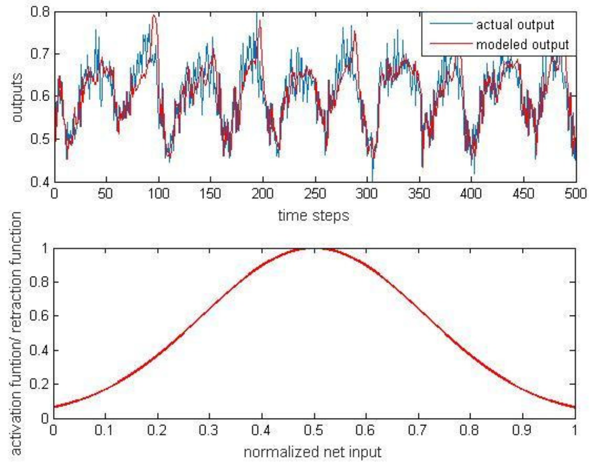
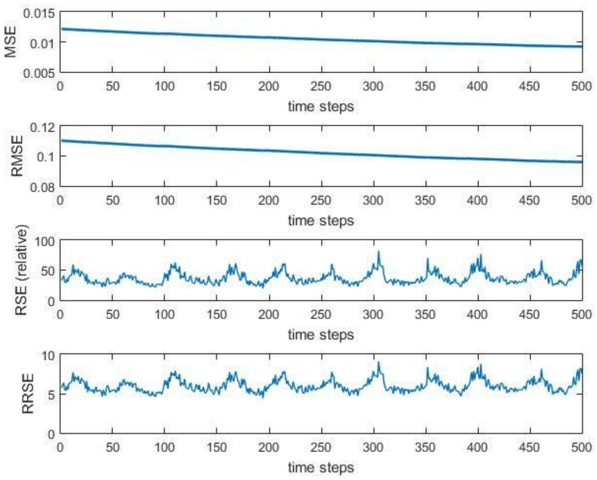


Fig. 4. Estimated Error terms when modelling of a MG time series under 15% input nominal noise, k=1 and without training of the basis functions.



depend mostly on the expected noisiness of the input data space. The input links w_o are initialized with random coefficients and they act as input gains to an RBF neuron net input. This type of links do not undergo training. To investigate

TABLE I. ESTIMATED ERROR TERMS

Time step	k=1		k=2		k=2 with RBF training	
	RMSE	*RRSE*	*RMSE*	*RRSE*	*RMSE*	*RRSE*
50	0.1070	5.2679	0.0993	4.9867	0.0955	4.8630
100	0.1053	5.9578	0.0978	5.6481	0.0940	5.2791
150	0.1035	5.1576	0.0962	4.8872	0.0926	4.6676
200	0.1021	5.6825	0.0949	5.3293	0.0916	5.0694
250	0.1005	6.0691	0.0935	5.1500	0.0905	5.0355
300	0.0993	7.5308	0.0923	6.2820	0.0896	6.1140
350	0.0979	6.3840	0.0914	6.3427	0.0889	5.8307
400	0.0970	7.7507	0.0903	6.9564	0.0879	6.8825
450	0.0957	6.2675	0.0894	6.0712	0.0870	5.6727
500	0.0948	7.9566	0.0885	6.4570	0.0862	6.1795

Fig. 5. Modelling of a MG time series under 15% input nominal noise, k=2 and without training of the basis functions.

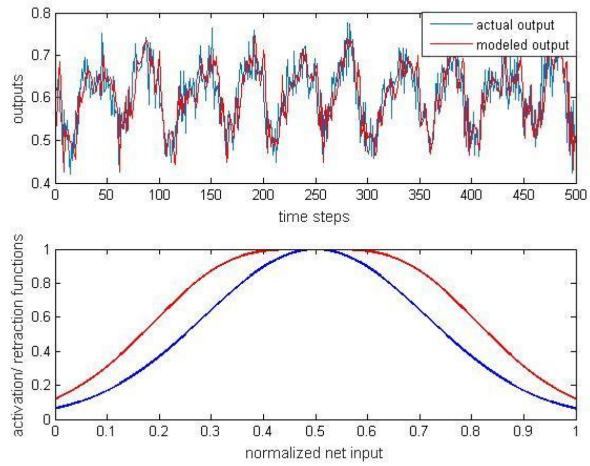
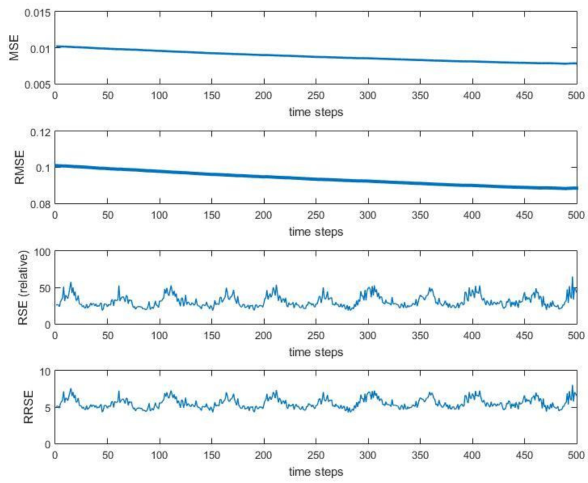


Fig. 6. Estimated Error terms when modelling of a MG time series under 15% input nominal noise, k=2 and without training of the basis functions.

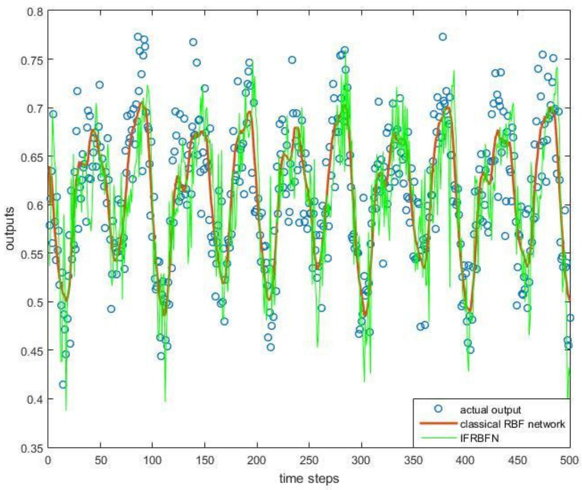


the potentials of the proposed neural networks, three different types of experiments under equal initial conditions are performed. Each network structure is trained for three epochs.

At first, a comparison on the influence of the parameter k is presented on Fig. 3 and Fig. 5, where the modelled signals and the respective shapes of the activation and retraction functions are depicted. It should be mentioned that, when $k=1$ this is a particular case of an RBF neural network, since the parameter $\pi=0$ for each neuron and the activation and retraction functions overlap. On Fig. 4 and Fig. 6 the variations of the MSE (*Means Squared Error*), the RMSE (*Root Mean Squared Error*), the RSE (*Relative Squared Error*) and the RRSE (*Root Relative Squared Error*) are investigated. In Table I are presented the obtained error terms in different sample time intervals. The right side of the table shows an additional comparison between networks with/without training of the basis functions.

The obtained results show the positive effect of the proposed IFS solution, which lead to increased absorption of the noise in the case when a value of $k=2$ is selected and the basis functions undergo training. The investigated MSE and RMSE error terms have a smooth nature leading to its decrease to values closer to zero. On the other hand, the observed relative error terms show the average of the actual signal variations, which are smaller when the parameter k is increased and the basis functions are being trained.

Fig. 7. Comparison between the proposed IFRBFN network and the classical RBF network for the same number of training cycles.



On Fig. 7 a comparison between the proposed IFRBFN network with training of the basis functions and a classical RBF neural network with Gaussian basis functions is shown. Both networks have equal number of neurons in the hidden basis layer and equal number of the training epochs. As can be seen from the proposed network based on Intutitionistic Fuzzy approach, it provides better generalization properties for signals with additive noise, covering more data points of the actual signal.

On Fig. 8 are shown the obtained results when modeling the Rossler chaotic time series in the adopted case of k=2 with training of the activation and retraction functions. As can be observed, the proposed modelling structure performs well again with minimal modeling error. Different error terms are studied on Fig. 9, where the respective transient responses are depicted. The terms show the smooth nature of decrease of

Fig. 8. Modelling of a Rossler time series under 15% input nominal noise, k=2 and with training of the basis functions.

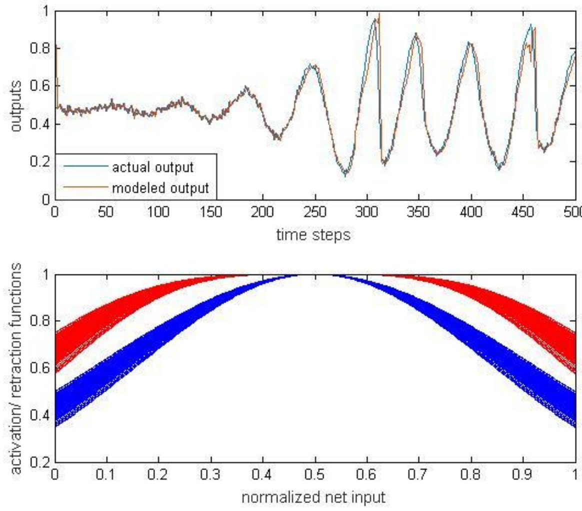
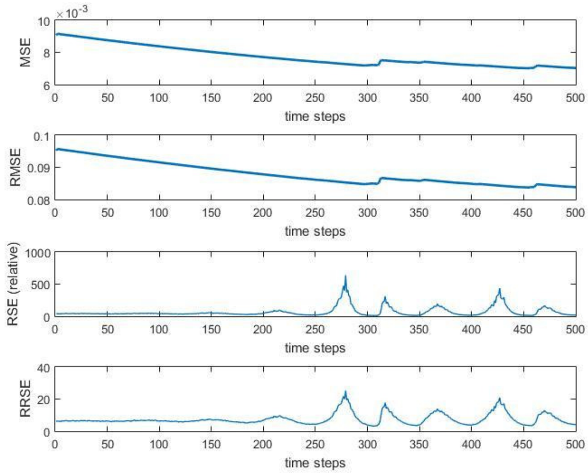


Fig. 9. Modelling of a Rossler time series unider 15% input nominal noise, k=2 and with training of the basis functions.



the MSE and RMSE, no matter the fact that the amplitude of the signal change sharply. Due to that phenomenon, the relative errors have large peak values, but they preserve their slight variations on the average.

Conclusions

It was presented in this paper a design methodology for an Intuitionistic Fuzzy Radial Basis Functions Network for modeling of nonlinear dynamical systems with uncertain input variations. The proposed approach combines the simplicity of the classical RBF networks to interpret nonlinear systems and the flexible mechanism of the Intuitionistic Fuzzy logic to deal with uncertainties. The network parameters are easily trained by employing a simple gradient learning procedure to adjust the network parameters: the synaptic links and the parameters of the basis functions.

The conducted experiments show the efficiency of the proposed approach over the classical RBF networks, since the error terms in modeling are reduced. Increasing of the design parameter k lead to wider hesitation bound, but it should be carefully selected depending on the expected uncertain conditions. The training of the parameters of the basis functions lead to additional reduction of the modeling errors. To address the problem of the computational burden in network topologies with a great number of basis neurons, the training of their parameters can be neglected.

A foreseen future extension of the approach is to extend the ideas for the purpose of modeling of nonlinear industrial processes under uncertain conditions, as well as exploiting the predictive features of the model within novel Model Predictive Control schemes, based on IFS.

References

[1] G. Eason, B. Noble, and I.N. Sneddon, "On certain integrals of Lipschitz-Hankel type involving products of Bessel functions," Phil. Trans. Roy. Soc. London, vol. A247, pp. 529-551, April 1955.

[2] V. Tresp, S. Ahmad, R. Neuner, "Training of neural networks with deficient data," Advances in Neural Information Processing Systems, vol. 6, 1994.

[3] L. Zadeh, "The concept of a linguistic variable and its applications to approximate reasoning-1," Information Sciences, vol. 8, pp. 199–249, 1975.

[4] N. Karnik and J. Mendel, "Introduction to type-2 fuzzy logic systems," In Proc. of the IEEE FUZZ Conference, Anchorage, pp. 915–920, 1998.

[5] N. Karnik and J. Mendel, "Operations on type-2 fuzzy sets," Fuzzy Sets and Systems, vol. 122, pp. 327–348, 2000.

[6] K. Atanassov, Intuitionistic Fuzzy Sets, Springr, Hielderberg, 1999 .

[7] V. Khatibi, H.Iranmanes and A Keramati, "A Neuro-IFS Intelligent System for Marketing Strategy Selection," Springer, Innovative Computing Technology: Proc. of First International Conference, INCT 2011, Tehran, pp. 61-70, 2011.

[8] L. Kuncheva and K. Atanassov, "An Intuitionistic Fuzzy RBFNetwork," In Proc. of EUFIT'96, pp. 777-781, 1996.

[9] X. Zhou, R. Zhao and L. Zhang, "An Intuitionistic Fuzzy Neural Network with Triangular Membership Function," In Proc. of 2013 Chinese Intelligent Automation Conference, Lecture Notes in Electrical Engineering, vol. 254, pp. 813-820, 2013.

[10] X. Zhou, R. Zhao, H. Shang and L. Zhang, "Intuitionistic Fuzzy Neural Networks based on Extended Kalman Filter Training algorithm," In Proc. of the 1st International Workshop on Cloud Computing and Information Security, 2013.

[11] P. Hájek and V. Olej, "Adaptive Intuitionistic Fuzzy Inference Systems of Takagi-Sugeno Type for Regression Problems," Artificial Intelligence Applications and Innovations, IFIP Advances in Information and Communication Technology, vol. 381, pp. 206-216, 2012.

[12] M. Terziyska, Y. Todorov, "Intuitionistic Neo-Fuzzy Network for modeling of nonlinear systems dynamics," In Proc. of 8th IEEE Conference on "Intelligent Systems'16", Sofia, Bulgaria, pp. 617-621, 2016.

[13] M. Terziyska, Y. Todorov, "Intuitionistic Neo-Fuzzy Predictive Control," In Proc. of 8th IEEE Conference on "Intelligent Systems'16", Sofia, Bulgaria, pp. 635-640, 2016.

[14] L. Li, J. Yang and W. Wu, "Intuitionistic fuzzy Hopfield neural network and its stability," Expert Systems with Applications, vol. 29, pp. 589-597, 2005.

[15] Q. Que, M. Belkin, "Back to the Future: Radial Basis Function Networks Revisited," Proceedings of the 19th International Conference on Artificial Intelligence and Statistics (AISTATS) 2016, Cadiz, Spain. JMLR: W&CP vol. 51, pp. 1375-1383, 2016.