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Analysis and Control of axial vibrations in tunnel drilling system ⋆

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Abstract: Resonant Sonic Head drilling (RSHD) induces axial vibrations along the drillstring system which affect the accuracy and quality of pre-reinforcement required to tunnels excavation in complex soils. The main goal of this paper is to control these vibrations in order to improve this process. First, an infinite dimensional equations as vibration model are elaborated with Neumann boundary conditions. After that, a reference axial vibration as target model is constructed with respect to the system dynamic and the RSHD model. An energy-based controller is proposed to ensure the convergence of both top and bottom axial displacements toward the target. Simulations are presented to highlight the control laws results.

Keywords: Distributed parameter model, tunnel drilling, axial vibration, Energy-based control.

1. INTRODUCTION

Fig. 1. The conventional Tunneling Method (CTM)

Underground tunneling is one of the most challenging tasks in the construction industry. The most widely used methods are the Tunnel Boring Machine (TBM) which is used to excavate tunnels with a circular cross section through a variety of soil and rock, the Conventional Tunneling Method (CTM), a cyclic construction process of repeated steps of excavation, mucking and placement of the primary support elements. The (CTM) remains the best one for projects with highly variable ground conditions or shapes and the only alternative for the renovation of existing tunnels and creating emergency exit. However, the use of this method in complex soils requires pre-reinforcement: tunnel face bolting and pipe umbrella.
During the drilling process, a wide variety of non-desired vibrations may arise. The main kinds of drilling vibrations are : Axial (bit-bouncing phenomenon), torsional (stick-slip oscillations) and lateral (whirling phenomenon). These vibrations have a major effect on drilling performance. They can affect the rate of penetration of the drill pipes as well as the drilling direction. Furthermore, they cause a damage to the drilling tools and reduce the drilling efficiency.
Therefore vibrations must be understood and their effects should be controlled by any approach. The modeling of drillstring dynamics is crucial for system analysis and control of detrimental vibrations. During the last half century, many research efforts have been handled to mathematically describe the physical phenomena that occur during the drilling process. Researchers started by lumped parameter models where the drillstring is regarded as a mass-spring-damper system which can be defined by an ordinary differential equation (ODE), this modeling strategy was adopted by Halsey et al. (1988), Sananikone (1993), Pavone et al. (1994), Jansen et al. (1995) and Serrarens et al. (1998). This finite-dimensional system representation did not respect the distributed nature of the drilling structure. Consequently, a distributed parameter models appeared and they provided a characterization of the drilling variables in infinite-dimension which added more accuracy to the model in reproducing the rod oscillatory behavior. This kind of models was used by Tucker et al. (1999a) Tucker et al. (1999b), Challamel (2000), Fridman et al. (2010), Saldivar et al. (2011) and Sagert et al. (2013). The drawback of this second type of modeling was the complexity involved in its analysis and simulations. Then, arose the Neutral-type time-delay models which were directly derived from the distributed parameter ones. The transformation of the partial differential equations (PDE) model to the time-delay system was first introduced by Cooke et al. (1968). Then this kind of modeling has been used by Mounier et al. (1995), Balanov et al. (2003), Blakely et al. (2004), Barton et al. (2007) and Saldivar et al. (2014).
Several approaches have been used to treat the vibrations control problem. Most of them focused on the suppression of stick-slip oscillations. Some of classical control strategies are : the introduction of steering torque feedback system to adjust the velocity in the top of the drillstring to torque

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variations by Halsey et al. (1988), this method was improved by the introduction of a Soft Torque Rotary System (STRS) in Sananikone (1993), a PID controller at the surface by Pavone et al. (1994), H_∞ controller by Serrarens et al. (1998), Active Vibration Damper (AVD) by Cobern et al. (2005) and Drilling OScillation KILLer (D-OSKILL) by Canudas-de-wit et al. (2005). In spite of these numerous methods to control drilling vibrations, the drilling process remains imperfect due to the non respect of the distributed nature of the system. Accordingly, other methodologies based on distributed parameter models have been proposed. In Sagert et al. (2013), Saldivar et al. (2014) the flatness and backstepping approaches were exploited to design a stabilizing controllers tackling the trajectory tracking problem. Lyapunov techniques Alli et al. (2000), Saldivar et al. (2011), Saldivar et al. (2013) was used to achieve the asymptotic, exponential and practical stability. Using sonic drilling in underground tunneling induced both axial and lateral vibrations. As we use short holes, we can neglect the lateral ones. Therefore in this paper, we focus on controlling axial vibrations. Our contribution is to design control laws based on Energy techniques to track a prescribed trajectory to improve the drilling direction as well as the performance of the drilling.
The paper is structured as follows. The second section is concerned by the problem statement, we present a distributed parameter model of the drillstring axial vibrations and the Resonant Sonic Head Drill model. In section 3 and 4, we tackle the vibration control problem. We start by, defining a reference axial vibration model taking into account a Resonant Sonic drilling.Then, we design controllers using the Energy's approach to ensure the tracking of the top and bottom axial displacements of drill pipes for the references computed from the target system. Simulations are proposed in section 5. Conclusions are presented in the last section.

2. PROBLEM STATEMENT

2.1 Distributed parameter model

The axial vibrations of a drillstring of length L, described by the axial displacement $u(x,t)$ can be modeled by the following hyperbolic partial differential equation (Saldivar et al. (2014))

$$AEu_{xx}(x,t) - \rho A u_{tt}(x,t) - \alpha u_t(x,t) = 0 \tag{1}$$

with the boundary conditions

$$\begin{array}{ll} AEu_x(0,t) = \alpha u_t(0,t) - H(t) & ,x=0 \\ AEu_x(L,t) + Mu_{tt}(L,t) = -F(u_t(L,t)) & ,x=L \end{array} \tag{2}$$

where E is the elasticity Young's modulus, A is the averaged section of the drill pipe, $\alpha u_t(0,t)$ represents a friction force of viscous type, H is the force applied by the drilling head on the top of the drillstring ($x = 0$) and considered as the control input. The other extremity ($x = L$), is subject to the friction force $F(u_t(L,t))$, which approximate the physical phenomenon at the bottom of the drill and is due to the interaction between the ground and the drill bit.
This friction force introduced in Navarro et al. (2007) is modeled by the following nonlinear expression

$$F(u_t(L,t)) = c_b u_t(L,t) + c_a\ \mu_{outil}(u_t(L,t)) sgn(u_t(L,t)) \tag{3}$$

the term $c_b u_t(L,t)$ is a viscous damping force at the bit and the term $c_a\ \mu_{outil}(u_t(L,t))sgn(u_t(L,t))$ is a dry friction force modeling the bit-rock contact

$$\mu_{outil}(u_t(L,t)) = \mu_{cb} + (\mu_{sb} - \mu_{cb})e^{-\gamma_b |u_t(L,t)|} \tag{4}$$

where $\mu_{sb}, \mu_{cb} \in (0,1)$ are the static and Coulomb friction coefficients and $0 < \gamma_b < 1$ is a constant defining the velocity decrease rate.
In Knuppel et al. (2013), the friction force is modeled as :

$$F(u_t(L,t)) = \frac{2\ \delta\ u_t(L,t)}{(u_t(L,t))^2 + \delta^2}, \quad \delta > 0 \tag{5}$$

the function above, enable to avoid the complexity of most of the proposed model of friction dynamics.

2.2 Resonant Sonic Head Drill model

The Resonant Sonic Head Drill (RSHD) (Fig. 2) is one of the most prominent drilling tools. It uses a percussion drilling to achieve higher efficiencies. It develops a sinusoidal force along the drillstring which is defined by:

$$G(t) = 2\ m_e\ e\ \omega^2 \sin(\omega t)$$

where m_e, e and ω are the mass of each unbalanced mass, the mass eccentricity with respect to its center of rotation and the angular velocity of the masses, respectively. The drill head is subject to a rubber isolator (spring/damper system) which generate a top axial displacement as a stationary solution

$$u(0,t) = A(\omega) sin(\omega t - \Phi(\omega)$$

with

$$A(\omega) = \frac{2\ m_e\ e\ \omega^2}{\sqrt{(k_s - M\ \omega^2)^2 + (c\ \omega)^2}}$$
$$\tan\Phi = \frac{c\ \omega}{k_s - M\omega^2}$$

and

- k_s : Spring constant for the rubber isolator
- c : Viscous damping coefficient for the rubber isolator

In order to develop a smart and efficient drilling systems, the RSHD produces and induces axial vibrations that we want to control. In our drilling system, an experiment is shown by figures 3 and 4 (Viroflay drilling site by Soletanche Bachy company), where the drillstring vibration frequency is between $90Hz$ and $120Hz$. These values depend on the drillstring length, and taken as resonant frequencies. It permits to the machine to generate the maximum amplitude of axial displacement. Moreover, we show in Fig. 4 the mud pressure and the bit velocity for 15 meters drilling depth. Figures 3 and 4 illustrate a random and manual work carried out on site, note that there is no vibration pressure and nor translational bit velocity regulations. The control problem of the RSHD displacement and the control of the mud pressure aren't considered in this paper. At present, let us define the vibration control problem.

3. AXIAL VIBRATION AND TARGET SYSTEM

To tackle the vibration control problem, we first define a reference axial vibration model that we will consider as

Fig. 2. RSHD tool in site

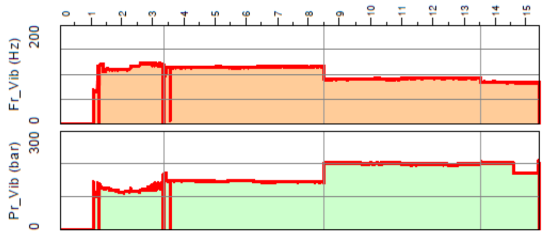

Fig. 3. Vibration pressure and frequency for 15m drilling depth (Viroflay site)

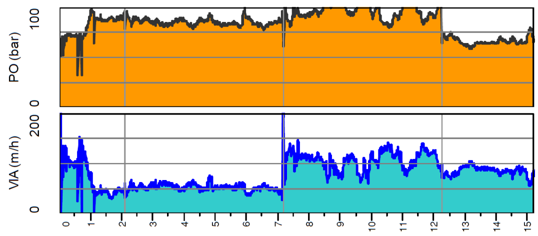

Fig. 4. Mud pressure and bit velocity for 15m drilling depth (Viroflay site)

a target system which allows us to improve the drilling process while taking into account the appropriate values of vibration frequencies. After normalization of the axial vibration model (1)-(2), we define the reference as following

$$u^r_{tt}(\sigma,t) = au^r_{\sigma\sigma}(\sigma,t) - bu^r_t(\sigma,t) \quad (6)$$
$$t > 0, 0 < \sigma < 1$$

with the boundary conditions

$$\begin{aligned} u^r_\sigma(0,t) &= \quad gu^r_t(0,t) - kH^r(t) \\ u^r_\sigma(1,t) &= \quad -hu^r_{tt}(1,t) - pF(u^r_t(1,t)) \quad (7) \\ H^r(t) &= 2\ m_e\ e\ \omega^2 \sin(\omega t) - k_s\ A(\omega) sin(\omega t - \Phi(\omega)) \\ &\quad -c\ \omega A(\omega) cos(\omega t - \Phi(\omega)) \end{aligned}$$

where $\sigma = \frac{x}{L}$, $a = \frac{E}{\rho L^2}$ and $b = \frac{\alpha}{\rho A}, g = \frac{\alpha L}{AE}$, $k = \frac{L}{AE}$, $h = \frac{ML}{AE}$ and $p = \frac{L}{AE}$.

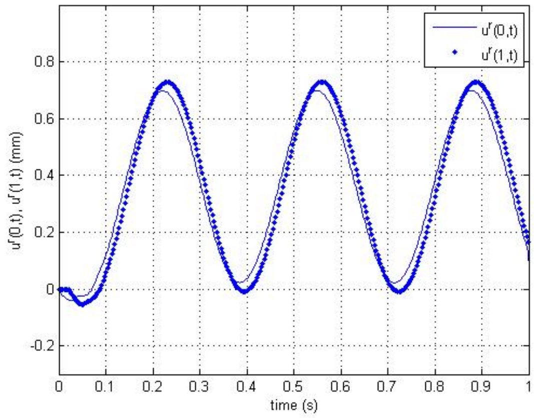

Fig. 5. Top and bottom reference axial displacements

A numerical resolution of the target system (6)-(7) using the numerical values of the physical parameters given in table 1, permits to get $u^r(0,t)$ and $u^r(1,t)$ as shown on Fig. 5.

4. TRACKING OF THE TARGET SYSTEM

4.1 Energy-based controller

In this section, we derive two Energy-based controllers. The control laws goal is to track a desired displacement established at the top and the bottom of the drillstring. Substituting (3) into (1)-(2), the normalized drilling system is:

$$u_{tt}(\sigma,t) = au_{\sigma\sigma}(\sigma,t) - bu_t(\sigma,t) \quad (8)$$
$$t > 0, 0 < \sigma < 1$$

where $\sigma = \frac{x}{L}$, $a = \frac{E}{\rho L^2}$ and $b = \frac{\alpha}{\rho A}$
with the boundary conditions

$$\begin{aligned} u_\sigma(0,t) &= gu_t(0,t) - kH(t) \\ u_\sigma(1,t) &= -hu_{tt}(1,t) - mu_t(1,t) \quad (9) \\ &\quad -q\mu_{outil}(u_t(1,t))sgn(u_t(1,t)) \end{aligned}$$

where $g = \frac{\alpha L}{AE}$, $k = \frac{L}{AE}$, $h = \frac{ML}{AE}$, $m = \frac{c_b L}{AE}$ and $q = \frac{c_a L}{AE}$.
We propose two control laws that enable the tracking of a prescribed reference displacement to ensure much drilling performance.

Proposition 1. The following controllers

$$\begin{aligned} H_d(t) = &\frac{g}{k}(1-c_1)u_t(0,t) + 2\frac{g}{k}c_1 u^r_t(1,t) \\ &- \frac{g}{k}c_1 \frac{(u^r_t(1,t))^2}{u_t(0,t)} \quad (10) \\ &- \frac{1}{k}[\frac{(u(1,t)-u^r(1,t))}{u_t(0,t)}(u_t(1,t)-u^r_t(1,t))] \end{aligned}$$

$$\begin{aligned} H_h(t) = &\frac{g}{k}(1-c_2)u_t(0,t) + 2\frac{g}{k}c_2 u^r_t(1,t) \\ &- \frac{g}{k}c_2 \frac{(u^r_t(1,t))^2}{u_t(0,t)} \quad (11) \\ &- \frac{1}{k}[\frac{(u(0,t)-u^r(0,t))}{u_t(0,t)}(u_t(0,t)-u^r_t(0,t))] \end{aligned}$$

where $H_d(t)$ and $H_h(t)$ are the down hole and head input control laws respectively, ensure the stability of the errors $e_d = u(1,t) - u^r(1,t)$ and $e_h = u(0,t) - u^r(0,t)$ between the bottom and top axial displacements respectively and the reference trajectories.

Proof. We consider the energy function

$$E_d(t) = \int_0^1 au_\sigma^2(\sigma,t)d\sigma + \int_0^1 u_t^2(\sigma,t)d\sigma + ahu_t^2(1,t) + a(u(1,t) - u^r(1,t))^2$$

Differentiating E(t) yields

$$\begin{aligned}\frac{dE_d(t)}{dt} &= 2\int_0^1 au_\sigma(\sigma,t)u_{\sigma t}(\sigma,t)d\sigma \\ &+2\int_0^1 u_t(\sigma,t)u_{tt}(\sigma,t)d\sigma \\ &+2ahu_t(1,t)u_{tt}(1,t) \\ &+2a(u(1,t) - u^r(1,t))(u_t(1,t) - u_t^r(1,t))\end{aligned}$$

Integrating by parts and plugging (9) gives

$$\begin{aligned}\int_0^1 u_t(\sigma,t)u_{\sigma\sigma}(\sigma,t)d\sigma &= [u_t(\sigma,t)u_\sigma(\sigma,t)]_0^1 \\ &- \int_0^1 u_{\sigma t}(\sigma,t)u_\sigma(\sigma,t)d\sigma \\ &= u_t(1,t)[-hu_{tt}(1,t) - mu_t(1,t) \\ &-q\ \mu_{outil}(u_t(1,t))sgn(u_t(1,t))] \\ &-u_t(0,t)[gu_t(0,t) - kH(t)] \\ &- \int_0^1 u_{\sigma t}(\sigma,t)u_\sigma(\sigma,t)d\sigma\end{aligned}$$

Hence,

$$\begin{aligned}\frac{dE_d(t)}{dt} &= 2a\int_0^1 u_\sigma(\sigma,t)u_{\sigma t}(\sigma,t) \\ &+2au_t(1,t)[-hu_{tt}(1,t) - mu_t(1,t) \\ &-q\ \mu_{outil}(u_t(1,t))sgn(u_t(1,t))] \\ &-2au_t(0,t)[gu_t(0,t) - kH(t)] \\ &-2a\int_0^1 u_\sigma(\sigma,t)u_{\sigma t}(\sigma,t)d\sigma \\ &-2b\int_0^1 u_t(\sigma,t)u_t(\sigma,t)d\sigma + 2ahu_t(1,t)u_{tt}(1,t) \\ &+2a(u(1,t) - u^r(1,t))(u_t(1,t) - u_t^r(1,t)) \\ &= -2ahu_t(1,t)u_{tt}(1,t) - 2amu_t(1,t)u_t(1,t) \\ &-2aq\ \mu_{outil}(u_t(1,t))u_t(1,t)sgn(u_t(1,t)) \\ &-2agu_t(0,t)u_t(0,t) + 2aku_t(0,t)H(t) \\ &-2b\int_0^1 u_t(\sigma,t)u_t(\sigma,t)d\sigma + 2ahu_t(1,t)u_{tt}(1,t) \\ &-2au_t(0,t)[gu_t(0,t) - kH(t)] \\ &+2a(u(1,t) - u^r(1,t))(u_t(1,t) - u_t^r(1,t))\end{aligned}$$

Since

$$\mu_{outil}(u_t(1,t))u_t(1,t)sgn(u_t(1,t)) = \mu_{outil}(u_t(1,t))|u_t(1,t)|$$

where | . | is the absolute value.
We have,

$$\begin{aligned}\frac{dE_d(t)}{dt} &= -2amu_t^2(1,t) - 2aq\ \mu_{outil}(u_t(1,t))|u_t(1,t)| \\ &- 2b\int_0^1 u_t^2(\sigma,t)d\sigma - 2agu_t(0,t)[u_t(0,t) - \frac{k}{g}H(t)] \\ &+ 2a(u(1,t) - u^r(1,t))(u_t(1,t) - u_t^r(1,t))\end{aligned} \quad (12)$$

To ensure the dissipativity of the system (8)-(9), the control law $H_d(t)$ should enable the negativity of (12).
Hence, the control law is as in (10)
where c_1 is a free design parameter satisfying $c_1 > 0$, $u_t^r(1,t)$ is the desired bottom velocity and $u^r(1,t)$ is the reference trajectory of the drill bit.

We obtain that

$$\begin{aligned}\frac{dE_d(t)}{dt} &= -2amu_t^2(1,t) - 2aq\ \mu_{outil}(u_t(1,t))|u_t(1,t)| \\ &-2b\int_0^1 u_t^2(\sigma,t)d\sigma - 2agc_1(u_t(0,t) - u_t^r(1,t))^2\end{aligned}$$

Taking into acount that, $\mu_{outil}(u_t(1,t)) > 0$, $u_t(0,t) \neq 0$ and a, g, q, m, k, h et $b > 0$ we find that $\frac{dE(t)}{dt} \leq 0$. Therefore, the proposed control law (10) ensures a convergence of the bottom axial displacement to the reference trajectory.
Similarly, for the control law $H_h(t)$, we consider the energy function

$$E_h(t) = \int_0^1 au_\sigma^2(\sigma,t)d\sigma + \int_0^1 u_t^2(\sigma,t)d\sigma + ahu_t^2(1,t) + a(u(0,t) - u^r(0,t))^2 \quad (13)$$

Then, we choose the control law (11), where c_2 is a free design parameter satisfying $c_2 > 0$, $u_t^r(0,t)$ is the desired top velocity and $u^r(0,t)$ is the topside drillstring reference trajectory.
Hence, the controller (11) enables us to establish a convergence of the top axial displacement to the reference trajectory.

5. SIMULATION RESULTS

5.1 Simulations

In this section, the efficiency of the Energy-based controller is highlighted through simulation results of the axial model (8)-(9) and the friction model (5). As top and bottom references, we use the trajectories computed numerically from the target system (6)-(7). In table 1, we give the numerical values of the physical parameters.

Fig. 6 shows the closed loop response of the system subject to the proposed Energy-based control approach for a vibration frequency equal to 120 Hz considered as

Table 1. Physical parameters

E	210 GPa	A	0.0146 m^2
ρ	7850 kg/m^3	L	3 m
M	150 kg	m_e	1.5 kg
α	0,02 $kg.s^{-1}$	ω	19.1 $rd.s^{-1}$
δ	0.03	c_1/c_2	4 / 0.4 10^6
k_s	$3.419 * 10^4$	c	$1.96 * 10^3$

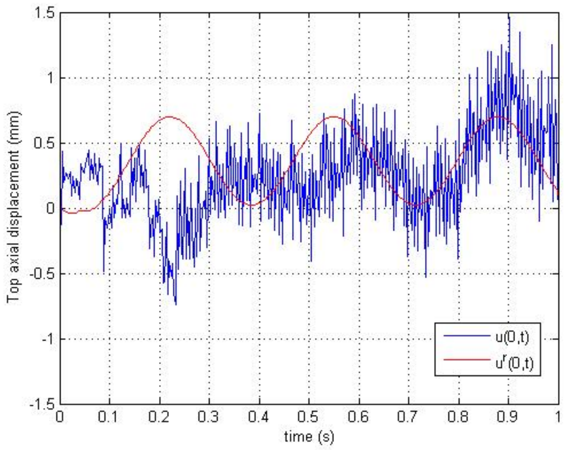

Fig. 6. The reference and real trajectories in the topside of the drillstring under H_d

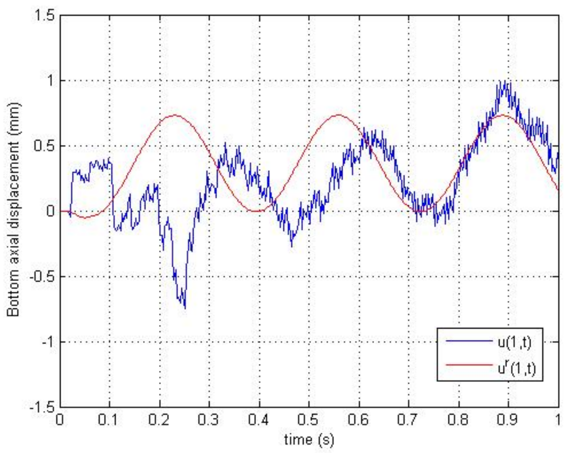

Fig. 7. The reference and real trajectories of the drill bit under H_d

the resonant frequency. The top axial displacement $u(0,t)$ reaches the reference trajectory $u^r(0,t)$.
In Fig. 7, The bottom axial displacement $u(1,t)$ attains the reference trajectory $u^r(1,t)$ under the Energy-based controller H_d for the resonant vibration frequency.
The behavior of the displacement in the top extremity of the drillstring under H_h and for a vibration frequency equal to 120 Hz is shown on Fig. 8.
Fig. 9 pictures a stabilization of the error between the bottom real and reference trajectories of the drillstring using the proposed Energy-based controller H_h.

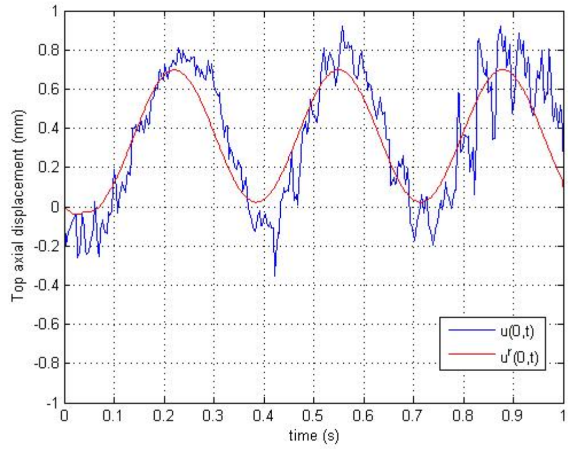

Fig. 8. Both top reference and real trajectories under the Energy-based controller H_h

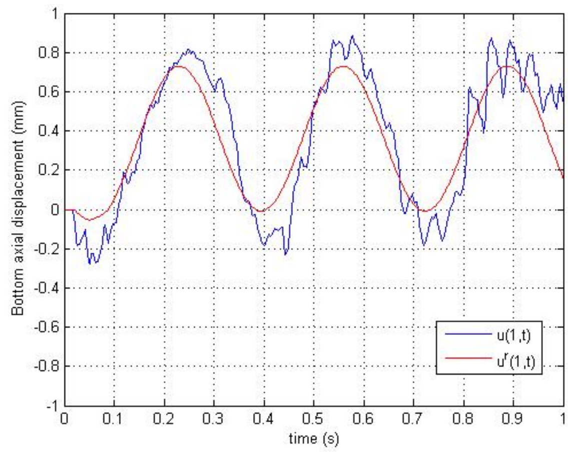

Fig. 9. Both bottom reference and real trajectories under the Energy-based controller H_h

5.2 Discussion

The controllers H_h and H_d ensure the stabilization of the downhole error $e_d = u(1,t) - u^r(1,t)$ and the head error $e_h = u(0,t) - u^r(0,t)$ between the real and the reference trajectories. It should be remarked that, the control law H_h allows to have faster convergence to the reference than H_d. As the input control H_d uses bottom's displacement and velocity measurements, which are not easy to get during drilling in tunnel construction field unlike in oil well one. For real implementation, in practice, it is interesting to estimate the system states at the bottom hole through boundary observer.

6. CONCLUSION

A distributed parameter model describing drillstring axial vibrations as well as a RSHD model have been presented. Taking into account the vibratory percussion conditions, an infinite dimensional target system was defined. The stabilizing control-inputs of the tunnel drilling system were stated at the boundary conditions, and an energy-based control scheme was elaborated. Consequently, a stabilizing error variable is constructed around the drillstring head and the bit behaviours. It was proved that the system of errors is stable. However, the downhole system's variables aren't easy to measure, despite the adequate simulation results in controlling the bit, in practice, these measurements can be difficult to obtain or expensive. Thereby, a boundary observer should be designed for experimental tests.

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