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Predictive Control of 5 DOF Robot Arm of Autonomous Mobile Robotic System

Motion Control Employing Mathematical Model of the Robot Arm Dynamics

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Abstract—**The paper deals with a design of model predictive control (MPC) as an example of the advanced local motion control of articulated robot arms in the scope of manipulation operations within the intradepartmental transportation among workplaces. Initially, the use of articulated robotic arms as a part of mobile robotic systems is discussed. Then, the convenient composition of mathematical models of kinematics and dynamics of the aforementioned robot arms is introduced. Thereafter, MPC design is explained. The proposed theoretical methods of the mathematical modeling and control design are demonstrated by the simulation of the 5 degrees of freedom robot arm composed of drive, joint and arm modules of the Schunk Co.**

Keywords—kinematics and dynamics of articulated robots; mathematical modeling; mobile robotic systems; model-based control design; motion control; predictive control

I. Introduction

Articulated robots or especially articulated robot arms constitute common auxiliary machines used for the wide range of operations such as (i) manipulation e.g. transport of raw materials and final products, or (ii) production e.g. painting, welding or assembling. They represent kinematics with good movability and dexterity and adequate workspace range [1].

The mentioned robots are constructed by the revolute joints and solid arms. It determines a spherical shape of their workspace. According to the number of degrees of freedom (DOF), the particular robot can achieve a specific range of positions. The positions, in which the robot begins or finishes required motion, are of specific importance. In these positions, the robot has to precisely maintain a given, typically orthogonal, orientation of a product or tool towards an appropriate product shelf or tool storage.

To extend action radius of the manipulation, the robot arms are fixed to the linear-moving platforms allowing the motion in additional axis. It is typical for the robots and manipulators employed in production lines, where one robot serves several neighboring production machines. Furthermore, the robot arm can be connected to some particular underframe that can extend the robot action radius substantially [2].

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If the robot arm is independent of a particular location and is equipped by some independent source of energy for both the underframe and robot arm, then such combination forms a specific autonomous mobile robotic system. In this paper, the basic control objectives of such systems are outlined. The main attention is focused on the modeling and the design of model predictive control (MPC) for a local motion control of the robot arm [3], [4].

The paper is organized as follows. Section II outlines the principal control objectives for the mobile robotic system shown in Fig. 1. Section III deals with suitable mathematical models for control design with the reference to the considered 5 DOF robot arm. It covers description of the kinematic model including direct and inverse kinematic transformations, and dynamic model derived from Lagrange equations. A specific model rearrangement that leads to linear-like state-space model is introduced as well. Section IV summarizes principles of MPC design. Finally, Section V shows one simulative example of the proposed MPC applied to the derived realistic mathematical model of the 5 DOF robot arm.

Fig. 1. Overall view of the mobile robotic system: 3 DOF mobile underframe with 5 DOF robot arm [5].

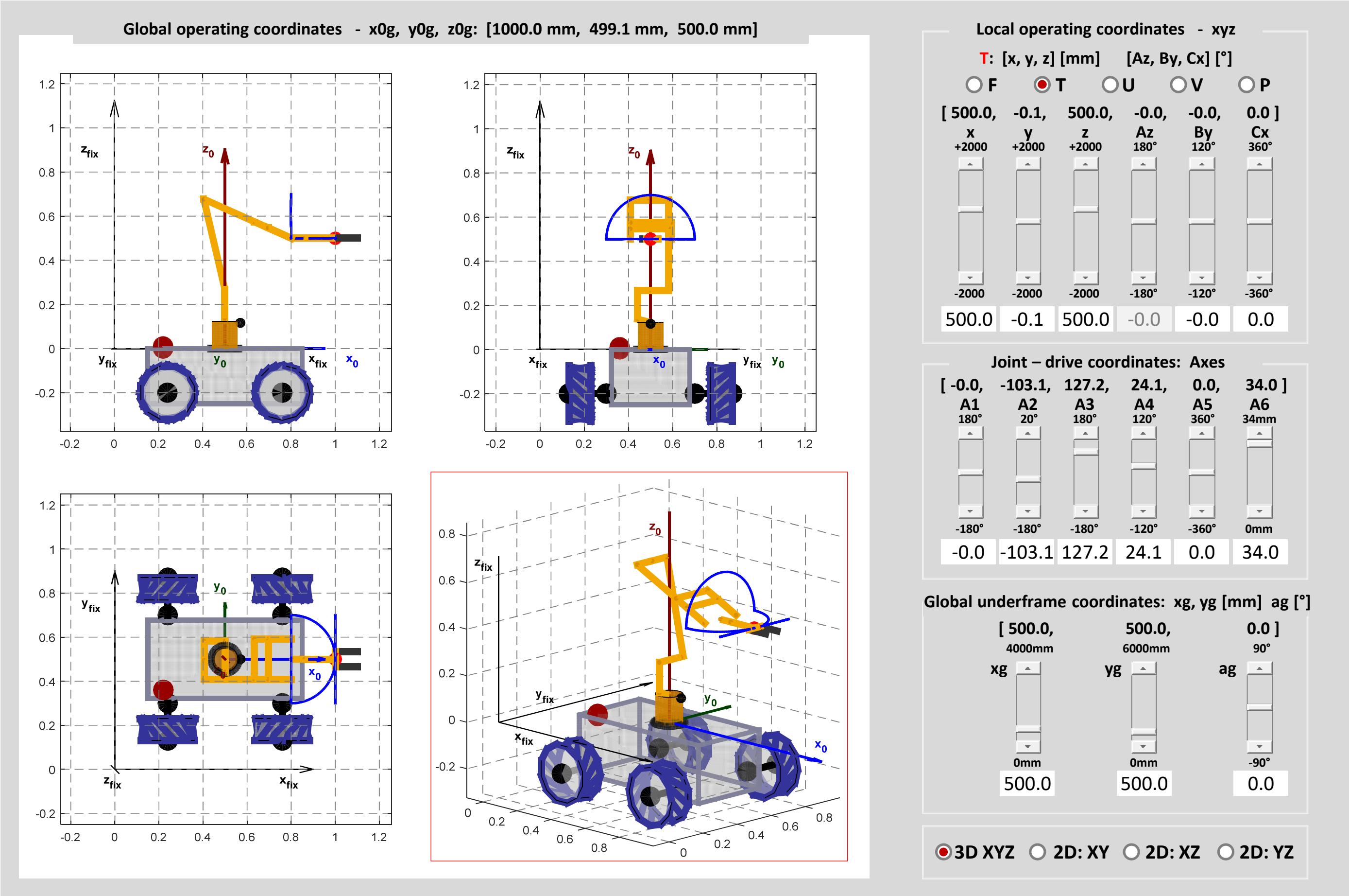


Fig. 2. Basic views: front, top plan, side and 3D view with definitions of global coordinate system (x_{fix}, y_{fix}, z_{fix}) and local coordinate system (x_0, y_0, z_0).

II. Control Objectives of Mobile Robotic System

In general, there are two main objectives for motion control of the mobile robotic system (MRS) [2] that is shown in Fig. 1. The first one is a global objective, i.e. the absolute motion of MRS within its workplace by means of the mobile underframe, which affords the base for the robot arm. The second one is a local objective, i.e. the relative motion of the robot arm end-effector with respect to the mobile underframe of the MRS.

Let us consider the cases where the underframe moves slower compared with the local motion of the robot arm, or the motion of the underframe and robot arm is sequentially arranged i.e. the arm motion does not begin before the underframe motion is finished and vice versa. In such cases, the two said control objectives can be considered as independent without mutual influencing each other. In this paper, the local objective is investigated and discussed.

To analyze the objective for the local motion control of the robot arm, let us focus on depicted plan with basic views in Fig. 2. It shows main key MRS elements and coordinate systems. The robotic arm is marked by a wireframe model, whereas the underframe is represented by solid model including wheels enabling planar omnidirectional motion. For further structural details and technical parameters see [5] and [6].

In Fig. 2, there are two independent coordinate systems global and local. The global coordinate system is a system with global fixed axes (x_{fix}, y_{fix}, z_{fix}) and the local coordinate system is a system with local relative axes (x_0, y_0, z_0). Several different coordinates are associated with them as follows.

Let the position of the underframe be given by coordinates xg, yg and ag that correspond to the directions along global axes x_{fix} and y_{fix} and the rotation angle around orthogonal axis z_{fix}, respectively. Furthermore, let the robot end-effector be given in the local system by local coordinates x, y and z that relate to one of specific operation points F, T, U, V and P, selected as a reference or operating point according to user requirement on the arm motion along a motion trajectory. The coordinates x, y and z are complemented by two angles By and Cx determining the orientation in the given reference point. The angles represent the rotation around axes y_0 and x_0, respectively. Thus, the coordinates (x, y, z, By, Cx) correspond to the 5 DOF of the arm as well as individual joint angles (q_1, q_2, q_3, q_4, q_5), i.e. robot axes A1-A5. Additionally, there is a coordinate s corresponding to the stroke of the robot gripper constituting the axis A6.

Let us consider that a local reference motion of the arm is given by Cartesian coordinates (x, y, z, By, Cx) and stroke s, and they are recomputed to the joint coordinates. Then, control actions (five torques in joints) are designed in the joint space.

Fig. 3. 5 DOF articulated robot arm in detail [2].

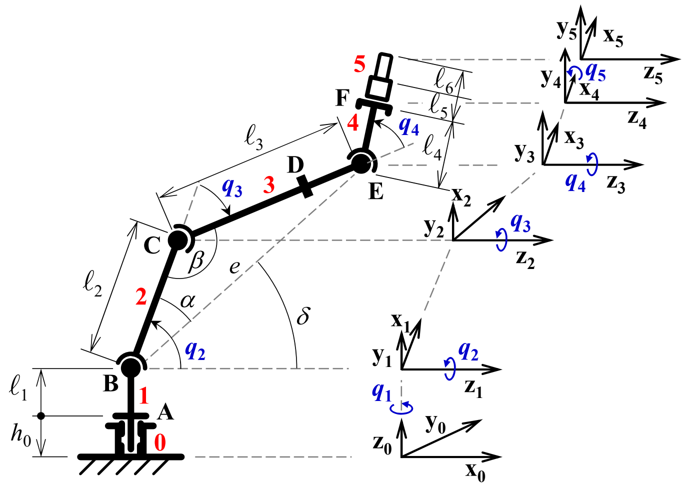


Fig. 4. Scheme of the DH coordinate frames.

III. Mathematical Models for Control Design

A mathematical model used for control design of the articulated arms is given by kinematic and dynamic model as follows.

A. Kinematic Model

Kinematic model arises from the Denavit-Hartenberg (DH) concept that defines the transformations of the coordinates using several homogenous matrices corresponding to individual kinematic pairs of the robot arm [1]. Kinematics of the robot represents the appropriate transformations of the coordinates between joint space (drives) and operation space (end-effector) in both directions: direct ($q \rightarrow y$) and inverse ($y \rightarrow q$), where joint and operation coordinates q and y are as follows

$$q = [q_1, q_2, q_3, q_4, q_5]^T, \quad y = [x, y, z, by, cx]^T \qquad (1)$$

• Direct Kinematic Transformation

For a considered 5 DOF articulated robot (Fig. 3, Fig. 4), the direct kinematic transformation (DKT) is defined as follows

$$\begin{aligned} F &= T_0^1(q_1)\, T_1^2(q_2)\, T_2^3(q_3)\, T_3^4(q_4)\, T_4^5(q_5)\, r_0 \\ &= T_0^5\, r_0, \quad F = [x_F, y_F, z_F, 1]^T, \quad r_0 = [0\,0\,0\,1]^T \end{aligned} \qquad (2)$$

where F is a coordinate vector of the robot flange (end-effector) and T_{i-1}^i are individual transformation matrices defied as:

$$T_{i-1}^i = \begin{bmatrix} \cos\vartheta_i & -\cos\alpha_i \sin\vartheta_i & \sin\alpha_i \sin\vartheta_i & a_i \cos\vartheta_i \\ \sin\vartheta_i & \cos\alpha_i \cos\vartheta_i & -\sin\alpha_i \cos\vartheta_i & a_i \sin\vartheta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

The matrices or set of algebraic equations represent the solution for the positions. The velocities and accelerations are computed by the appropriate derivatives of (2). The particular DH parameters for the considered 5 DOF robot arm are summarized in the TABLE I.

TABLE I. DH Parameters

Links	**Parameters**			
	ϑ_i	d_i	a_i	α_i
link 1	q_1	ℓ_1	0	90°
link 2	$-q_2$	0	ℓ_2	0°
link 3	$-q_3$	0	ℓ_3	0°
link 4	q_4	0	ℓ_4	0°
link 5	*0*	0	0	q_5

• Inverse Kinematic Transformation

Inverse kinematic transformation (IKT), needed for computation of joint (drive) coordinates from operation ones, can be determined from the said transformation matrices. However, for simple use, IKT can be expressed straightforwardly considering the scheme in Fig. 4 in the following way

$$\begin{aligned} &\alpha = \arccos \frac{\ell_2^2 + e^2 - \ell_3^2}{2\,\ell_2\, e}, \quad \beta = \arccos \frac{\ell_2^2 + \ell_3^2 - e^2}{2\,\ell_2\, \ell_3} \\ &\delta = \arccos \frac{\sqrt{x_E^2 + y_E^2}}{e}, \quad e = \sqrt{x_E^2 + y_E^2 + (z_E - \ell_1 - h_0)^2} \end{aligned} \qquad (4)$$

where α, β and δ are auxiliary angles determined from a scalene triangle BCE with one variable side e. Then, the searched joint angles can be determined as follows

$$\begin{aligned} &q_1 = \arccos \frac{x_E}{\sqrt{x_E^2 + y_E^2}}, \quad q_2 = \alpha + \delta \\ &q_3 = \pi - \beta, \ q_4 = by - (q_2 + q_3), \ q_5 = cx \end{aligned} \qquad (5)$$

Note that joint angles are determined for known xyz coordinates of the point $E = [x_E, y_E, z_E, 1]^T$ from the system x_0, y_0, z_0.

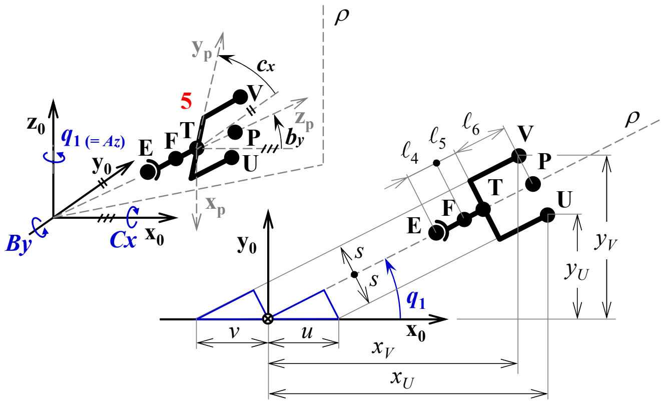


Fig. 5. Detail of the link 5 representing the robot griper with stroke s.

If the coordinates of the robot motion is related to another point, it is necessary to take such a point into account. Then, computations of the point E from the other points (see Fig. 5) are as follows. At first, let us consider the points F, T and P, all lying in one plane ρ, which is a vertical plane containing axis z_0 and central points of all robot joints (A, B, C, E and F):

$$\begin{aligned} &T_g = T_{6c}(q_1)\, T_{5b}(-by)\, T_{4a}(cx) \\ &F: \; E = T_g\, T_{1x}(-\ell_4)\, r_0 + F \\ &T: \; E = T_g\, T_{1x}(-\ell_4 - \ell_5)\, r_0 + T \\ &P: \; E = T_g\, T_{1x}(-\ell_4 - \ell_5 - \ell_6)\, r_0 + P \end{aligned} \tag{6}$$

For the remaining points U and V, the angle q_1 cannot be determined according to (5), since these points do not belong to the plane ρ (see Fig. 5). However, the angle q_1 can be determined by means of the following trigonometric laws:

$$\begin{aligned} &\tan q_1 = \frac{y_U}{x_U - u} = \frac{\sin q_1}{\cos q_1}, \quad \frac{s}{\sin q_1} = \frac{u}{\sin \frac{\pi}{2}} \\ &\cos^2 q_1 + \sin^2 q_1 = 1 \end{aligned} \tag{7}$$

It leads to the quadratic equation with the following roots

$$\begin{aligned} &q_{1,(1,2)} = \arccos \frac{x_U\, s \pm \sqrt{x_U^2\, s^2 - \left(x_U^2 + y_U^2\right)\left(s^2 - y_U^2\right)}}{x_U^2 + y_U^2} \\ &\textit{if} \;\; y_U \geq 0 \wedge x_U \geq 0, \;\; q_1 = q_{1,(1)}, \;\; \textit{else} \;\; q_1 = q_{1,(2)} \end{aligned} \tag{8}$$

Then, coordinates of the point E can be determined as follows

$$\begin{aligned} &T_g = T_{6c}(q_1)\, T_{5b}(-by)\, T_{4a}(cx) \\ &U: \; E = T_g\, T_{1x}(-\ell_4 - \ell_5 - \ell_6)\, T_{2y}(s)\, r_0 + U \\ &V: \; E = T_g\, T_{1x}(-\ell_4 - \ell_5 - \ell_6)\, T_{2y}(-s)\, r_0 + V \end{aligned} \tag{9}$$

where T_{1x}, T_{2y}, T_{4a}, T_{5b}, T_{6c} in (6) and (9) are the homogenous transformation matrices for translation along the axes x and y and for rotation around the axes x, y and z [1].

B. Dynamic Model

The model of the robot dynamics employs both kinematic and dynamic quantities. It can be derived by means of Lagrange equations of the second type [1], [7]:

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}}\right)^T - \left(\frac{\partial E_k}{\partial q}\right)^T + \left(\frac{\partial E_p}{\partial q}\right)^T = \tau \tag{10}$$

These equations lead to the equations of motion that can be written in the following form

$$M(q,\dot{q})\,\ddot{q} + N(q,\dot{q})\,\dot{q} + g(q) = \tau \tag{11}$$

They are composed using transformation matrices defined in DKT and dynamic parameters as weights, moments of inertia relative to the coordinate frames shown in Fig. 4. The equations of motion (11) can be rewritten with a separate highest (second) derivative as follows

$$\ddot{q} = -M^{-1} N\, \dot{q} - M^{-1} g + M^{-1}\tau \tag{12}$$

where $M = M(q,\dot{q})$ and $N = N(q,\dot{q})$ are matrices relating to the effects of inertia, $g = g(q)$ is a vector corresponding to the effects of gravitation, parameters of which are lengths, weights and moments of inertia of arms, see [5]. τ is a vector of torques expected in the particular joints of the robot arm.

The model (12) is suitable for further modification that enables us to obtain the standard linear-like state-space model for control design. It can be written as follows

$$\begin{aligned} &\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & -M^{-1}N \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u \\ &q = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} \end{aligned} \tag{13}$$

$$\begin{aligned} \dot{x} &= A(x)\, x + B\, u \\ y &= C\, x \end{aligned} \tag{14}$$

It is supplemented with an algebraic equation for the calculation of joint torques τ from an auxiliary vector of control actions u

$$\tau = M\, u + g \tag{15}$$

A utilization of the newly introduced vector of the control actions u and appropriate model rearrangement (13) enable us to compose the mentioned linear-like state-space model (14). This model contains the variable matrix $A(x)$ that depends on the state vector $x = [q, \dot{q}]^T$. The vector x consists of the five joint angles and their derivatives considering the 5 DOF robot arm in Fig. 3 (scheme Fig. 4) as follows

$$x = [q_1, q_2, q_3, q_4, q_5, \dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4, \dot{q}_5]^T.$$

IV. Design of Model Predictive Control

Let us consider that the predictive control should generate the control actions with respect to the required motion of the robot arm. In general, the desired motion is given either by the motion trajectories represented by an ordered set of time and coordinates or only by the selected points such as starting, terminal and key transit points and possible limits for obstacles. In this paper, the usual way of the motion representation via time-parametrized coordinates is considered [8]. The second way with selected points is a specific generalization of this tasks only, see [9], [10]. The MPC design follows from the assumption that the trajectories are parametrized with the equivalent sampling Ts as used discrete state-space model:

$$\begin{aligned} \hat{x}_{k+1} &= A_k x_k + B_k u_k \\ y_k &= C\, x_k \end{aligned} \tag{16}$$

This model is obtained from the model (14) that is updated according to regular state measurement x_k: $A(x)|x=x_k$ and discretized in every new computation of the control actions u_k .

A. Criterion and Cost Function

The basis of MPC design consists in a minimization of the criterion with the appropriate cost function:

$$\min_{U_k} J_k(\hat{Y}_{k+1}, U_k, W_{k+1}) \tag{17}$$

$$\text{subject to: } \begin{aligned} \hat{x}_{k+i} &= A\, x_{k+i-1} + B\, u_{k+i-1} \\ \hat{y}_{k+i} &= C\, \hat{x}_{k+i} \quad \forall\, i = 1,\cdots,N \end{aligned}$$

where $A = A_{k+i-1} = A_k$, $B = B_{k+i-1} = B_k, \forall\, i = 1,\cdots,N$ from (16) and N is a prediction horizon. The cost function is defined as

$$\begin{aligned} J_k &= \sum_{i=1}^{N} \{\| Q_{yw}(\hat{y}_{k+i} - w_{k+i}) \|_2^2 + \| Q_u\, u_{k+i-1} \|_2^2\} \\ &= (\hat{Y}_{k+1} - W_{k+1})^T Q_{YW}^T Q_{YW} (\hat{Y}_{k+1} - W_{k+1}) + U_k^T Q_U^T Q_U U_k \end{aligned} \tag{18}$$

where U_k , $\hat{Y}_{k+1}$ and W_{k+1} stand for the sequences of control actions, output predictions and references, respectively

$$U_k = [u_k^T, \cdots, u_{k+N-1}^T]^T \tag{19}$$

$$\hat{Y}_{k+1} = [\,\hat{y}_{k+1}^T, \cdots,\, \hat{y}_{k+N}^T\,]^T \tag{20}$$

$$W_{k+1} = [w_{k+1}^T, \cdots, w_{k+N}^T]^T \tag{21}$$

and Q_{YW} and Q_U are matrices of penalizations defined as

$$Q_\Diamond^T Q_\Diamond = \begin{bmatrix} Q_*^T Q_* & & 0 \\ & \ddots & \\ 0 & & Q_*^T Q_* \end{bmatrix} \begin{array}{l} \text{subscripts } \Diamond, *: \\ \Diamond \in \{\, YW, U\,\} \\ * \in \{\, yw, u\,\} \end{array} \tag{22}$$

B. Equations of Predictions

The equations of predictions express functional estimates of the output elements in $\hat{Y}_{k+1}$ relative to the searched vector of the control actions U_k within the prediction horizon N. The equations arise from the state-space model (16) considering constant matrices A and B within one horizon N as follows

$$\begin{aligned} \hat{y}_{k+1} &= CA\;\; x_k + CB\, u_k \\ \hat{y}_{k+2} &= CA^2\;\; x_k + CAB\, u_k \quad + CB\, u_{k+1} \\ &\;\;\vdots \qquad \vdots \\ \hat{y}_{k+N} &= CA^N\; x_k + CA^{N-1}B\, u_k + \quad \cdots \quad + CB\, u_{k+N-1} \end{aligned} \tag{23}$$

The equations (23) can be rewritten in the matrix form

$$\hat{Y}_{k+1} = F_k\; x_k \;+\; G_k\; U_k \tag{24}$$

$$F_k = \left.\begin{bmatrix} CA \\ \vdots \\ CA^N \end{bmatrix}\right|_k, \quad G_k = \left.\begin{bmatrix} CB & \cdots & 0 \\ \vdots & \ddots & \vdots \\ CA^{N-1}B & \cdots & CB \end{bmatrix}\right|_k$$

C. Minimisation of the Criterion

Minimization of the criterion (17) can be provided efficiently by means of the square-root form as follows

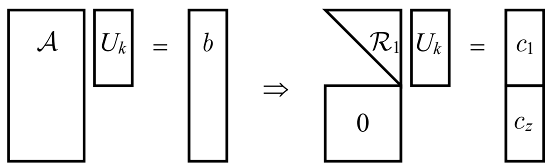
$$\min_{U_k} J_k = \min_{U_k} \mathbb{J}_k^T\, \mathbb{J}_k \;\rightarrow\; \min_{U_k} \mathbb{J}_k \tag{25}$$

$$\min_{U_k} \mathbb{J}_k = \min_{U_k} \left\{ \begin{bmatrix} Q_{YW} & 0 \\ 0 & Q_U \end{bmatrix} \begin{bmatrix} \hat{Y}_{k+1} - W_{k+1} \\ U_k \end{bmatrix} \right\} \tag{26}$$

Evaluation of (26) can be transformed into the solution of algebraic equations with respect to the unknown vector U_k

$$\begin{bmatrix} Q_{YW}\, G_k \\ Q_U \end{bmatrix} U_k = \begin{bmatrix} Q_{YW}\,(W_{k+1} - F_k\, x_k) \\ 0 \end{bmatrix} \tag{27}$$

$$\begin{aligned} \mathcal{A} \quad U_k &= \quad b \\ Q^T \quad \mathcal{A} \quad U_k &= \quad b \quad \text{for} \quad \mathcal{A} = Q\,\mathcal{R} \\ \mathcal{R}_1 \quad U_k &= \quad c_1 \end{aligned} \tag{28}$$



The indicated solution arises from the orthogonal triangular decomposition of the matrix $\mathcal{A}$, which is used in the least-squares problems [11]. Vector c_z represents square-root of the global minimum of the cost function $J_k = c_z^T\, c_z$ within the horizon N. Note that the final control actions u_k are included in the first sub-vector of the overall vector U_k, as indicated in (19). Then, the control actions u_k are recomputed into the vector of expected joint torques τ_k according to (15).

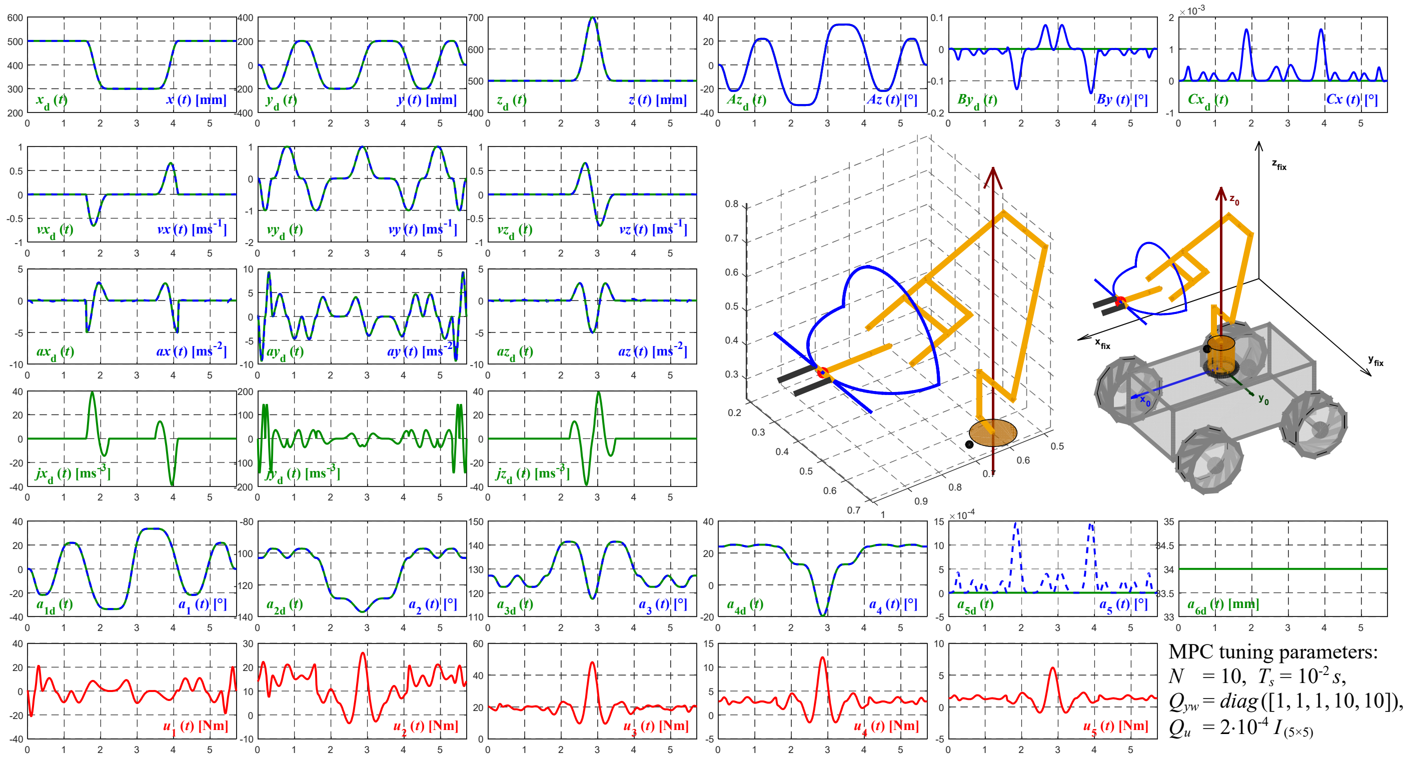


Fig. 6. Time histories of kinematic quatitites and control actions presenting in local motion control of 5 DOF articulated robot arm including used MPC parameters.

V. Demonstration of Motion Control

In this section, the proposed MPC concept for motion control of the robot arm is demonstrated. The used reference motion trajectory was composed of arc and line segments, which were time-parametrized according to [8]. Appropriate time histories of the kinematic quantities and control actions associated with the desired arm motion for the selected reference point T are shown in Fig. 6. The first four rows are for Cartesian coordinates and their derivatives, i.e. velocities, accelerations and jerks. The last but one row shows the time histories of joint coordinates. Finally, in the last row, there are time histories of control actions designed by proposed MPC and used MPC parameters. It is obvious that the actual joint coordinates (dashed blue lines) have a good coincidence with the reference coordinates (solid green lines). In case of mobile robotic systems with the robot arm intended especially for manipulation operations, such behavior is sufficient enough even for varying load. It is given by the ratio of the overall robot mass (permanent load about 12 kg) and useful load (up to 0.25 kg).

VI. Conclusion

This paper presents the analysis of the control objectives of one mobile robotic system that is equipped with the 5 DOF articulated robot arm. The predictive control design concept was introduced for the local motion control of the robot arm. The proposed solution represents a suitable energy-optimal centralized control that takes into account the robot arm motion as a one complex task. It is in contrast with the standard decentralized concept based on local PID controllers that take the mutual relations into account as external disturbances only.

Future work will be aimed at an experimental verification of the proposed solution as well as at a modeling of the mobile underframe with inclusion of the obtained model into the global control objective. Thus, we will focus on the problem of positioning of the whole mobile robotic system, i.e. underframe and articulated robot arm, in the global coordinate system.

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