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Optimization of Oil Field Production Under Gas Coning Conditions Using the Optimal Closed-Loop Estimator

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Abstract: In an oil field that has reached its maximum gas handling capacity, and where wells are producing under gas coning conditions such that the gas-oil ratio (GOR) depends on the wells production rate, oil production is maximized if the marginal GOR (mGOR) is equal for all wells. The GOR and the mGOR are not readily available measurements, but they can be predicted using detailed reservoir models. In this paper we propose predicting the mGOR for each well from measurements like pressure and valve position by using the static linear *optimal closed-loop estimator*. The estimator can be generated for each well individually, and it is not necessary to consider the production network as a whole. The *optimal closed-loop estimator* is intended for use in combination with feedback. Based on a simple two-well case study, we show that the method is effective and results in close-to-optimal production.

Keywords: Production optimization, static estimators, feedback

1. INTRODUCTION

Optimal production from an oil field, for example maximum oil rates, involves finding an optimal combination of individual well rates. Usually, wells produce oil, gas and water. The gas-oil ratio (GOR) becomes an important factor in the later stages of a field's production if the maximum gas handling capacity of the facility has been reached. In such a case, assuming no other constraints are in effect, it is beneficial to produce from wells with low GOR. The optimal production problem can be formulated as

$$\max \Sigma\, m_{o,i}, \tag{1}$$

subject to the constraint

$$\Sigma\, m_{g,i} = m_g^{max}. \tag{2}$$

That is, we want to maximize the total oil production at the maximum gas handling capacity. In this paper, we have chosen to use mass as the basis for the calculations, and we define the gas-oil ratio (GOR) and marginal GOR (mGOR) as

$$\text{GOR} = (m_g/m_o) \quad \text{and} \quad \text{mGOR} = (\partial m_g/\partial m_o). \tag{3}$$

The GOR can depend on the production rate. This is the case for wells producing under gas coning conditions. For optimal production, assuming that the wells are independent, it is required that

$$\text{mGOR}_{well\, i} = \text{mGOR}_{well\, j}. \tag{4}$$

is satisfied for all wells. A simple proof can be found in (Downs and Skogestad, 2011, p. 107). The optimality criterion in (4) is well known, and Urbanczyk and Wattenbarger (1994) used the mGOR in an iterative procedure to obtain optimal well rates for gas coning wells. However, they did not mention that the criterion in (4) only holds for independent wells.

At present, there exist reservoir simulators capable of predicting the GOR for different well rates. Gunnerud and Foss (2010) used such simulators to generate well performance curves, and presented an efficient real time optimizer (RTO) for solving large scale well allocation problems. However, execution times still range from minutes to tens of minutes depending on the size of the problem. If there is a sudden change (disturbance) in for example GOR or reservoir pressure between each execution of the RTO, the production will be sub-optimal.

In this paper, we propose to use reservoir simulators to make simple linear static estimators. The goal is to predict the mGOR for each well using available process measurements, e.g. pressure and valve position. Next, we propose to use simple feedback controllers, like proportional-integral (PI) controllers, to adjust the well rates such that the predicted mGOR are equal for all wells. Because we use the estimate in a feedback loop, we use the optimal closed-loop estimator presented in Ghadrdan et al. (2013). One of the benefits of this estimator is that it ensures that the gain of the estimator, from measurements to prediction, is not corrupted by measurement noise, which is essential for control purposes.

By using feedback, we can continuously reject disturbances and thus minimize the deviation from optimal operation. This control structure can also be used in combination with RTO. In such a scheme, the RTO finds optimal allocation of well rates based on detailed models and adjusts the setpoints for the feedback layer. Between each

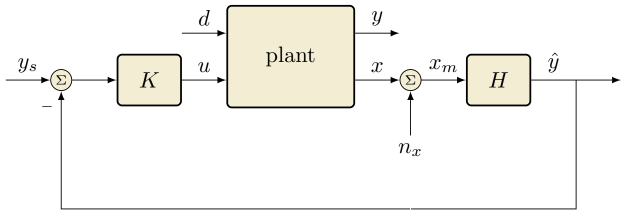

Fig. 1. Control of the predicted control variable $\hat{y}$ by K such that $\hat{y} = y_s$ by adjusting the plant input u. The estimator is used in closed-loop.

execution of the RTO, the feedback layer will attenuate disturbances and keep the production optimal.

The structure of the paper is as follows: The optimal closed-loop estimator will be presented, followed by the basis for and assumptions behind the case study model, and ending with a simple case study demonstrating the method.

2. THE OPTIMAL CLOSED-LOOP ESTIMATOR

In this section we give a brief overview of the closed-loop estimator and the assumptions behind it. For the complete derivation, please refer to Ghadrdan et al. (2013). Consider the system given in Figure 1, and a linear static estimator H on the form,

$$\hat{y} = Hx_m. \tag{5}$$

Here, $\hat{y}$ is the predicted output (controlled variable) and x_m is the measurements with measurement noise. We assume that the measurements x and the outputs y, which we want to estimate, can be expressed as linear static models;

$$x = G_x u + G_x^d d, \tag{6}$$
$$y = G_y u + G_y^d d. \tag{7}$$

The measurements x_m, which are influenced by noise n_x, are expressed as

$$x_m = x + n_x. \tag{8}$$

It is also assumed that $\dim(y) = \dim(u)$.

In closed-loop operation, the manipulated variables u are adjusted to keep the estimated controlled variables $\hat{y}$ at its setpoint y_s,

$$\hat{y} = y_s.$$

In this case, the optimal closed-loop estimator H can be found by solving the optimization problem (Ghadrdan et al., 2013)

$$H = \arg\min_H \|H\,(FW_d\ W_{n_x})\|_F \tag{9}$$
$$s.t. \quad HG_x = G_y.$$

Here, F is the optimal sensitivity,

$$F = (\tfrac{\partial x}{\partial d})_{y=y_s} = \left(G_x^d - G_x G_y^{-1} G_y^d\right), \tag{10}$$

and W_d and W_{n_x} are diagonal scaling matrices, representing the expected disturbance and noise. If these are worst case magnitudes, the estimator will minimize the worst case prediction error (Halvorsen et al., 2003). If they are average normally distributed deviation, the estimator will minimize the expected prediction error (Kariwala et al., 2008).

The gain of some estimators tends to approach zero when the measurement noise approaches infinity. This becomes a problem when using feedback control. Here the manipulated variable u is used to adjust $\hat{y}$. If the gain of the estimator is zero, the required u to reach setpoint approaches infinity. The closed-loop estimator avoids this problem with the constraint $HG_x = G_y$. This ensures that the gain is unaffected even in the presence of large measurement noise.

The optimization problem (2) can be written as

$$\min_H \|H\widetilde{F}\|_F$$
$$\text{s.t.} \quad HG_x = G_y,$$

where $\widetilde{F} = (FW_d\ W_{n_x})$. Under the assumption that $\widetilde{F}\widetilde{F}^T$ is of full rank, the optimal closed-loop estimator H has the following analytical solution (Alstad et al., 2009),

$$H^T = \left(\widetilde{F}\widetilde{F}^T\right)^{-1} G_x \left(G_x^T \left(\widetilde{F}\widetilde{F}^T\right)^{-1} G_x\right)^{-1} G_y^T. \tag{11}$$

3. A SIMPLE MODEL

A simple steady-state pressure drop model has been developed for the well allocation problem (Figure 2). The model consists of three parts: model for reservoir inflow, model for pressure drop in a vertical pipe, and model for flow across a valve. The multiphase fluid of oil, water and gas is treated as a one phase pseudo fluid. These three parts can be combined to create a network of wells, manifolds, and clusters. This is a significantly simplified model, and is only intended as a demonstration.

3.1 Reservoir inflow model

The inflow relations for oil and water are assumed to follow the quadratic deliverability equation proposed by Fetkovich (1973):

$$\dot{m}_o = k_o\left(p_r^2 - p_{wf}^2\right), \tag{12}$$
$$\dot{m}_w = k_w\left(p_r^2 - p_{wf}^2\right). \tag{13}$$

The flow of gas is given by the gas-oil ratio

$$\dot{m}_g = \text{GOR} \times \dot{m}_o. \tag{14}$$

To represent gas coning conditions, we have assumed the GOR to have the following relation to pressure,

$$\text{GOR} = \frac{k_g}{k_o}\left(p_r - p_{wf}\right)^2. \tag{15}$$

This implies a rapid increase in GOR with increasing production.

3.2 One phase pseudo fluid

To simplify our model, we approximate the multiphase fluid (liquid and gas) as a one-phase pseudo fluid. Neglecting mixing volumes, the density of the pseudo fluid is approximated by its volumetric average

$$\rho_{mix} = v_g \rho_g^{ig} + v_o \rho_o + v_w \rho_w. \tag{16}$$

where ρ is the density and v is the volume fraction of the respective phase. In terms of mass flows, the overall density becomes

$$\rho_{mix} = \frac{\dot{m}_o + \dot{m}_g + \dot{m}_w}{\dot{m}_g/\rho_g^{ig} + \dot{m}_o/\rho_o + \dot{m}_w/\rho_w}\,. \tag{17}$$

We assume that oil and water are incompressible, and that the gas behaves as an ideal gas,

$$\rho_g^{ig} = \frac{p\,M_g}{R\,T}, \tag{18}$$

where M_g is the molar weight of the gas, R is the ideal gas constant and T the temperature.

3.3 Pressure drop through a vertical pipe

We estimate the pressure drop for multiphase flow in a vertical pipe using the stationary mechanical energy balance. Assuming no slip between the phases and neglecting friction, work and kinetic energy, the energy balance becomes

$$\mathrm{d}p = \rho_{mix}\,g\,\mathrm{d}h. \tag{19}$$

Integrating (19) between the limits (p_1, h_1) and (p_2, h_2) gives

$$\alpha\,\ln(p_2/p_1) + \beta\,(p_2 - p_1) = \dot{m}_{tot}\,g\,\Delta h, \tag{20}$$

where $\Delta h = h_2 - h_1$,

$$\alpha = \dot{m}_g\,RT/M_g, \quad \text{and} \quad \beta = \dot{m}_o/\rho_o + \dot{m}_w/\rho_w.$$

The equation cannot be solved directly for the pressure p_2. By using a serial expansion of the natural logarithm,

$$\ln\left(p_2/p_1\right) = \ln\left(p_1 + \Delta p/p_1\right) = \ln\left(1 + \Delta p/p_1\right) \approx \Delta p/p_1, \tag{21}$$

the pressure drop over the pipeline can be expressed as a function of the pipe length inlet pressure and height Δh,

$$\Delta p = p_2 - p_1 = \frac{\dot{m}_{tot}\,g\,p_1\,\Delta h}{\alpha + p_1\,\beta}\,. \tag{22}$$

3.4 Pressure drop across a valve

The mass flow across a valve is assumed given by a standard valve equation,

$$\dot{m} = f(z)\,C_d\,A\sqrt{\rho\,(p_2 - p_1)}, \tag{23}$$

where C_d is the valve constant, A is the cross section area, and p_1 and p_2 are the pressures on each side of the valve. $f(z)$ is the valve characteristics, with the valve opening z ranging between 0 when fully closed and 1 when fully open. For simplicity, we assume linear valve characteristics

$$f(z) = z, \quad \text{where} \quad z \in [0,\,1]. \tag{24}$$

We assume a one-phase pseudo fluid and the density in (23) is calculated as the average density for the two sides of the valve;

$$\rho = \tfrac{1}{2}\left(\rho_{mix,1} + \rho_{mix,2}\right). \tag{25}$$

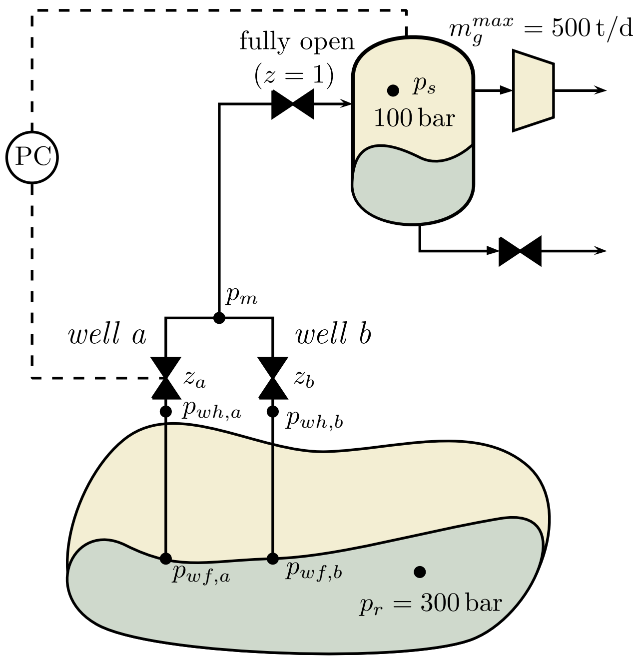

Fig. 2. Sketch of the two-well case study with all relevant nomenclature. Pressure control is shown varying *well a*, but also other options are studied.

4. CASE STUDY

We consider a two-well case (*well a* and *well b*) with a separator as shown in Figure 2. We assume the separator is operated at its maximum pressure (100 bar), which together with a given maximum compressor work, give a maximum gas handling capacity of 500 t/d. The flow characteristics as a function of the well flow pressure p_{wf} for the two wells are shown in Figures 3 and 4. Notice the sharp increase in gas production with higher production (lower flow pressure p_{wf}). For simplicity, only the well valves can be manipulated, and the top valve is fixed to fully open ($z = 1$). For nominal operation, the optimal production is given in Table 1.

For each well the predicted output (controlled variable) is

$$\hat{y} = \mathrm{mGOR} = H x_m,$$

and for each well, the manipulated variable, measurements, and disturbances are as follows,

$$u = z, \quad x = \begin{pmatrix} p_{wf} \\ p_{wh} \\ p_m \\ z \end{pmatrix}, \quad \text{and} \quad d = \begin{pmatrix} k_g \\ p_r \\ p_m \end{pmatrix}. \tag{26}$$

The disturbance k_g represents a shift in the GOR, and p_m represents an upstream disturbance e.g. a change in gas handling capacity.

4.1 The method

To obtain a simple method, we evaluate each well individually. This can be done by assuming that the manifold pressure p_m is a disturbance and independent of the other flow to the manifold. Because we control the separator pressure

Table 1. Optimal well allocation under nominal operation (no disturbances)

		well a	*well b*	*total*
z	—	0.5177	0.8505	—
p_{wf}	bar	214.6	204.5	—
p_{wh}	bar	190.9	173.8	—
p_m	bar	—	—	151.1
m_o	t/d	289.9	265.0	555.0
m_g	t/d	263.7	236.3	500.0
m_w	t/d	145.0	496.3	641.3

p_s, this is not quite true. The manifold pressure p_m will depend on the flow from the other well, and the optimality criterion in (4) does not hold; However, the effect of this interaction is thought to be quite small. The more wells connected to the manifold, the less influence the individual well will have on the manifold pressure. For an infinite number of wells, the manifold pressure will be independent of the individual well flows and the optimality criterion (4) holds. The wells can always be made independent of each other by controlling the manifold pressure instead of the separator pressure.

In addition, inputs and disturbances are assumed to be independent of each other in the derivation of the closed-loop estimator. This is clearly not the case for the input z and the upstream disturbance p_m. Nevertheless, we have

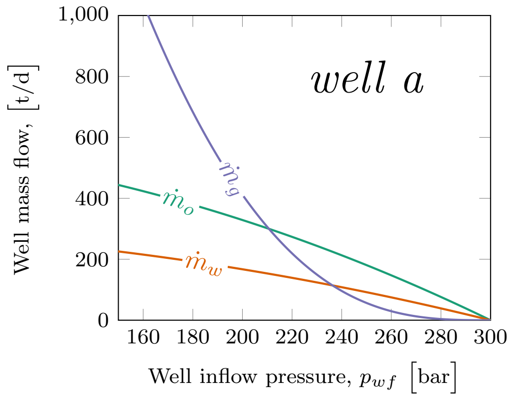

Fig. 3. Well inflow characteristics for *well a*

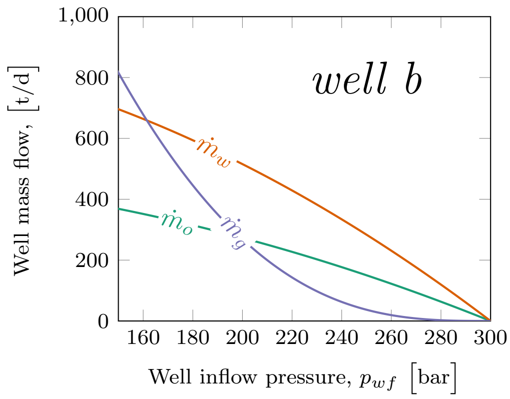

Fig. 4. Well inflow characteristics for *well b*

neglected this interaction in order to evaluate each well individually. We could have used the separator pressure p_s to represent the upstream disturbances. However, when finding the estimator we would have to consider the whole well network.

4.2 The closed-loop estimator

The linear static model for each well was approximated by subjecting the respective well to a 1% positive change in the manipulated variable u, and disturbances d. The mGOR was approximated by linear approximation $^{\Delta m_g}/_{\Delta m_o}$ from the nominal to the new steady state. The expected disturbance is assumed to be ±10%, and the measurement noise is assumed to be ±0.1 bar for pressures and ±0.01 for valve position.

This gives the following optimal closed-loop estimators for *well a* and *well b*,

$$H_{well\,a} = (-1.7900\ 2.0128\ 0.3735\ 35.7994)\,, \tag{27}$$

$$H_{well\,b} = (-1.0521\ 1.1297\ 0.4626\ 24.3815)\,. \tag{28}$$

For this two-wells case, we have two manipulated variables and one operational constraint (maximum separator pressure). One of the manipulated variables must control the pressure, leaving only one free manipulated variable. This gives three different control structures:

Open-loop a
well a valve is fixed (z_a fixed) and *well b* valve controls separator pressure p_s.

Open-loop b
well b valve is fixed (z_b fixed) and *well a* valve controls separator pressure p_s (Figure 2).

Closed-loop mGOR
one well controls the optimality condition, the other controls the pressure p_s.

In general, for closed-loop mGOR with n wells and n valves, one manipulated variable would be used to control pressure. The remaining $n-1$ valves would be used to ensure equal mGOR for all wells.

We compared the different control structures when subjecting the system to sets of disturbances. Sub-optimality of a structure is quantified in terms of loss. We define loss as the difference between oil production with a given control structure J and the optimal oil-production $J^\star$, for a given disturbance. Mathematically this becomes

$$loss(d) = J(d) - J^\star(d). \tag{29}$$

It is clearly seen from the substantially lower oil-loss that the closed-loop estimator performs better than the open-loop strategies for combined disturbances (Table 2).

For individual disturbances, the open-loop policies had a smaller loss if the disturbance is in the other well. In some cases they also performed better than the closed-loop mGOR policy based on (4). This may seem surprising; one would expect the open-loop strategies to always have worse performance as there is no correction for disturbances. At a closer inspection, the worst control strategy is to fix the valve position on wells that are subjected to disturbances; For example, fixing the valve on *well a* when there is a disturbance down hole in *well a*. Naturally, the best

Table 2. Comparison of control strategies with combinations of disturbances affecting the system.

Disturbance		*Optimal*	*Open-loop a*	*Open-loop b*	*Closed-loop mGOR*
well a: 10% inc. k_g *well b*: 10% inc. k_g	z_a	0.4872	0.5177	0.4482	0.4778
	z_b	0.7682	0.7080	0.8505	0.7873
	oil-loss	—	0.3030 t/d	0.4995 t/d	**0.0284 t/d**
well a: 10% inc. k_g *well b*: 1% dec. p_r	z_a	0.4868	0.5177	0.5077	0.5001
	z_b	0.9063	0.8250	0.8505	0.8703
	oil-loss	—	0.3112 t/d	0.1426 t/d	**0.0582 t/d**
well a: 10% inc. k_g 5% dec. m_g^{max}	z_a	0.4576	0.5177	0.4373	0.4615
	z_b	0.8018	0.6703	0.8505	0.7927
	oil-loss	—	1.3579 t/d	0.1554 t/d	**0.0057 t/d**
well a: 10% inc. k_g *well b*: 5% inc. k_g 5% dec. m_g^{max}	z_a	0.4589	0.5177	0.4176	0.4555
	z_b	0.7582	0.6395	0.8505	0.7656
	oil-loss	—	1.3009 t/d	0.6467 t/d	**0.0045 t/d**

Table 3. Comparison of control strategies with different isolated disturbances affecting the system.

Disturbance		*Optimal*	*Open-loop a*	*Open-loop b*	*Closed-loop mGOR*
well a: 10% inc. k_g	z_a	0.4846	0.5177	0.4882	0.4913
	z_b	0.8593	0.7817	0.8505	0.8430
	oil-loss	—	0.3562 t/d	**0.0042 t/d**	0.0146 t/d
well b: 10% inc. k_g	z_a	0.5202	0.5177	0.4744	0.5031
	z_b	0.7604	0.7652	0.8505	0.7931
	oil-loss	—	**0.0018 t/d**	0.6033 t/d	0.0835 t/d
well a: 1% dec. p_r	z_a	0.5342	0.5177	0.5366	0.5318
	z_b	0.8557	0.8916	0.8505	0.8608
	oil-loss	—	0.0686 t/d	0.0015 t/d	**0.0014 t/d**
well b: 1% dec. p_r	z_a	0.5199	0.5177	0.5388	0.5272
	z_b	0.8970	0.9027	0.8505	0.8789
	oil-loss	—	**0.0014 t/d**	0.1001 t/d	0.0148 t/d
10% dec. m_g^{max}	z_a	0.4597	0.5177	0.4110	0.4565
	z_b	0.7422	0.6269	0.8505	0.7491
	oil-loss	—	1.2854 t/d	0.9176 t/d	**0.0040 t/d**

Table 4. Comparison of closed-loop estimators with reduced number of measurements.

Disturbance		$x^T = (p_{wf}\ p_{wh}\ p_m\ z)$	$x^T = (p_{wh}\ p_m\ z)$	$x^T = (p_{wh}\ z)$
well a 10% inc. k_g *well b* 10% inc. k_g	z_a	0.4778	0.4788	0.4786
	z_b	0.7873	0.7853	0.7858
	oil-loss	0.0284 t/d	**0.0227 t/d**	0.0241 t/d
well a 10% inc. k_g *well b* 1% dec. p_r	z_a	0.5001	0.5066	0.5058
	z_b	0.8703	0.8533	0.8554
	oil-loss	**0.0582 t/d**	0.1282 t/d	0.1183 t/d
well a 10% inc. k_g 5% dec. m_g^{max}	z_a	0.4615	0.4678	0.4672
	z_b	0.7927	0.7782	0.7797
	oil-loss	**0.0057 t/d**	0.0391 t/d	0.0344 t/d
well a 10% inc. k_g *well b* 5% inc. k_g 5% dec. m_g^{max}	z_a	0.4555	0.4591	0.4585
	z_b	0.7656	0.7579	0.7591
	oil-loss	0.0045 t/d	**0.0000 t/d**	0.0001 t/d

solution appears when there is control action on the well that is subjected to disturbance. The closed-loop strategy is always close to the optimal solution.

4.3 The effect of number of measurements

We have so far used 4 measurements for each well, see (26). Because p_m is the same for both wells, closed-loop mGOR uses 7 independent measurements.

To investigate the effect of fewer available measurements, closed-loop estimators were made for the two following cases for each well,

$$x_3 = \begin{pmatrix} p_{wh} \\ p_m \\ z \end{pmatrix} \quad \text{and} \quad x_2 = \begin{pmatrix} p_{wh} \\ z \end{pmatrix}. \tag{30}$$

The resulting closed-loop estimators for three available measurements for each well (5 measurements overall) are

$$H_{well\,a,3} = (0.1949 \;\; 0.1709 \;\; 69.7064)\,,$$
$$H_{well\,b,3} = (0.2419 \;\; 0.1230 \;\; 43.0890)\,.$$

In the case of two available measurements for each well (4 measurements overall), the closed-loop estimator are

$$H_{well\,a,2} = (0.2117 \;\; 70.3486)\,,$$
$$H_{well\,b,2} = (0.2719 \;\; 43.7314)\,.$$

The performance of the estimators with a reduced number of measurements is surprisingly good in some cases (Table 4). For the three measurement estimator, one of the disturbance scenarios actually results in zero loss, outperforming the four measurement estimator. This is a coincidence due to the particular numerical values, and shows that one must be careful about drawing conclusions. Nevertheless, the good result indicates that the method is still applicable when few measurements are available. However, if the measurements contain noise, the estimators using more measurements would most likely have better performance.

5. CONCLUSION

In this paper, we have used the optimal closed-loop estimator to predict the mGOR for different wells using pressure and valve-opening measurements. By adjusting the respective well rates, these mGOR estimates have been controlled such that they are equal for all wells. We have shown that this method gives close-to-optimal production despite disturbances in GOR, reservoir pressure and gas handling capacity. One important advantage of this method is that the estimators can be generated for each well independently of the other wells in the field. The method is also well suited for use with simple feedback controllers, like a PI controller, for efficiently rejecting disturbances.

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Appendix A. CASE STUDY MODEL PARAMETERS

Table A.1. Parameters for the case study model

		well a	*well b*
p_r	bar	300	300
k_o	t/bar^2	6.576×10^{-3}	5.462×10^{-3}
k_g	t/bar^4	8.239×10^{-7}	5.373×10^{-7}
k_w	t/bar^2	3.344×10^{-3}	1.031×10^{-2}
h	m	1000	1000
T	K	373	373
M_g	kg/kmol	16.04	16.04
ρ_o	kg/m^3	800	800
ρ_w	kg/m^3	1000	1000
R	J/(kmol K)	8314	8314
C_d	(kg/m bar d^2)$^{0.5}$	84600	84600
A	m^2	5.66×10^{-4}	5.66×10^{-4}
		riser model	
h	m	1500	
C_d	(kg/m bar d^2)$^{0.5}$	84600	
A	m^2	0.0011	