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Robust Point-to-Point Control with Velocity Limitation

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***Abstract*—In this contribution, the original modification of a special kind of robust point-to-point (fixed target) motion control with the intentional limitation of maximal velocity is presented. Theoretical background of the provided control algorithm is given by the variable structure control theory yielding the fast and accurate time sub-optimal control. This ensures the desired quality and integrity of the motion control despite the presence of significant parametric and signal disturbances. Simulation results show the reliability of the presented control method.**

***Keywords*—fixed target position control; velocity limitation; robust variable structure control (VSC); time sub-optimal control**

I. INTRODUCTION

Time sub-optimal control in motion systems [1], [2] represents a non-deterministic and technically feasible equivalent of the time optimal control – the fastest possible bang-bang control with driving forces/torques boundary and with both the maximal allowed acceleration and the maximal braking in a motion process. The specific feature of time sub-optimal control – the sliding mode [3] – forces any system's trajectory to follow the switching function, guaranteeing the insensitivity of the control process to internal and external disturbances (parameter variation, interaction among the DOFs, Coriolis and centrifugal forces, gravitational forces, etc.) [4]. The input of control algorithm is given by the linear combination of easily accessible position and velocity errors; therefore the implementation of time sub-optimal control in real motion systems is very simple. In fixed-target position control, the maximal velocity of the motion is proportional both to the driving force/torque value and to the system gain; nevertheless, the necessary intentional limitation of the maximal velocity by the force or gain reduction that yields the decrease of a motion dynamics, is inacceptable. Via the sliding mode application, next chapters offer an elegant answer to this problem in variable structure motion control. Furthermore, to avoid chattering [5] – the consequence of discontinuous sliding mode utilization in real control systems – we provide a smooth modification of the fixed-target control algorithm – the equivalent time sub-optimal control [1], [6] with velocity limitation. Numeric simulation outputs of both the discontinuous control and the continuous one are compared and discussed at the end of this contribution.

II. TIME SUB-OPTIMAL CONTROL OF THE MOTION SYSTEM

Let the phase model of one DOF's motion system in the control error phase plain $(e,\dot{e})$ be given by the system of differential equations

$$\begin{aligned}\frac{de}{dt} &= \dot{e}\\ \frac{d\dot{e}}{dt} &= -\frac{1}{T}\left(Ku+\dot{e}\right)\end{aligned} \tag{1}$$

where T represents the system's time constant and K is the system's gain, u refers to the system's input – driving force/torque (actuating variable), e and $\dot{e}$ stand for the position and velocity errors in that order.

Two main features of time sub-optimal (TSO) motion control [7] are a) the linear switching function (switching line)

$$F(\mathbf{e}) = \dot{e} + \alpha e = 0 \tag{2}$$

where $\mathbf{e} = (e,\dot{e})^T$ stands for the control error vector and α denotes the switching line slope; and b) the relay form of actuating variable u generator with a driving level boundary M > 0 (In (3) sgn(.) stands for sign function)

$$u = M\,\mathrm{sgn}(F) \tag{3}$$

The only parameter of the control algorithm to be designed is the slope α of a switching line (2) that should cross two points in the control error phase plane $(e,\dot{e})$ [7]: the origin (0,0) of the phase plane, i.e. the set-point, and the point (in the quadrant of the phase plane with $e>0$ and $\dot{e}<0$) given by the intersection of the speed limitation line and the dynamically worst braking (decelerating, i.e. with $u=-M$) trajectory of system (1)

$$e = -T\dot{e} + TKu\ln\left(1+\frac{\dot{e}}{Ku}\right) \tag{4}$$

Denote $v_{\lim} < KM$ the value of the velocity limitation and $T_{\max}$ the maximal time constant of the motion system. The speed limitation line in the control error phase plane is given by

$$\dot{e} = -v_{\lim} \tag{5}$$

After some algebra, using (4) and (5), $T = T_{\max}$ and $u = -M$, the coordinates of the upper mentioned intersection point can be expressed by

$$(e, \dot{e}) = \left(T_{\max} v_{\lim} - T_{\max} KM \ln\left(1 + \frac{v_{\lim}}{KM}\right), v_{\lim} \right) \tag{6}$$

and consequently, parameter α is specified by

$$\alpha = \frac{1}{T_{\max}\left(1 - \frac{KM}{v_{\lim}} \ln\left(1 + \frac{v_{\lim}}{KM}\right)\right)} \tag{7}$$

In presence of an external disturbance d acting against the driving torque/force (actuating variable u), the value of the boundary M in (7) should be replaced by the value

$$M_d = M - D_{\max} \tag{8}$$

where $D_{\max}$ stands for the maximal value of the external disturbance d.

To include the velocity limitation $v_{\lim}$ in a control algorithm, an additional switching line given by

$$F_1(\mathbf{e}) = \dot{e} + v_{\lim} = 0 \tag{9}$$

should be considered. In case of the reverse direction of motion, plus sign in (9) has to be replaced by the minus sign.

Combination of switching lines (2) and (9) with the switching algorithm (3) yields the common robust controller structure shown in Fig. 1. The advantage of this original structure is that it represents the simultaneous implementation of both the switching line (2) and the velocity limitation switching line (9) despite the absence of any switching logic (if / else structure).

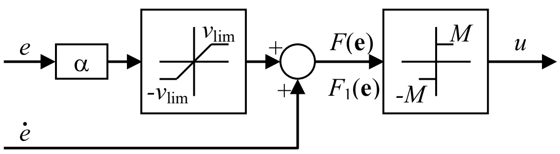

Fig. 1. Time sub-optimal controller structure with velocity limitation

III. Equivalent Time Sub-Optimal Control with Velocity Limitation

Robust time sub-optimal controller (2), (3), (7) and (9) belongs to the category of discontinuous or relay control algorithms. Such a type of controller in real motion systems with parasitic dynamics and/or parasitic non-linearity would suffer from chattering – an undesirable low-frequency oscillation. To avoid chattering, many techniques are utilized in engineering practice [3] – continuous approximation of the discontinuous element, high-frequency dither injection, superposition of the continuous and discontinuous signals, high-frequency bypass via the state observer or the reaching law application [6], [7].

The latter yields the smooth, continuous control preserving the sliding mode benefits (robustness, dynamics and accuracy) therefore we use this kind of modification to show the principle of the velocity limitation.

Applying the simplified version of the reaching law [6]

$$\frac{dF(\mathbf{e})}{dt} = -kF(\mathbf{e}) \tag{10}$$

to the switching function (2) and substituting (1) yields the desired continuous equivalent time sub-optimal (ETSO) control algorithm taking the boundary M into account [7]

$$u_{ETSO} = \begin{cases} u_{rl} & \text{for } |u_{rl}| < M \\ M \operatorname{sgn}(u_{rl}) & \text{for } |u_{rl}| \geq M \end{cases} \tag{11}$$

where u_{rl} is given by

$$u_{rl} = \frac{1}{K}\left[(kT_{\max} - 1)\dot{e} + \alpha T_{\max}(\dot{e} + ke)\right] \tag{12}$$

Similarly, applying reaching law (10) to switching line (9), we get

$$u_{rl1} = \frac{1}{K}\left[(kT_{\max} - 1)\dot{e} + kT_{\max} v_{\lim}\right] \tag{13}$$

To get the motion control accuracy in ETSO control algorithm comparable with the one in TSO control, the value of parameter k should meet the condition

$$k >> 1 \tag{14}$$

Let us adopt the notation

$$LIM = kT_{\max} v_{\lim} \tag{15}$$

Combination of expressions from (11) to (15) yields the structure of equivalent time sub-optimal control with velocity limitation depicted in Fig. 2. Again, this structure represents the implementation of both the control algorithms (12) and (13) despite the absence of any switching logic.

IV. Simulation Results

Numeric simulation of rotating motion system with both the time sub-optimal (TSO) and the equivalent time sub-optimal (ETSO) fixed target control has been performed for parameters given in Table I. To proof the robustness of the motion control, the parametric disturbance $T \in \langle T_{\min}, T_{\max} \rangle$ and the harmonic external disturbance $d = D_{\max} \sin(\omega_d t)$ have been taken into consideration. The parametric disturbance represents a 425% change of the system's time constant and the maximal value of the external disturbance represents 50% of the driving torque boundary, thus, the motion system suffers from severe turbulences.

To avoid the numeric integration stuck in sliding mode, the fixed step of numeric integration 0.1ms has been chosen. This value is satisfactory for the given dynamics of the motion system and for the desired accuracy of simulation.

TABLE I. System parameters

Parameter	Value
System's gain K	0.0883 $\text{radN}^{-1}\text{m}^{-1}$
System's minimal time constant $T_{\min}$	16 ms
System's maximal time constant $T_{\max}$	68 ms
System's maximal velocity (KM)	4.415 rads^{-1}
Velocity boundary $v_{\lim}$	3 rads^{-1}
Driving torque boundary M	50 Nm
Disturbance amplitude $D_{\max}$ (50% of M)	25 Nm
Disturbance frequency ω_d	20 rads^{-1}
Control algorithm parameter α (7)	39.91
Control algorithm parameter k (14)	5000
Fixed target set-point x_d (angular position)	1 rad

The goal of the presented robust control is to reach the set-point in a shortest time without an overshoot, considering the driving torque boundary M and preserving the specified velocity limitation, despite the influence of parametric and external disturbances. As can be seen in Fig. 3 showing the system's phase portrait, despite the fact that the transient process suffers from the signal disturbance (cf. time interval 0.2s – 0.3s in Fig. 5), system's trajectory is forced to follow switching lines (2) and (9), and reaches the set-point (0,0) keeping the desired quality. We can see the interval of the maximal acceleration, the interval of the maximal velocity (velocity boundary $v_{\lim}$) and the final interval of the maximal braking. It is inevitable that this control algorithm is the one with the shortest settling time. Corresponding graphs of angular position are in Fig. 4.

Velocity diagrams in Fig. 5 show the influence of the external disturbance in that part of the graph, where the desired value of the driving torque (Fig. 6) reaches the given boundary M (time interval 0.2s – 0.3s). However, the most important phase of the control process guaranteeing the overshoot free response – the braking phase – is robust against both the parametric and the signal disturbances. Furthermore, within the whole control process, the velocity doesn't exceed the specified velocity boundary. In Fig. 6 we can see the main difference between the TSO and ETSO control. Only time sub-optimal control shows the intervals of sliding mode – the high frequency oscillation of actuating variable u. In equivalent time sub-optimal control we can see the smooth performance of driving torque representing the mean value of the oscillation in sliding mode. To keep the desired quality of the fixed target control, the actuating variable in ETSO shows the low frequency oscillation – the response to the signal disturbance oscillation.

To illustrate the influence of parameter k on the accuracy of the control process, Fig. 7 displays the system's phase portrait for k value ten times lower than the one in previous simulation, i.e. $k = 500$ versus $k = 5000$ (Table I). As can be seen, particularly the interval of the velocity limitation is sensitive to parameter k value. It is evident that this control parameter should meet the condition (14) with a significant reserve. Corresponding graphs of angular velocity and position are in Fig. 8 and Fig. 9 in that order.

V. Conclusions

The aim of this contribution has been to provide a method of the velocity limitation in variable structure control using either the sliding mode or the reaching law concept. The latter, guaranteeing the chattering elimination, seems to be more acceptable in real motion control systems. Two original structures with in-build velocity boundary have been presented. The viability of the proposed control algorithm has been illustrated by means of the numeric simulation for one DOF's motion system with parametric uncertainty and external disturbance influence.

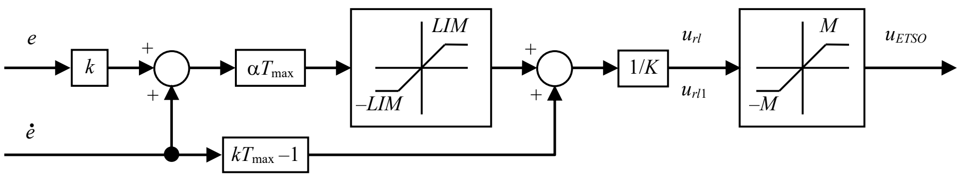

Fig. 2. Equivalent time sub-optimal controller structure with velocity limitation

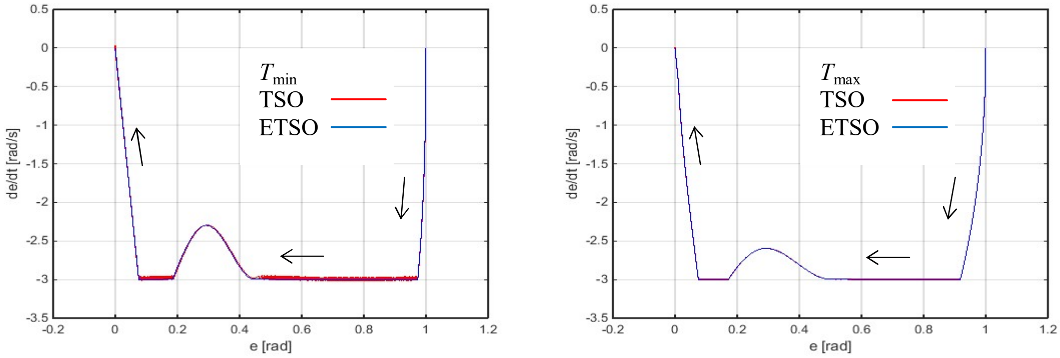

Fig. 3. System's phase portrait in TSO and ETSO motion control

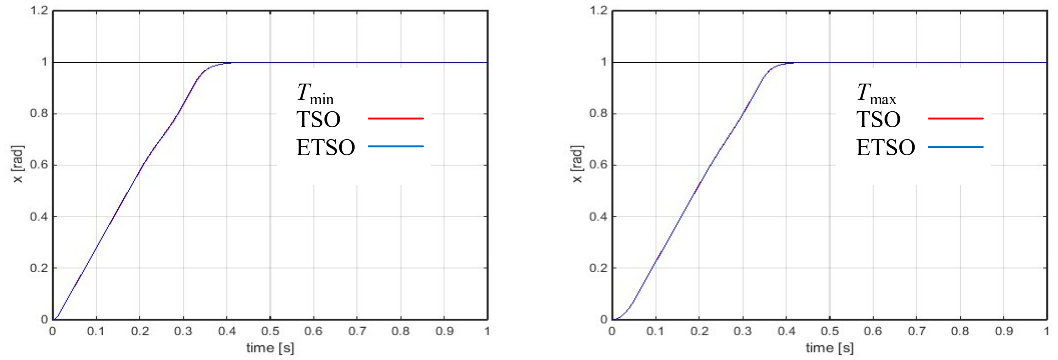

Fig. 4. Position versus time diagram in TSO and ETSO motion control

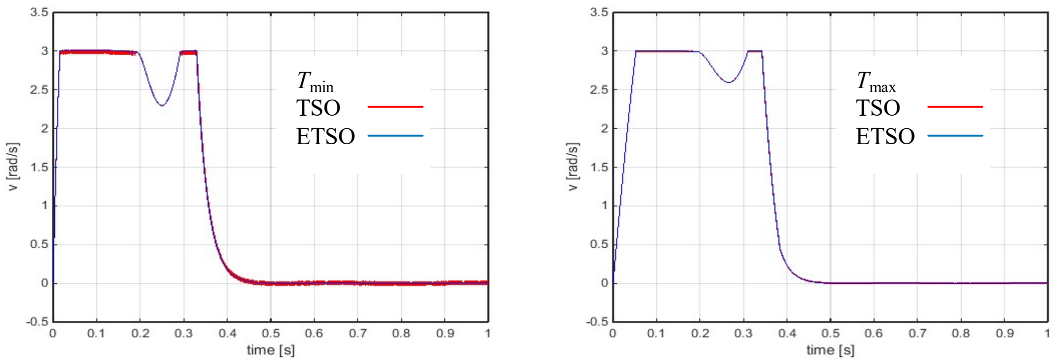

Fig. 5. Velocity limitation in TSO and ETSO motion control

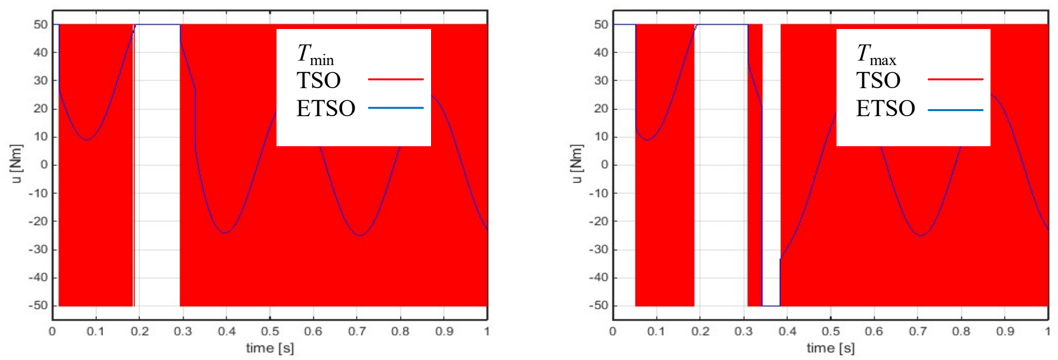

Fig. 6. Driving torque graph in TSO and ETSO motion control

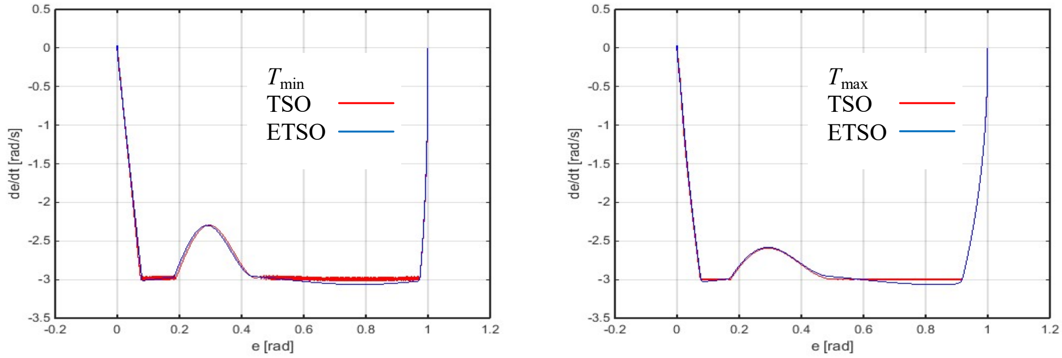

Fig. 7. System's phase portrait in TSO and ETSO motion control (the case of lower accuracy)

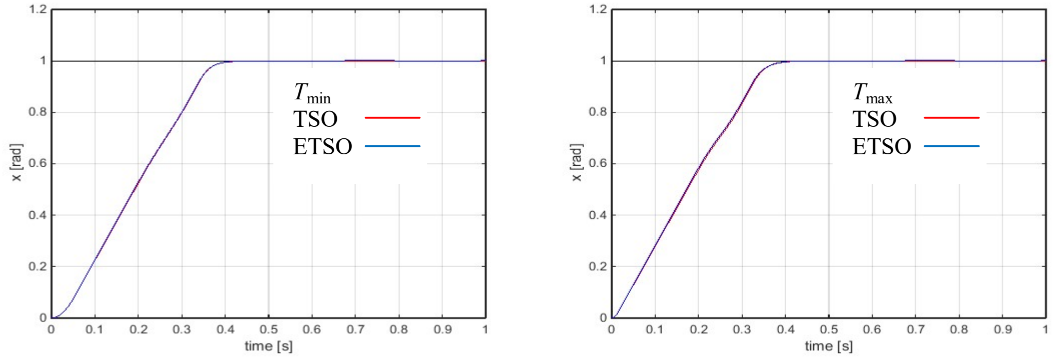

Fig. 8. Position versus time diagram in TSO and ETSO motion control (the case of lower accuracy)

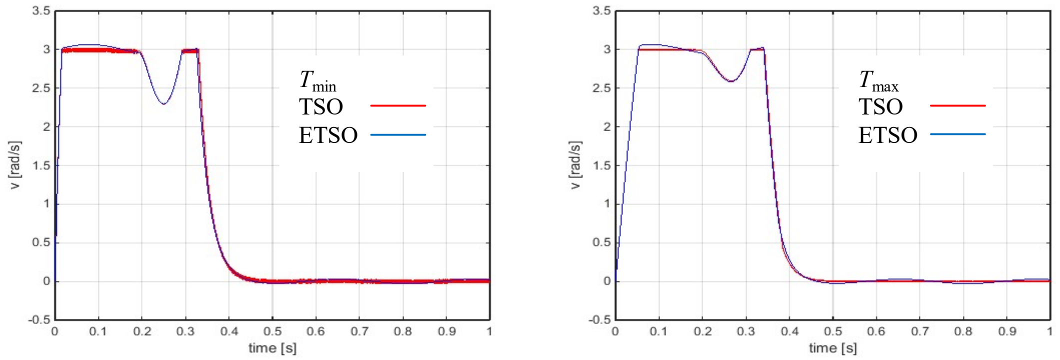

Fig. 9. Velocity limitation in TSO and ETSO motion control (the case of lower accuracy)

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