# Design of Robust MPC with Integral Action for a Laboratory Continuous Stirred-Tank Reactor

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Abstract—The paper presents a design of the robust model predictive control (RMPC) for a laboratory continuous stirredtank reactor (CSTR). A neutralization reaction ran in the CSTR, and the reactants were acetic acid and sodium hydroxide. The controlled variable was pH of the reaction mixture. The control input was the volumetric flow-rate of the base. The system was modeled using experimental data of several step-responses. Measurement noise was reduced using the Hebky filter. The robust model-based control strategy was implemented to assure good control performance. The offset-free reference tracking was ensured by the implementation of RMPC with integral action.

## I. INTRODUCTION

Industry highly appreciates the implementation of the model predictive control (MPC). MPC represents the state-ofart optimal strategy for complex multiple-inputs and multipleoutputs (MIMO) systems. The main benefit of MPC lies in the possibility to design optimal control action in the presence of various real-world constraints, see e.g. [1]. The closed-loop control performance depends on the accuracy and the precision of the plant model, i.e., minimization of the well-known process-model mismatch. From the robust control viewpoint, the main disadvantage of MPC is that there are not explicitly incorporated system uncertainties into the predictions of the future system behavior. The robust model predictive control (RMPC) overcomes this drawback, as it evaluates an optimal control action subject to the bounded uncertain parameters, see e.g. [2]. RMPC attracted the high interest of researchers in past two decades when the possibility to design RMPC via the convex optimization was proposed in [3]. The non-convex constraints were formulated in the form of the semidefinite programming (SDP, [4]) using the linear matrix inequalities (LMI, [5]).

Continuous stirred-tank reactors (CSTRs) represent key devices in chemical, petrochemical, and pharmaceutical industries. CSTRs are complex processes due to their non-linear behavior, multiple steady-states, heat effect of the chemical reactions, time delay effect, and effect of various time-varying uncertainties [6, chap. 1]. In [7] LMI-based RMPC was designed for the uncertain system with time-varying linear fractional perturbations. The designed RMPC was validated using the simulation of the closed-loop control of CSTR with first-order, irreversible, exothermic kinetics. The MIN-MAX-based RMPC design strategy presented in [7] was improved in [8]. Based on the work [9], conservativeness of RMPC design was reduced by introducing parameter-dependent Lyapunov functions. On the other hand, the overall computational burden increased. The approach was validated by simulation

case study of CSTR stabilization for exothermic, irreversible chemical reaction. LMI-based explicit constrained RMPC was developed in [10]. The RMPC was designed using a concept of an asymptotically stable invariant ellipsoid. The closedloop control performance was investigated also considering a non-isothermal CSTR. The same benchmark of CSTR was considered in [11]. The conservativeness of ellipsoidal robust positively invariant sets was reduced using more tight polytopic approximations. These approximations led to a larger region of feasible conditions.

In [12] was designed RMPC for non-linear CSTR, in which ran an exothermic, irreversible reaction. The operating point was locally linearised for each steady-state, and the local control strategy was designed. Simulation of LMIbased RMPC for the same CSTR benchmark was investigated in [13]. Constrained RMPC was designed based on quasilinear parameter varying systems with bounded disturbance. The iterative algorithm was implemented for the control law, that was parametrized via parameter-dependent dynamic output feedback. Partial enumeration approach for fast computation of a suboptimal solution to RMPC design problems was introduced in [14]. The properties for the proposed RMPC were investigated using simulation case study of CSTR, in which irreversible exothermic reaction took place. In [15] the general balance-based adaptive control methodology to the control of the nonlinear neutralization process was designed to improve the robustness of the control performance. An adaptive nonlinear control strategy for a bench-scale pH neutralization system was experimentally investigated in [16], and the control performance was significantly improved compared to compared to a non-adaptive nonlinear controller and conventional PI controller. The real-time implementation of a set of multilinear model-based control design methodologies was analyzed using a bench-scale pH neutralization system in [17].

There is a lack of real-world implementations of LMIbased RMPC for CSTRs reported in the literature. This paper presents LMI-based RMPC design for the laboratory CSTR of Armfield PCT40 [18]. It was not straightforward to implement the offset-free LMI-based RMPC strategy, see e.g. [19], where RMPC design for a laboratory heat exchanger was investigated. In this paper, we designed RMPC with integral action to overcome this obstacle. Moreover, the influence of noise was reduced by the Hebky filter. RMPC was implemented using our freely available MUP toolbox [20]. The plant model was identified using experimental data, and the real-time closedloop control performance of the designed RMPC was judged using a quadratic criterion.

## II. NOTATION

The following notation has been used in the paper:

- $\mathbb{R}^n$  denotes the *n*-dimensional space of real-valued 1) vectors,  $\mathbb{R}^{n \times m}$  represents the  $(n \times m)$ -dimensional space of real-valued matrices. For a real-valued matrix  $A, A^{\top}$  denotes its transpo-
- 2) sition and  $A^{-1}$  denotes its inverse, if exists.
- I denotes the identity matrix, and 0 denotes zero 3) matrix of appropriate dimensions.
- 4) Symbol  $\star$  used as an element of a matrix, denotes symmetric structure of the matrix.
- 5) For a real-valued vector  $\boldsymbol{x}$  and positively defined matrix A,  $|| \mathbf{x} ||_A^2 = \mathbf{x}^\top A \mathbf{x}$ .
- Function convhull :  $\mathbb{R}^n \mapsto \mathbb{R}^n$  denotes convex hull 6) that maps original set to the smallest-volume convex set that includes original set, i.e., for the set  $\mathcal{T}$  holds  $\operatorname{convhull}(\mathcal{T}) \supseteq \mathcal{T}.$
- Function diag :  $\mathbb{R}^n \to \mathbb{R}^{n \times n}$  maps n-dimensional 7) vector to the square symmetric matrix with the vector elements placed on the principal diagonal and zeros elsewhere.
- Function  $V : \mathbb{R}^n \to \mathbb{R}^1$  denotes the Lyapunov 8) function in the discrete time domain, i.e., following statements simultaneously hold:
  - $\begin{aligned} &\forall \pmb{x} \neq 0 \Rightarrow V(\pmb{x}) > 0, \quad \pmb{x} = 0 \Rightarrow V(\pmb{x}) = 0, \\ &V(\pmb{x}(k+t_{\rm s})) V(\pmb{x}(k)) \leq 0, \quad \forall k \geq 0. \end{aligned}$

The controlled system was the laboratory continuous stirred-tank reactor (CSTR) of Armfield PCT40.

## A. Specifications of CSTR

The process control teaching station Armfield PCT40 has of several parts, see Fig. 1. The controlled system was the laboratory CSTR. The reaction vessel (Fig. 1, (I)) had the volume  $V = 1.5 \,\mathrm{dm^3}$ . In the reaction vessel ran a chemical reaction, and reactants were acetic acid (CH<sub>3</sub>COOH) and sodium hydroxide (NaOH). The neutralization reaction is described in (1)

$$NaOH (aq) + CH_3COOH (aq) \rightarrow CH_3COONa (aq) + H_2O (l),$$
(1)

where the products of neutralization are sodium acetate  $(CH_3COONa)$  and water  $(H_2O)$ . The controlled output was pH in the reaction vessel defined as

$$pH = -\log[H^+], \tag{2}$$

where  $[H^+]$  is the concentration of hydrogen cations. pH in the reactor was measured using a pH sensor (Fig. 1, (II)).

Two tanks with volumes  $V_{\rm A}, V_{\rm B} = 100 \, {\rm dm}^3$  were used to store acid and base. Peristaltic pump A (Fig. 1, (III)) and peristaltic pump B (Fig. 1, (IV)) dosed acid and base into the reactor, respectively. The input concentrations were  $c_{\rm A} = 0.01 \, {\rm mol/m^3}$  (acid) and  $c_{\rm B} = 0.01 \, {\rm mol/m^3}$  (base). Input voltage of peristaltic pumps  $U_{\rm A}$ ,  $U_{\rm B}$  was [0,5] V, and the corresponding volumetric flow rates were  $q_A$ ,  $q_B$  within [0, 10] mL/s.



Fig. 1. Controlled reactor of Armfield PCT 40: (I) CSTR, (II) pH sensor, (III) pump A, (IV) pump B.

The input voltage of pump A was set to the constant value  $U_{\rm A} = 2.5 \, \text{V}$ , i.e.,  $q_{\rm A} \approx 5 \, \text{mL/s}$ . The control input was the volumetric flow rate  $q_{\rm B}$ . The aim of control was to implement the RMPC to ensure set-point tracking in the presence of the interval uncertainties and the measurement noise.

#### B. Experimental model of CSTR

The laboratory CSTR of Armfield PCT40 represents a complex system, as its behavior is non-linear and asymmetric. For RMPC design, we considered the controlled uncertain system in the discrete-time domain. The family of uncertain systems A was expressed as the convex hull of the set of linear state-space systems with polytopic uncertainty given by

$$\boldsymbol{x}(k+1) = \boldsymbol{A}\,\boldsymbol{x}(k) + \boldsymbol{B}\,\boldsymbol{u}(k), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0, \qquad (3a)$$

$$\mathbf{y}(k) = \mathbf{C}\mathbf{x}(k),\tag{3b}$$

$$\left|\boldsymbol{A}^{(v)}, \boldsymbol{B}^{(v)}\right| \in \mathbb{A},\tag{3c}$$

$$\mathbb{A} = \operatorname{convhull}\left(\left\{\left[\boldsymbol{A}^{(v)}, \boldsymbol{B}^{(v)}\right], \forall v\right\}\right), \quad (3d)$$

where  $k \ge 0$  denotes element of the discrete-time domain,  $\boldsymbol{x}(k) \in \mathbb{R}^{n_{\mathrm{x}}}$  is the real-valued vector of system states,  $\boldsymbol{u}(k) \in$  $\mathbb{R}^{n_{\mathrm{u}}}$  are control inputs,  $\mathbf{y}(k) \in \mathbb{R}^{n_{\mathrm{y}}}$  are system outputs,  $\mathbf{x}_0$  is the measured or estimated vector of system initial conditions,  $A \in \mathbb{R}^{n_{\mathrm{x}} \times n_{\mathrm{x}}}$  denotes a system-state matrix,  $B \in \mathbb{R}^{n_{\mathrm{x}} \times n_{\mathrm{u}}}$  is a matrix of system inputs,  $m{C} \in \mathbb{R}^{n_{\mathrm{y}} imes n_{\mathrm{x}}}$  is a matrix of system outputs. Parameter  $n_v$  represents the total number of uncertain system vertices. The matrix superscript (v) denotes the v-th vertex system of A.

For RMPC design, it was necessary to consider the normalized state-space system, i.e., the steady-state values were shifted into the origin. Then, the manipulated variable was defined as  $\boldsymbol{u}(k) = [q_{\rm B}^{\rm s} - q_{\rm B}(k)]$ , system state  $\boldsymbol{x}(k) = [{\rm pH}^{\rm s}$ pH(k)], and system output y(k) = x(k). The superscript s denotes the steady-state value. The steady-state values corresponded to the operation point:  $q_B^s = 5 \text{ mL/s}$  and  $pH^s = 7$ .

Model of the laboratory CSTR in the form of the statespace system in (3) was identified using experimental data of

TABLE I. OUTPUT STEADY STATE VALUES ASSOCIATED TO THE INPUT STEP CHANGES.

$q_{\rm B}$ [mL/s]	pH
$4.0 \rightarrow 4.5$	$6.1 \rightarrow 6.6$
$4.5 \rightarrow 5.0$	$6.6 \rightarrow 7.0$
$5.0 \rightarrow 5.5$	$7.0 \rightarrow 7.6$
$5.5 \rightarrow 6.0$	$7.6 \rightarrow 8.6$
$6.0 \rightarrow 4.5$	$8.6 \rightarrow 7.8$
$5.5 \rightarrow 5.0$	$7.8 \rightarrow 7.0$
$5.0 \rightarrow 4.5$	$7.0 \rightarrow 6.6$
$4.5 \rightarrow 4.0$	$6.6 \rightarrow 6.1$

several step-responses. As the process behavior is significantly non-linear and asymmetric, we performed the set of upwards and downwards step changes to investigate the system performance in the neighborhood of operating conditions. The range of the control input  $q_{\rm B}$  was within [4.00, 6.00] mL/s. The realized step changes are summarized in Tab. I. The normalized process step responses are depicted in Fig. 2, where  $\Delta pH$  denotes the normalized value of pH, i.e., measured data of pH shifted into the origin, and divided by the size of the input step change  $\Delta q_{\rm B} = 0.5$  mL/s.

The normalized step responses were individually identified in the form of the first-order transfer function

$$G_{\rm s}(s) = \frac{\Delta \rm pH}{\Delta q_{\rm B}} = \frac{Z}{T\,s+1}e^{-Ds},\tag{4}$$

where  $G_{\rm s}$  is the transfer function, Z is the gain, T is the time constant, and D is the time delay. We have identified the system parameters in the form of interval uncertainties. The computed limit and mean values are summarized in Tab. II, where the mean values represent the nominal system, i.e., an idealized system without uncertainties. The values of time delay D were neglected, as they were sufficiently small compared to the time constant T.

Each system in (4) was transformed into the state-space system in the discrete-time domain (3) considering the sampling time  $t_s = 10 \text{ s}$ . The parameters obtained for the system in (3) are summarized in Tab. III.



Fig. 2. Normalized step responses.

# IV. RMPC DESIGN WITH INTEGRAL ACTION

The model of CSTR was considered in the form (3). The aim was to design a state-feedback control law:

$$\boldsymbol{u}(k) = \boldsymbol{F}(k) \, \boldsymbol{x}(k) \,, \tag{5}$$

where  $F \in \mathbb{R}^{n_u \times n_x}$  is the gain matrix of the feedback controller. Moreover, the control law had to satisfy the constraints on control inputs and system outputs

$$\boldsymbol{u}_{\min} \preceq \boldsymbol{u}(k) \preceq \boldsymbol{u}_{\max}, \quad \boldsymbol{y}_{\min} \preceq \boldsymbol{y}(k) \preceq \boldsymbol{y}_{\max}, \quad \forall k \ge 0, \quad (6)$$

where  $u_{\min}, u_{\max} \in \mathbb{R}^{n_u}, y_{\min}, y_{\max} \in \mathbb{R}^{n_y}$  represent the boundary values. In this work we considered just symmetric constraints

$$-\boldsymbol{u}_{\text{sat}} \preceq \boldsymbol{u}(k) \preceq \boldsymbol{u}_{\text{sat}}, \quad -\boldsymbol{y}_{\text{sat}} \preceq \boldsymbol{y}(k) \preceq \boldsymbol{y}_{\text{sat}}, \quad \forall k \ge 0,$$
(7)

where  $u_{\text{sat}} \in \mathbb{R}^{n_{\text{u}}}$  and  $y_{\text{sat}} \in \mathbb{R}^{n_{\text{y}}}$  stand for the symmetric boundary values.

The quadratic quality criterion was used

$$J_{0 \to n_{k}} = \sum_{k=0}^{n_{k}} J(k) = \sum_{k=0}^{n_{k}} \left( \| \boldsymbol{x}(k) \|_{\boldsymbol{\varrho}}^{2} + \| \boldsymbol{u}(k) \|_{\boldsymbol{R}}^{2} \right), \qquad (8)$$

where  $Q \in \mathbb{R}^{n_x \times n_x} \succeq 0$ ,  $R \in \mathbb{R}^{n_u \times n_u} \succ 0$  are the weight matrices, and  $n_k$  is the total number of control steps. RMPC was designed in receding horizon control fashion, i.e., the control control law (10) was updated in each control step. The aim of RMPC design is to minimize the total value of the quality criterion in (8). LMI-based RMPC strategy introduced in [3] as well as our strategy considered the infinity prediction horizon, to predict the future behaviour of controlled system, i.e.,  $n_k \rightarrow \infty$ .

We designed RMPC with integral action to assure the offset-free reference tracking by following the analogous procedure of LQ optimal controller design with integral action, e.g., see [22]. The extended vector of system states  $z \in \mathbb{R}^{(2n_x)}$  had the form:

$$\mathbf{z}(k) = \begin{bmatrix} \mathbf{x}(k) \\ \sum_{i=0}^{k} \mathbf{x}(i) \end{bmatrix}.$$
(9)

Then, the state-feedback control law in (5) was extended subject to the integral action into the form:

$$\boldsymbol{u}(k) = \begin{bmatrix} \boldsymbol{F}_{\mathrm{P}}(k) & \boldsymbol{F}_{\mathrm{I}}(k) \end{bmatrix} \boldsymbol{z}(k) = \boldsymbol{F}(k) \ \boldsymbol{z}(k) , \qquad (10)$$

where  $F_{\rm P}$ ,  $F_{\rm I}$  are the proportional and integral parts of the feedback control law. The proportional and integral parts were considered in a compact form of  $\tilde{F}$ .

TABLE II.	IDENTIFIED PARAMETERS OF THE SYSTEM TRANS	SFER
FUNCTION.		

value	Z [s/mL]	T [s]	<i>D</i> [s]
minimal	0.7	94.6	3.2
nominal	1.1	134.3	4.6
maximal	1.5	174.0	5.4

TABLE III. IDENTIFIED PARAMETERS OF THE STATE-SPACE SYSTEM.

value	A	В	C
minimal	0.8997	0.1574	1
nominal	0.9282	0.2168	1
maximal	0 9441	0.2763	1

The system in (3) extended subject to integral action in (9) was given by:

$$\mathbf{z}(k+1) = \tilde{\mathbf{A}} \mathbf{z}(k) + \tilde{\mathbf{B}} \mathbf{u}(k), \quad \mathbf{z}(0) = \mathbf{z}_0, \quad (11a)$$
$$\mathbf{y}(k) = \tilde{\mathbf{C}} \mathbf{z}(k) \quad (11b)$$

$$\begin{bmatrix} \tilde{\boldsymbol{A}}^{(v)}, \tilde{\boldsymbol{B}}^{(v)} \end{bmatrix} \in \tilde{\mathbb{A}},$$
(11c)

$$\tilde{\boldsymbol{\lambda}} = \operatorname{convhull}\left(\int \left[\tilde{\boldsymbol{\lambda}}^{(v)} \ \tilde{\boldsymbol{B}}^{(v)}\right] \ \forall v \right)$$
(11d)

$$\tilde{\mathbb{A}} = \text{convhull}\left(\left\{\left[\tilde{\boldsymbol{A}}^{(t)}, \tilde{\boldsymbol{B}}^{(t)}\right], \forall v\right\}\right), \quad (11d)$$

where:

$$\tilde{\boldsymbol{A}}^{(v)} = \begin{bmatrix} \boldsymbol{A}^{(v)} & \boldsymbol{\theta} \\ -t_{s} \boldsymbol{C} & \boldsymbol{I} \end{bmatrix}, \quad \tilde{\boldsymbol{B}}^{(v)} = \begin{bmatrix} \boldsymbol{B}^{(v)} \\ \boldsymbol{\theta} \end{bmatrix}, \quad \tilde{\boldsymbol{C}} = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{\theta} \end{bmatrix}. \quad (12)$$

For the purposes of RMPC design with integral action, the quadratic criterion (8) was modified subject to the extended system in (11):

$$\tilde{J}_{0 \to n_{k}} = \sum_{k=0}^{n_{k}} \tilde{J}(k) = \left( \|\boldsymbol{z}(k)\|_{\tilde{\boldsymbol{\varrho}}}^{2} + \|\boldsymbol{u}(k)\|_{\boldsymbol{R}}^{2} \right), \quad (13)$$

where  $\tilde{\boldsymbol{Q}} \in \mathbb{R}^{2n_x \times 2n_x} \succeq 0$  is the weighting matrix subject to the extended vector of states  $\boldsymbol{z}$ , i.e.,  $\tilde{\boldsymbol{Q}} = \begin{bmatrix} \boldsymbol{Q} & \boldsymbol{\theta} \\ \boldsymbol{\theta} & \boldsymbol{Q}_{\mathrm{I}} \end{bmatrix}$ .

Based on [3], RMPC with integral action was designed subject to the extended system in (11) using the solution of the following SDP:

$$\min_{\gamma, \boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{U}} \gamma \tag{14a}$$

s.t.: 
$$\begin{bmatrix} 1 & \star \\ z(k) & X \end{bmatrix} \succeq 0,$$
 (14b)

$$\begin{vmatrix} \mathbf{A} & \star & \star & \star \\ \tilde{\mathbf{A}}^{(v)}\mathbf{X} + \tilde{\mathbf{B}}^{(v)}\mathbf{Y} & \mathbf{X} & \star & \star \\ \tilde{\mathbf{Q}}^{1/2}\mathbf{X} & \mathbf{0} & \gamma \mathbf{I} & \star \\ \mathbf{R}^{1/2}\mathbf{Y} & \mathbf{0} & \mathbf{0} & \gamma \mathbf{I} \end{vmatrix} \succeq 0, \quad (14c)$$

$$\begin{bmatrix} \boldsymbol{X} & \star \\ \boldsymbol{Y} & \boldsymbol{U} \end{bmatrix} \succeq 0, \quad \boldsymbol{U}_{i,i} \leq \boldsymbol{u}_{\mathrm{sat},i}, \forall i \in \mathbb{N}_1^{n_{\mathrm{u}}}, \text{ (14d)}$$
$$\begin{bmatrix} \boldsymbol{X} & \star \\ \tilde{\boldsymbol{C}} \left( \tilde{\boldsymbol{A}}^{(v)} \boldsymbol{X} + \tilde{\boldsymbol{B}}^{(v)} \boldsymbol{Y} \right) \quad \boldsymbol{y}_{\mathrm{sat}}^2 \boldsymbol{I} \end{bmatrix} \succeq 0, \quad (14e)$$

where  $v = 1, ..., n_v$ , and the decision variables are:  $\boldsymbol{X} = \boldsymbol{X}^\top \in \mathbb{R}^{n_x \times n_x}$  the weighted inverse-matrix of the quadratic Lyapunov function,  $\boldsymbol{Y} \in \mathbb{R}^{n_u \times n_x}$ ,  $\boldsymbol{U} \in \mathbb{R}^{n_u \times n_u}$ , and  $\gamma \in \mathbb{R}$ the weighting parameter of  $\boldsymbol{X}$ . To match the theory of [3], we considered the quadratic Lyapunov function  $V(\boldsymbol{x}(k)) = \gamma \boldsymbol{x}(k)^\top \boldsymbol{X}^{-1} \boldsymbol{x}(k)$ . We used simplified notation in SDP (14), but all the decision variables are discrete-time-dependent, i.e., they are  $\boldsymbol{X}(k), \boldsymbol{Y}(k), \boldsymbol{U}(k)$ , and  $\gamma(k)$ .

In SDP (14), LMI (14b) represents the robust positively invariant ellipsoid that ensures the recursive feasibility of the optimization problem, and the objective function in (14a) maximizes its volume, see [23]. LMIs (14b), (14c) assure the asymptotic stability in the sense of Lyapunov, with the convergence rate subject to the weight matrices in the quality criterion in (13). The constraints on control inputs and system outputs in (7) are formulated using LMIs in (14d), (14e).

Using the feasible solution of SDP in (14), the state-feedback controller of the control law (10) was given by

$$\tilde{\boldsymbol{F}} = \boldsymbol{Y}\boldsymbol{X}^{-1}.$$
(15)

Finally, RMPC with integral action was implemented using Algorithm 1.

Algorithm I Design of RMPC with integral action.
<b>Require:</b> measured/estimated $x(k)$ , cost function weight ma-
trices $Q$ , $R$ and constraints $\mathbb{U}$ , $\mathbb{Y}$ .
<b>Ensure:</b> Control action $u(k)$
1: solve SDP (14)
2: $\tilde{\boldsymbol{F}}(k) \leftarrow \boldsymbol{Y}(k) \boldsymbol{X}(k)^{-1}$
3: $\boldsymbol{u}(k) \leftarrow \tilde{\boldsymbol{F}}(k)\boldsymbol{z}(k)$

## V. DESIGN OF HEBKY FILTER

The controlled variable during experiments was affected by the measurement noise. To overcome this obstacle, we implemented the Hebky filter, see [24]. The Hebky filter is based on the linear regression for multiple measurements during each sampling period. From the control viewpoint, the main benefit was that the filtered signal approximated the measured noisy data without delays, compared to the recursive filters, e.g., Butterworth filter, etc. Consider the measurement of the noise signal  $w \in \mathbb{R}^1$  is given by

$$w(k) = y(k) + q(k),$$
 (16)

where  $y, q \in \mathbb{R}^1$  are real values of the controlled variable and noise, respectively. The task was to estimate the original value of y(k) with zero mean value of measurement noise. The measured variable w was measured *n*-times within each sampling period, i.e., we had measurements  $w_1$  in time  $t_1, w_2$ in time  $t_2$ , etc., up to  $w_n$  in time  $t_n$ .

The filtered signal was evaluated by

$$\tilde{y}(k) = \alpha \, \boldsymbol{h}^{\top} \left[ 1, k \right], \tag{17}$$

where  $\tilde{y} \in \mathbb{R}^1$  is the estimated value of the controlled variable in time k based on the n measurements of the noisy controlled variable  $w_i$ . Parameter  $\alpha$  in (17) is

$$\alpha = \frac{1}{n \sum_{i=0}^{n} t_i^2 \left(\sum_{i=0}^{n} t_i\right)^2},$$
(18)

and  $\boldsymbol{h} \in \mathbb{R}^2$  is given by

$$\boldsymbol{h} = \begin{bmatrix} \sum_{i=0}^{n} t_i^2 \sum_{i=0}^{n} w_i - \sum_{i=0}^{n} t_i \sum_{i=0}^{n} t_i w_i \\ n \sum_{i=0}^{n} t_i \sum_{i=0}^{n} w_i - \sum_{i=0}^{n} t_i \sum_{i=0}^{n} w_i \end{bmatrix}.$$
 (19)

In (18)–(19),  $t_i$  denotes *i*-th time instant, when the noisy controlled variable  $w_i$  was measured. For each estimation of  $\tilde{y}(k)$  it is necessary to evaluate *n* samples of  $w_i$  measured in time  $t_i$ . Finally, based on (17)  $\tilde{y}(k) \approx y(k)$  should hold for sufficiently frequent measurements of  $w_i$ . For  $\tilde{y} \in \mathbb{R}^{n_y}$ , each element of the vector *y* can be approximated using Hebky filter individually.

## VI. RESULTS AND DISCUSSION

RMPC for a laboratory CSTR of Armfiled PC40 was provided by CPU i7 3.4 GHz and 8 GB RAM. RMPC was designed in MATLAB R2012b environment. The communication with the plant was ensured using the Real-Time Windows Target v4.1 toolbox. The Algorithm 1 of RMPC design was evaluated using MUP toolbox [20], the SDP in (14) was formulated using toolbox YALMIP [25], and solved using a solver SeDuMi [26]. The Hebky filter was designed using the RMPC sampling time  $t_{\rm s}=10.0\,{\rm s}$  and the plant-computer communication time  $t_{\rm c}=0.2\,{\rm s}$ , i.e., the linear regression was evaluated based on the 50 measurements of pH in each control step. The closed-loop control performance was evaluated considering  $t_{\rm RMPC}=800\,{\rm s}$ , i.e., the  $n_{\rm k}=80$  control steps.

The considered RMPC setup was as follows: the weighting matrices of quadratic criterion in (13) were set to respect :

$$\tilde{\boldsymbol{Q}} = \begin{bmatrix} 1 & 0\\ 0 & 0.5 \end{bmatrix}, \quad \boldsymbol{R} = [10]. \tag{20}$$

The constraints on the control input and the system output in (7) were set to respect the natural constraints of the plant, i.e., flow rate  $q_{\rm B}$  within interval [0, 10] mL/s, and pH value within interval (0, 14]:

$$u_{\rm sat} = 5.0, \quad y_{\rm sat} = 7.0.$$
 (21)

We performed various experiments on the CSTR to validate the control performance of the designed RMPC with the integral action and results of four of them are presented. The set-point tracking was investigated and the considered stepchanges of the set-point are summarized in Tab. IV as control cases RMPC I – RMPC IV. We implemented the same RMPC design setup in (20), (21) for all control cases RMPC I – RMPC IV to obtain fully comparable results.

The closed-loop control performance of the laboratory CSTR reached in RMPC I - RMPC IV control cases are depicted in Figs. 3-6, respectively. The same RMPC design setup was used for all presented results. The main goal, i.e., the offset-free control performance, was successfully achieved using RMPC with integral action in all control cases. Different dynamics of the control performance was caused by the nonlinear and asymmetric behavior of the laboratory CSTR as well as the properties of the neutralization reaction of the strong base with the weak acid. The control cases RMPC I and RMPC II compare set-point tracking for the upward step changes of the set-point. The control response in RMPC I shows the overshoot and fast dynamics. The controlled output settled approximately in 200 s. The control response does not have overshoot and is slow in the case RMPC II. The controlled output settled approximately in 450 s. The control cases RMPC III and RMPC IV compare set-point tracking for the downward step changes of the set-point. The control response in RMPC III shows the undershoot and fast dynamics. The controlled output settled approximately in 250 s. The control response is smooth, i.e. without undershoot, and slow in the case RMPC IV. The controlled output settled approximately in 400 s. As can be observed in Figs. 3-6, under the same RMPC design setup, controlled output settled faster at the reference value that corresponded to the operating point of pH = 7, see Fig. 3 (RMPC I) and Fig. 5 (RMPC III). On the other hand, the controlled variable settled at the reference slowly, when we considered pH = 6 or pH = 8, cf. Fig. 4 (RMPC II) and Fig. 6 (RMPC IV). These effects relate to the fact, that the operating point pH=7 was chosen in the steepest part of the titration curve for the strong base and weak acid, where the dynamics of the neutralization is the fastest.



Fig. 3. Control performance assured by RMPC I: pH (blue), reference (red).



Fig. 4. Control performance assured by RMPC II: pH (blue), reference (red).

#### VII. CONCLUSION

The paper presents the successful implementation of LMIbased RMPC approach with integral action for the laboratory CSTR of Armfield PCT40. The Hebky filter was designed to eliminate the influence of measurement noise. Four control cases of set-point tracking were considered to investigate the control performance assured by the designed RMPC. The designed integral action ensured the offset-free control performance in all control cases. The next research will be

TABLE IV. RMPC CLOSED-LOOP CONTROL SCENARIOS.

case	reference
RMPC I	$6.0 \rightarrow 7.0$
RMPC II	$7.0 \rightarrow 8.0$
RMPC III	$8.0 \rightarrow 7.0$
RMPC IV	$7.0 \rightarrow 6.0$



Fig. 5. Control performance assured by RMPC III: pH (blue), reference (red).



Fig. 6. Control performance assured by RMPC IV: pH (blue), reference (red).

focused on the implementation of the advanced LMI-based RMPC strategies for the neutralization.

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